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ISyE 2030 — Practice Test 2

Summer 2005

This test is open notes, open books. You have *exactly 75 minutes.*

1. Short-Answer Questions

   (a) TRUE or FALSE? If arrivals occur according to a Poisson process with rate λ, then the probability that the first arrival will take place after time t is $e^{-\lambda t}$.

   ANSWER: True. $\diamondsuit$

   (b) In queueing theory, the symbol "ρ" usually denotes...

   i. Traffic intensity.
   ii. Server utilization.
   iii. Service rate.
   iv. Both (i) and (ii).
   v. Both (i) and (iii).

   ANSWER: (*iv*). $\diamondsuit$

   (c) How many servers does an M/G/∞ queue have?

   ANSWER: ∞. $\diamondsuit$

   (d) Consider an M/M/1 queueing system. Customer interarrival times have an average of 5 minutes, and service times have an average of 4 minutes. What will be the average number of customers waiting in line?

   ANSWER:

$$\rho = \frac{\lambda}{\mu} = \frac{1/5}{1/4} = \frac{4}{5} \quad \Rightarrow \quad L_Q = \frac{\rho^2}{1-\rho} = \frac{(4/5)^2}{1/5} = \frac{16}{5}. \quad \diamondsuit$$

(e) TRUE or FALSE? $L = \lambda w$ is commonly referred to as the Pollaczek-Khintchine equation.

ANSWER: False. (It's Little's law.) $\diamondsuit$

(f) Cars show up at a parking lot at the rate of 100 per hour. On average, a car stays for two hours. How many cars on average will be in the parking lot?

ANSWER: $L = \lambda w = 100(2) = 200$. $\diamondsuit$

(g) What's the maximum possible size of the line an M/M/4/9 queue can have?

ANSWER: $9 - 4 = 5$. $\diamondsuit$

(h) Consider an M/M/3 queueing system having 10 arrivals per hour and a mean service time of 10 minutes. What is the traffic intensity of this system?

ANSWER:

$$\lambda = 10, \mu = 6, c = 3 \quad \Rightarrow \quad \rho = \frac{\lambda}{c\mu} = \frac{10}{18} = \frac{5}{9}. \quad \diamondsuit$$

(i) Suppose customers arrive at a single-server queue via a Poisson process with rate 5/hour, and service times are *exactly* 10 minutes (no variation). What's the steady-state expected number of customers in the system?

ANSWER:

$$\text{M/D/1} : \lambda = 5, \mu = 6, \rho = 5/6 \quad \Rightarrow \quad L = \rho + \frac{\rho^2}{2(1-\rho)} = \frac{35}{12} = 2.917. \quad \diamondsuit$$

(j) Customers again arrive at a single-server queue via a Poisson process with rate 5/hour. Now suppose that service times are *normally* distributed with a mean of 10 minutes and a standard deviation of 1 minute. What is the steady-state expected number of customers in the system?

ANSWER: M/G/1 : $\lambda = 5, \mu = 6, \sigma = 1/60$.

$$L = \rho + \frac{\rho^2(1+\sigma^2\mu^2)}{2(1-\rho)} = \frac{5}{6} + \frac{(25/36)(1+36/3600)}{2/6} = 2.9375. \quad \diamondsuit$$

(k) Consider an M/G/2 queue with a arrival rate of 10 customers an hour. What is the smallest service rate that the system's servers must have before the system becomes unstable?

ANSWER:

$$\rho = \frac{\lambda}{c\mu} = \frac{10}{2\mu} < 1 \quad \Rightarrow \quad \mu > 5. \quad \diamondsuit$$

2. (20 points) Mickey Mouse Questions.

(a) Customers arrive at Space Mountain in Disneyworld at the rate of 1000/hr according to a Poisson process. They form a long line and are served by one "server" at the rate of 1200/hr. Just for the heck of it, suppose that service times are exponential and first-come-first-served. Find the steady-state expected waiting time.

ANSWER: This is an M/M/1 system for which $\rho = \lambda/\mu = 5/6$. Therefore,

$$L_Q = \frac{\rho^2}{1-\rho} = 4.167.$$

Thus,

$$w_Q = L_Q/\lambda = 0.004167 \text{ hrs} \approx 15.00 \text{ sec.} \quad \diamondsuit$$

(b) Now suppose that Disney has decided to hire a two *slow* "servers", each of whom can perform services at the rate of 600/hr. Again, assume services are exponential. Now find the steady-state expected waiting time for this alternative system.

ANSWER: We see that $\rho = \lambda/(2\mu) = 5/6$, as in the previous part of the problem. Then we have

$$P_0 = \left\{ \left[\sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!} \right] + \left[\frac{(c\rho)^c}{(c!)(1-\rho)} \right] \right\}^{-1} = 0.0909$$

so that

$$L_Q = \frac{(c\rho)^{c+1} P_0}{c(c!)(1-\rho)^2} = 3.788.$$

Thus,

$$w_Q = L_Q/\lambda = 0.003788 \text{ hrs} \approx 13.64 \text{ sec.} \quad \diamondsuit$$

3. (20 points) Joey owns "Nerds 'R Us," a small computer consulting company. Today he is interviewing ten candidates for jobs at his company. The interarrival times (in minutes) for the candidates are as follows:

11 8 1 3 6 12 10 6 10 6

(Assume that the first customer arrives at time 11.) Suppose it takes Joey exactly 10 minutes to complete any particular candidate's interview; but for some reason, he processes the queue in a last-in-first-out manner.

customer	arrive	start service	leave	wait
1	11	11	21	0
2	19	101	111	82
3	20	21	31	1
4	23	91	101	68
5	29	31	41	2
6	41	41	51	0
7	51	51	61	0
8	57	61	71	4
9	67	71	81	4
10	73	81	91	8

(a) When does the last candidate leave the system?

ANSWER: 111. ♢

(b) When does candidate 3 start getting served?

ANSWER: 21. ♢

(c) What is the average waiting time for the candidates?

ANSWER: 16.9. ♢

(d) What is the maximum length of the waiting line?

ANSWER: 3 (in line). ♢

4. Suppose that we are simulating the sales of turkeys at a local market. The owner starts off with 10 turkeys. The daily demand for turkeys for the next 6 days is as follows.

$$9 \quad 3 \quad 8 \quad 5 \quad 2 \quad 7$$

Each turkey costs the owner \$7 to buy, and can be sold to a customer for \$13. It costs the owner \$1 to store each turkey overnight (i.e., if it wasn't sold that day). It also costs the owner \$2 per turkey if he is unable to fill a demand — maybe the irate customer does damage to the store. If, at the end of a particular day, the owner has less than or equal to 3 turkeys in stock, he places an order to bring the inventory level back up to 10 the next morning. He gets the turkeys the first thing in the morning before any customers arrive. Compute his total profit/loss for this 6-day period. How many turkeys are left at the end of the last day?

ANSWER: There are a number of interpretations for this problem, all of which are perfectly acceptable, as long as you state the assumptions you used. In the answer that I'll now display, let's assume:

- We have to pay for the initial 10 turkeys.
- We have no holding cost for the first day.
- We don't have to pay holding costs for any turkeys left over after the last day.

So here's what happens...

Day t	Demand D_t	Inventory at End of Day I_t	Order Quantity
0	–	10	
1	9	1	9
2	3	7	0
3	8	-1	10
4	5	5	0
5	2	3	7
6	7	3	0

Our earnings are therefore

Turkeys bought (7 units)	−\$252
Holding cost (16 units)	−\$16
Stockout cost (1 unit)	−\$2
Turkeys sold (33 units)	\$429
TOTAL	\$159

Further, note that 3 turkeys are left. ♢

5. Consider a queueing system with customers arriving according to a Poisson process at the rate of 5 per hour. Which of the following systems gives the lower steady-state expected time in system (i.e., waiting in line plus service time)? (A) Two "slow" parallel servers, each of whom can serve customers with Exp(3/hour) service times, or (B) One "fast" server, who can serve customers with Exp(6/hour) services? Show all work.

   ANSWER: (A) Here we have an M/M/2 queueing system with $\lambda = 5$, $\mu = 3$, and $c = 2$. Thus, $\rho = \lambda/(c\mu) = 5/6$.

   Going to the M/M/2 tables, we do a little calculation and come up with $P_0 = 1/11$, and with a little more algebra, we have $L = 60/11$.

   Finally, $w = L/\lambda = 1.1$ hours.

   (B) Meanwhile, now we have we have an M/M/1 queueing system with $\lambda = 5$ and $\mu = 6$. Thus, $\rho = \lambda/\mu = 5/6$ (still).

   Going to the M/M/1 tables, we do a teensy calculation and come up with

$$w = \frac{1}{\mu(1-\rho)} = 1.$$

   Thus, choose the M/M/1. $\diamondsuit$

6. Miscellaneous Easy Arena Questions — Just know what blocks we've gone over and what you did in lab last week. Nothing difficult.