ISYE 3104: Manufacturing Systems
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Final Exam
April 30, 2013

Name: SOLUTIONS

Answer the following questions (**8 points each**):

1. What are the primary *two* reasons for taking a data-aggregating approach at the aggregate planning phase?

1) It helps controlling the complexity of the planning process that would have resulted by considering each single SKU as a distinct entity.

2) By considering demand forecasts that concern the cumulate demand across product families, we control better the error in those forecasts (since we might be over-estimating the demand of some SKUs in those families while under-estimating the others...)

2. A company performing its aggregate planning with respect to a particular product family has decided that it may use subcontracting but the subcontracted quantity at any period should not exceed 20% of its internal production during that period. Furthermore, the company has decided to allow for no backlogs. Write a set of constraints for the LP formulation of the corresponding aggregate planning problem that will express the above conditions in this formulation.

In this case, the material balance equation becomes:

$$\forall t, \quad I_{t-1} + P_t + S_t = D_t + I_t$$

which is equivalent to

$$\forall t, \quad I_t = I_{t-1} + P_t + S_t - D_t$$

In the above equation:

1) I_t = Inventory carried from period t to period $t+1$
2) P_t = Internal production in period t
3) S_t = Subcontracted quantity for period t
4) D_t = Demand for period t.

We have omitted the variables relating to backlogs, since we do not allow for backlogs in this case.

Of course, $I_t, P_t, S_t \geq 0, \ \forall t$

Finally, we also need the constraint

$$\forall t, \quad S_t \leq 0.2 P_t$$

that enforces the ceiling that we impose on subcontracting.

3. The forecasted demand (in aggregate units) during the next six months is as follows:

$$250, 300, 360, 350, 330, 300$$

Assuming that the current availability is 100 (aggregate) units, and that the production capacity of a single worker is 40 units per month, what is the *minimum* size of a *constant* workforce over the considered planning horizon that will support the expected demand without experiencing any backlogs?

i. 7

ii. 8

iii. 9

iv. 10

Explain your answer.

A calculation similar to that in the solution of Problem 14 in HW 5 will reveal that we need 8 workers.

Some of you answered (iii - 9) but this answer fails to account for the anticipatory inventories that can be built by the employed workforce in periods that present slack capacity.

4. The only reason that heuristics like the Silver-Meal, the Least Unit Cost and the Part-Period Balancing are used in the contemporary MRP planners is that they have been entrenched in these software platforms from the past. With the perspectives that have been provided by contemporary Operations Research and the current IT capabilities, one should not use anything else than the Wagner-Whitin algorithm for the relevant lot-sizing problem.

(A) TRUE (B) FALSE

Explain your answer.

While it is true that the W-W algorithm is indeed an algorithm that provides optimal solutions for the uncapacitated dynamic lot sizing problem and the other methods lack such a guarantee (being just heuristics for this problem), the significance of this fact is mitigated by the "rolling horizon" nature of the overall control scheme that is implemented in the MRP computation.

In other words, the overall decision making process is based only on a partial view of the future demand that is defined by the length of the employed planning horizon, and this fact renders the notion of optimality for the subproblems that are addressed in each period less important.

On the other hand, recognizing the volatility of the derived plans due to replanning in the subsequent periods, sometimes the heuristics mentioned in the question tend to be preferred over the W-W algorithm because their decisions tend to be more robust to demand info that is introduced as the rolling horizon extends to future periods. This robustness implies more stable schedules for the closer periods.

* For PULL-type of systems, like KANBAN or CONWIP the correct answer would have been (iv), since in that case, the blocking that might be experienced by the jobs moving between two consecutive stations implies that each station impacts also the dynamics of its upstream stations.

5. Consider a single-server workstation with very high processing-time variability. If you were able to choose the position of this workstation within an asynchronous transfer line, where would you place it?

i. At the beginning of the line.

ii. In the middle of the line.

iii. At the end of the line.

iv. It doesn't matter; anywhere is equally bad.

Briefly explain your answer.

The equations:

$$\begin{cases} C^2_{a_{i+1}} = C^2_{d_i} \\ C^2_{d_i} = (1-u_i^2)\, C^2_{a_i} + u_i^2\, C^2_{p_i} \end{cases}$$

implies that the variability in the proc. times of the considered station has the tendency to propagate to its downstream station(s). This is especially true for highly utilized stations.

On the other hand, nothing in the presented analysis of the (steady-state) dynamics of Asynchronous Transfer Lines (ATLS) considered in class indicates that the variability of a certain station affects the dynamics of its upstream stations. The reason for that is that jobs can move freely from station to station in these lines, i.e., there is ample capacity to accommodate the jobs arriving at each station and no blocking (this is the nature of a "push" scheme)

Problem 1 (20 points): A large producer of household products purchases a glyceride used in one of its deodorant soaps from outside of the company. It uses the glyceride at a fairly steady rate of 40 pounds per month, and the company uses a 23 percent annual interest rate to compute holding costs. The chemical can be purchased from two suppliers, A and B. A offers the following *all-units* discount schedule:

order size	price per pound ($)
$0 \le Q < 500$	1.30
$500 \le Q < 1{,}000$	1.20
$1{,}000 \le Q$	1.10

On the other hand, B offers the following *incremental* discount schedule: $1.25 per pound for all orders less than or equal to 700 pounds, and $1.05 per pound for all incremental amounts over 700 pounds. Assume that the cost of order processing for each case is $150. Which supplier should be used and what should be the replenishment order size?

First we evaluate the options provided by Supplier A:

$$Q_1^* = \sqrt{\frac{2 \cdot 150 \cdot 40 \cdot 12}{0.23 \cdot 1.3}} = 693.98 \Rightarrow Q_1^{best} = 499$$

$$TAC_1(Q_1^{best}) = 1.3 \cdot 40 \cdot 12 + 150\,\frac{40 \cdot 12}{499} + 1.3 \cdot 0.23 \cdot \frac{499}{2} = 842.888$$

$$Q_2^* = \sqrt{\frac{2 \cdot 150 \cdot 40 \cdot 12}{0.23 \cdot 1.2}} \simeq 722 \Rightarrow Q_2^{best} = Q_2^*$$

$$TAC_2(Q_2^{best}) = 1.2 \cdot 40 \cdot 12 + 150\,\frac{40 \cdot 12}{722} + 1.2 \cdot 0.23\,\frac{722}{2} = 775.359$$

$$Q_3^* = \sqrt{\frac{2 \cdot 150 \cdot 40 \cdot 12}{0.23 \cdot 1.10}} \simeq 754 \Rightarrow Q_3^{best} = 1000$$

$$TAC_3(Q_3^{best}) = 1.1 \cdot 40.12 + 150\,\frac{40 \cdot 12}{1000} + 1.1 \cdot 0.23 \cdot \frac{1000}{2} = 726.5$$

For supplier B we have:

$$OC(Q) = \begin{cases} 1.25\,Q, & 0 \le Q \le 700 \\ 1.25 \times 700 + 1.05\,(Q-700) = \\ \quad = 1.05\,Q + 140, & Q > 700 \end{cases}$$

and

$$C(Q) = \frac{OC(Q)}{Q} = \begin{cases} 1.25, & 0 \le Q \le 700 \\ 1.05 + \frac{140}{Q}, & Q > 700 \end{cases}$$

Hence,

$$\left.\begin{aligned} TAC_1(Q) &= C_1(Q)\,D + \frac{AD}{Q} + i\,C_1(Q)\,\frac{Q}{2} = \\ &= 1.25 \cdot 40.12 + 150\,\frac{40.12}{Q} + 0.23 \cdot 1.25 \cdot \frac{Q}{2} \end{aligned}\right\} \Rightarrow$$

and $Q_1^* = \sqrt{\dfrac{2 \cdot 150 \cdot 40.12}{0.23 \cdot 1.25}} \simeq 707 \Rightarrow Q_1^{best} = 700$

$$\Rightarrow TAC_1(Q_1^{best}) = 1.25 \cdot 40.12 + 150\,\frac{40.12}{700} + 0.23 \cdot 1.25 \cdot \frac{700}{2} =$$

$$= 803.482$$

$$TAC_2(Q) = \left(1.05 + \frac{140}{Q}\right) \cdot 40.12 + 150\,\frac{40.12}{Q} + 0.23\left(1.05 + \frac{140}{Q}\right)\frac{Q}{2} =$$

$$= \left(1.05 \cdot 40.12 + 0.23 \cdot \frac{140}{2}\right) + (140+150)\,\frac{40.12}{Q} + 0.23 \cdot 1.25 \cdot \frac{Q}{2}$$

So, $Q_2^* = \sqrt{\dfrac{2 \cdot (140+150) \cdot 40.12}{0.23 \cdot 1.05}} \simeq 1074 \Rightarrow Q_2^{best} = Q_2^*$

and

$$TAC_2(Q_2^{best}) = \left(1.05 \cdot 40.12 + 0.23 \cdot \frac{140}{2}\right) + (140+150)\,\frac{40.12}{1074} + 0.23 \cdot 1.25\,\frac{1074}{2}$$

$$= 779.39$$

$\Rightarrow$ Best option: Order 1000 pounds from Supplier A.

Problem 2 (20 points): Customers arriving at a local office get a ticket that routes them to one of the two office counters. The clerk at the first counter is quite skilled and her service times are distributed according to a normal distribution with a mean of 5 minutes and a standard deviation of 1 minute. The clerk at the second counter is a novice, and as a result, his service times are distributed according to a normal distribution with a mean of 7.5 minutes and a standard deviation of 4 minutes.

i. (5pts) If customers arrive according to a Poisson process with a rate of 15 customers per hour, what is the smallest percentage of them that must be directed to the second counter described above, in order to establish a utilization of 95% for the first counter?

ii. (5pts) Under the routing scheme determined in part (i), is the operation of the second counter stable?

iii. (5pts) What is the routing probability p_1, for routing an arriving customer to the first counter described above, that will balance the workload of the two clerks? What is the corresponding utilization for each clerk?

iv. (5pts) If the office operates under the balancing routing policy determined in part (iii) above, what is the expected sojourn time for a customer arriving at this office?

(i) $\frac{15}{60}(1-p)\cdot 5 = 0.95 \Rightarrow (1-p) = 0.76 \Rightarrow p = 0.24$

(ii) $\frac{15}{60} \cdot 0.24 \cdot 7.5 = 0.45 < 1 \Rightarrow$ stable

(iii) In the considered context, equal workload implies equal utilizations, i.e.,

$U_1 = U_2 \Leftrightarrow \frac{15}{60} p \cdot 5 = \frac{15}{60}(1-p) \cdot 7.5 \Rightarrow$

$\Rightarrow p = 0.6$

and $U_1 = U_2 = \frac{15}{60} \, 0.6 \cdot 5 = 0.75$.

(iv)

$$CT_1 = \frac{1+0.2^2}{2}\,\frac{0.75}{1-0.75}\,5+5 = 12.8 \text{ min}$$

$$CT_2 = \frac{1+(4/7.5)^2}{2}\,\frac{0.75}{1-0.75}\,7.5+7.5 = 21.95 \text{ min}$$

$$CT = p\,CT_1 + (1-p)\,CT_2 =$$

$$= 0.6 \times 12.8 + 0.4 \times 21.95 = 16.46 \text{ min}$$

Remark 1: In the above computation we have taken $c_{a_1} = c_{a_2} = 1$ because we know (from 3232) that if you split ~~any~~ Poisson arrival stream to two sub-streams by routing each arrival two the two streams with corresponding probabilities p and $1-p$, then each of the two substreams is a Poisson process.

Remark 2: The routing scheam that is described in the problem implies that each counter maintains its own queue (perhaps not the most pertinent idea, but this is what is suggested by the problem data). For this reason, we cannot consider the entire office as a $G/G/2$ queue, but ~~only~~ as two $G/G/1$ queues with the corresponding arrival processes defined by the substreams mentioned in Remark 1.

Problem 3 (20 points): An end-item widget is assembled from 4 subassemblies of type A36, 2 parts of type B11s, and 1 part of type C35. Each A36 is made from a B22 and 2 B19. Each B22 is made using a C7 and a D19.

i. (5pts) Express the BOM of the above end-item as a tree and classify all the aforementioned parts into levels.

ii. (5pts) The nominal production lead time for a lot of the considered end-item is 2 weeks, and its net requirements over an 8-week planning horizon are estimated as follows:

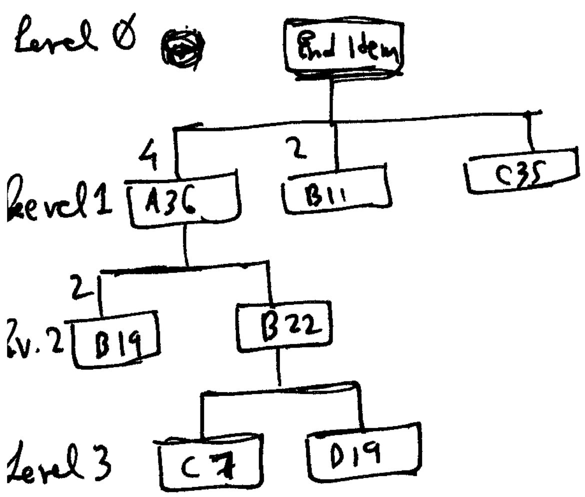
$$0, 0, 0, 10, 50, 40, 60, 50$$

Assuming that the production of the end-item is organized on a lot-for-lot basis, determine the gross requirements for the level-1 items that you determined in step (i).

iii. (5pts) The current inventory position of subassembly A36 is 50 units, and there is an initiated production lot of 100 parts of this item that will be ready at the end of the first week. Compute the net requirements for A36 over the considered planning horizon.

iv. (5pts) The nominal production lead time for a lot of A36 is 2 weeks. Also, the set-up cost for the production of a lot for A36 is estimated at \$120 and the holding cost for this item is \$2 per unit per week. Compute the size of the production lot for this item that must be started in the upcoming week, (i.e., in week 1 of the planning horizon) using your results from item (iii) above and the lot-sizing policy that is based on the Silver-Meal heuristic.

(i)



(ii)

Period	1	2	3	4	5	6	7	8
A36								
B11								
C35								

(ii)

	Period	1	2	3	4	5	6	7	8
	End Item NR	0	0	0	10	50	40	60	50
L4L	End Item PSRec	0	0	0	10	50	40	60	50
Lead Time = 2	End Item PSRel	0	10	50	40	60	50	0	0
×4	A36 GR	0	40	200	160	240	200	0	0
×2	B11 GR	0	20	100	80	120	100	0	0
×1	C35 GR	0	10	50	40	60	50	0	0

(iii)

Period	0	1	2	3	4	5	6	7	8
A36 GR	0	0	40	200	160	240	200	0	0
A36 SR		100							
A36 IP	50		110	-90					
A36 NR				90	160	240	200	0	0

(iv) Since the prod. lead time for A36 is two weeks and the first net requirement (of 90 units) appears in week 3, we need to start a lot of at least 90 units in week 1. To select the size of this lot according to the SM heuristic, we perform the following computation:

Period	Setup Cost	Holding Cost	Total Cost	Total Cost / Period
1	120	—	120	120
2	120	2×160	440	440/2 = 220 ↑

⇒ lot size = 90.