HW 3 Solution

Problem 1

c_s=100 $/pound. (over time production cost)

c_o=200 $/pound. (regular time cost)

$$G(Q^*) = \frac{100}{100+200} = 0.33$$

From normal distribution table, we get $Z_{0.33} = -0.44$.

$Q^* = 100{,}000 - 0.44 * 5000 = 97{,}800$ pounds per month.

Therefore, the optimal selection of the plant capacity equals 97,800 pounds per month.

Problem 2

(a) Casting the considered problem to the newsvendor model, in this case we have:

$c_o = \$40 \times \frac{0.35/yr}{52weeks/yr} \approx \$0.27/week,$ i.e., the holding cost per week.

$c_s = \$65 - \$40 = \$25$, i.e., the lost profit.

Since the demand follows normal distribution, and sales are lost when Tammi runs out of stock, we can find the amount of order as follows.

$\phi\left(\frac{S-35}{10}\right) = \frac{25}{25+0.27} = \frac{25}{25.27} = 0.989.$

Thus we have $\frac{S-35}{10} = 2.29 \Rightarrow S = 35 + 22.9 \approx 58$

(b) Let f be the fill rate, i.e., the proportion of demand that is met from stock. In class we showed that f is given by:

$$f = \frac{E[X^S]}{E[X]} \text{ where}$$

E[*X*] is the expected weekly demand and $E[X^S]$ is expected demand met during a weekly interval. Hence,

$$E[X^S] = \int min\{x, S\}\, g(x)dx$$

$$= \int_{-\infty}^{S} xg(x)dx + \int_{S}^{\infty} Sg(x)dx$$

$$= \int_{-\infty}^{\infty} xg(x)dx - \int_{S}^{\infty} (x - S)g(x)dx$$

$$= E[X] \ - B(S)$$

where $B(S)$ is the *"loss"* function for the considered normal distribution. Hence,

$$E[X^S] = E[X] - \sqrt{var[X]}\, L\left(\frac{S - E[X]}{\sqrt{var[X]}}\right)$$

and

$$f = 1 - \frac{\sqrt{var[X]}\, L\left(\frac{S - E[X]}{\sqrt{var[X]}}\right)}{E[X]}$$

$$= 1 - \frac{10L\left(\frac{58 - 35}{10}\right)}{35}$$

$$= 1 - \frac{10L(2.3)}{35} = 1 - 10 \times \frac{0.0037}{35} = 0.9989$$

(c) Since unmet demand is backordered, now c_s = b = \$12, and we have:

$\phi\left(\frac{S-35}{10}\right) = \frac{12}{12+0.27} = \frac{12}{12.27} = 0.978.$

Thus we have $\frac{S-35}{10} = 2.015 \Rightarrow S = 35 + 10 * 2.015 = 55.15 \approx 55.$

Problem 3

The chassis stage has 20 parallel stations with a constant processing time of 15 minutes

The amplifier stage has 15 parallel stations, with their processing time following $Exp(\frac{1}{20})$ (since the expected value is 20, the parameter of exponential distribution should be 1/20).

According to the relationship between the Poisson distribution and the exponential distribution, one can calculate the demand rate for chassis as follows:

Demand for chassis: Poisson with rate $\lambda = 15 \times \frac{1}{20} = \frac{3}{4} min^{-1}$

Lead time l = 15 mins (the time for operator to build a chassis)

Demand per a lead time interval is Poisson with parameter $\lambda l = 0.75 \times 15 = 11.25$

The considered system of maintaining the inventory of completed chassis is essentially a Basestock system with its basestock level defined by the number of cards, m. So, we want

$G(m-1) \geq 0.99 \Rightarrow m-1 = 20 \Rightarrow m = 21$

Problem 4

Let L be the lead time. Then,

$E[L] = 4$ months

$st.dev[L] = 1.5$ months.

Also the monthly demand (r.v. X_m) follows the normal distribution $N(15, 6^2)$, i.e.,

$X_m \sim N(15, 6^2)$.

Replenishment order Q=100.

Since the fill rate is 90 percent, using the Type-2 approximation for the fill rate S(Q,r), we get that expected backorder level, $B(r)$, is

$B(r) = (1-a)Q = 0.1 \times 100 = 10$

Also, the lead time demand X_l follows normal distribution; one can calculate the parameters of the distribution for this demand using the formulae for computing the expectation and the variance of compound random variables presented in class:

$X_l \sim N(4 \times 15, 4 \times 6^2 + 15^2 \times 1.5^2) = N(60, 25.5^2)$. Hence, we have:

$$B(r) = 25.5L\left(\frac{r-60}{25.5}\right) = 10$$

$$\Rightarrow L\left(\frac{r-60}{25.5}\right) = 0.3921$$

$$\Rightarrow \frac{r-60}{25.5} \approx 0.015$$

$$\Rightarrow r = 60 + 0.015 \times 25.5 = 60.38 \approx 60$$

Problem 5

Weekly demand $\sim U[500.600]$

A=$250 (order cost)

C=$ 2.75 (item cost)

I=8% (interest rate to be used in the estimation of the holding cost)

Expected annual demand = $52 \times 550(expected\ value\ of\ uniform\ dis.) = 28{,}600$

By EOQ formula, one can get Q:

$$Q = \sqrt{\frac{2AD}{h}} = \sqrt{\frac{2 \times 250 \times 28600}{0.08 \times 2.75}} = 8062$$

Now, to ensure a fill rate of 97%,

$S(Q,r) \approx 1 - \frac{B(r)}{Q} = a \Rightarrow (1-a)Q = B(r) \Rightarrow B(r) = 0.03 \times 8062 = 241.86 \quad Eq.(1)$

Next we compute *B*(*r*) for the considered problem. This function has three different branches. The first branch spans the interval [0,500] i.e., it concerns *r* values that are below the minimum value of demand. The second branch spans the interval [500,600], i.e., exactly the region over which the demand varies. The last branch concerns r values greater than 600, i.e., greater than the maximum value of the demand. Combining three cases, one can get the following definition of *B*(*r*).

$$B(r) = \int_r^{\infty} (x-r)g(x)dx, \text{ for } r \geq 0$$

$$= \begin{cases} \int_{500}^{600} (x-r)\frac{1}{100}dx \ , for\ 0 \leq r \leq 500 \\ \int_{r}^{600} (x-r)\frac{1}{100}dx \ , for\ 500 \leq r \leq 600 \\ 0, for\ r \geq 600 \end{cases}$$

$$= \begin{cases} 550 - r, for\ r \leq 500 \\ \frac{1}{200}(r-600)^2, for\ 500 \leq r \leq 600 \\ 0, for\ r \geq 600 \end{cases}$$

Notice that over the first branch, *B*(*r*) drops from the value of 550 to the value of 50, while over the second branch I further drops from 50 to 0. Hence, to get the desired value of 241.86 (c.f., Eq. (1) above) we need to work with the first branch of this function. Then,

$$B(r) = 241.86 = 550 - r$$

$$\Rightarrow r = 550 - 241.86 = 308.14 \approx 309$$