MANAGING INVENTORY IN THE FACE OF UNCERTAINTY

The Newsvendor Problem

IE2030

You are the GaTech Barnes & Noble Merchandise Buyer



<u>#BB12 - Final Four tee</u>

It's Sunday, March 28. The Jackets just made it to the Final Four.

Before heading out to celebrate, you need to call your T-shirt supplier and order Final Four merchandise.

How many do you order?

100% cotton -shirt with Final Four logo **\$21.98**.



Need to think about

Cost and price

Lead time

Uncertainty in demand What happens to Final Four T-shirt sales if the Jackets lose the semifinal? win the semifinal? win the final?

Value of leftovers after Monday? After this season?

The Newsvendor Framework

- One chance to decide on the stocking quantity for the product you're selling
- Demand for the product is uncertain
- Known marginal profit for each unit sold and known marginal loss for the ones that are bought and not sold
- Goal: Maximize expected profit

Examples where the Newsvendor framework is appropriate

- Perishable goods
 - Meals in a cafeteria
 - Dairy foods
- Short selling season
 - Christmas trees, toys
 - Flowers on Valentine's day
 - Fashion clothes, seasonal clothes
 - Newspapers

The Newsvendor Problem

- Newsvendor selling the New York Times
- Sells the papers for \$1.00
- Buys them for 70 cents
- Leftover papers sold at discount: 20 cents each.
- He will definitely sell at least 35 papers, but no more than 40:
 - 35 papers with probability 0.10
 - 36 papers with probability 0.15
 - 37 papers with probability 0.25
 - 38 papers with probability 0.25
 - 39 papers with probability 0.15
 - 40 papers with probability 0.1

How Many Papers Should He Buy To Maximize His Expected Profit?

X	Probabili- ty that demand = x	P = Prob. of selling the xth unit	(1-P)=Prob. of NOT selling the xth unit	P x \$0.3 =Expected profit from selling xth unit	(1-P) x \$0.5 =Expected loss from NOT selling xth unit	Expected NET profit from stocking xth unit		
<35	0							
35	0.1	1.0	0	\$0.30	\$0	\$0.30		
36								
37								
38								
39								
40								
>40								

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<35	0					
35	0.1	1.0	0	\$0.30	\$0	\$0.30
36	0.15	0.9	0.1	\$0.27	\$0.05	\$0.22
37						
38						
39						
40						
>40						

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36	0.15	0.9	0.1	\$0.27	\$0.05	\$0.22		
37	0.25	0.75	0.25	\$0.225	\$0.125	\$0.10		
38								
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36	0.15	0.9	0.1	\$0.27	\$0.05	\$0.22		
37	0.25	0.75	0.25	\$0.225	\$0.125	\$0.10		
38	0.25	0.50	0.50	\$0.15	\$0.25	\$-0.10		
39	0.15	0.25	0.75	\$0.075	\$0.375	\$-0.30		
40	0.1	0.1	0.9	\$0.03	\$0.45	\$-0.42		
>40	0	0	1	\$0.0	\$0.50	\$-0.50		

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	39	0.15	0.25	0.75	\$0.075	\$0.375	\$-0.30
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	>40	0	0	1	\$0.0	\$0.50	\$-0.50

Decreasing marginal returns to each additional unit.

Solution

Start from 35, buy an additional paper only if you expect to make extra profits.

Let's generalize!

c: cost

r: selling price

s: salvage value

MP: marginal profit from selling a stocked unit = r-c*ML:* marginal loss from NOT selling a stocked unit = c-sx: the number of newspapers you buy. P(x): the probability that the xth newspaper is sold.

Solution

Buy one more unit only if the expected net profit of doing so is positive.

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Buying x is profitable if

P(x) MP - (1-P(x)) ML = 0

P(x) = ML / (MP + ML)

Critical ratio:

Pc=ML / (MP + ML)

Optimal solution: Buy largest x such that

P(x) = Pc
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Solution

In this problem: *MP* = \$1-\$0.70 = \$0.30 *ML* = \$0.70 -\$0.20 = \$0.50

Pc=ML / (MP+ML) = 50 / (50+30) = 0.625

Buy largest x such that P(selling xth unit) = Pc

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Now Let's Assume that Demand is Normally Distributed



How do we find order quantity X?*



 $P(Demand = x) = P(ALL \ x \ new spapers \ are \ sold)$

Choose X^* such that P (*Demand* = X^*) = Pc



Newsvendor: Solution with Normal Distribution

- Remember: MP = 0.30; ML = 0.50
- Then *Pc* is

Pc=ML/(MP+ML)=50/(50+30)=0.625

- Look at the *z* table to find the *z* corresponding to *Pc*
- $X^* = m + z s$



Area	Z	Area	z	Area	z	Area	z		Area	Z		Area	Z		Area	z		Area	Z
0.005	2.576	0.130	1.126	0.255	0.659	0.380	0.305		0.505	-0.013		0.630	-0.332		0.755	-0.690		0.880	-1.175
0.010	2.326	0.135	1.103	0.260	0.643	0.385	0.292		0.510	-0.025		0.635	-0.345		0.760	-0.706		0.885	-1.200
0.015	2.170	0.140	1.080	0.265	0.628	0.390	0.279		0.515	-0.038		0.640	-0.358		0.765	-0.722		0.890	-1.227
0.020	2.054	0.145	1.058	0.270	0.613	0.395	0.266		0.520	-0.050		0.645	-0.372		0.770	-0.739		0.895	-1.254
0.025	1.960	0.150	1.036	0.275	0.598	0.400	0.253		0.525	-0.063		0.650	-0.385		0.775	-0.755		0.900	-1.282
0.030	1.881	0.155	1.015	0.280	0.583	0.405	0.240		0.530	-0.075		0.655	-0.399		0.780	-0.772		0.905	-1.311
0.035	1.812	0.160	0.994	0.285	0.568	0.410	0.228		0.535	-0.088		0.660	-0.412		0.785	-0.789		0.910	-1.341
0.040	1.751	0.165	0.974	0.290	0.553	0.415	0.215		0.540	-0.100		0.665	-0.426		0.790	-0.806		0.915	-1.372
0.045	1.695	0.170	0.954	0.295	0.539	0.420	0.202		0.545	-0.113		0.670	-0.440		0.795	-0.824		0.920	-1.405
0.050	1.645	0.175	0.935	0.300	0.524	0.425	0.189		0.550	-0.126		0.675	-0.454		0.800	-0.842		0.925	-1.440
0.055	1.598	0.180	0.915	0.30:	Ara	a - b	D_{c-}	\mathbf{n}	625	-									-1.476
0.060	1.555	0.185	0.896	0.31	AIC	a - I	<i>L</i> –	0.0	02J										-1.514
0.065	1.514	0.190	0.878	0.31:	$z \equiv$	-0.32	2												-1.555
0.070	1.476	0.195	0.860	0.32		0.01			~ -			<i>(</i>)		-		•	_	<u> </u>	-1.598
0.075	1.440	0.200	0.842	0.32:	X^* =	= m +	z S	: =	: 37	′.5 I	⊢ (′-0 . .	32)	1.	44	= 3	7.	04	-1.645
0.080	1.405	0.205	0.824	0.330	0.440	0.455	0.113		0.580	-0.202	,	0. 10	-0.539		0.830	-0.954		0.955	-1.695
0.085	1.372	0.210	0.806	0.335	0.426	0.460	0.100		0.585	-0.215		0/10	-0.553		0.835	-0.974		0.960	-1.751
0.090	1.341	0.215	0.789	0.340	0.412	0.465	0.088		0.590	-0.228		0.715	-0.568		0.840	-0.994		0.965	-1.812
0.095	1.311	0.220	0.772	0.345	0.399	0.470	0.075		0.595	-0.240		0.720	-0.583		0.845	-1.015		0.970	-1.881
0.100	1.282	0.225	0.755	0.350	0.385	0.475	0.063		0.600	-0.253		0.725	-0.598		0.850	-1.036		0.975	-1.960
0.105	1.254	0.230	0.739	0.355	0.372	0.480	0.050		0.605	-0.266		0.730	-0.613		0.855	-1.058		0.980	-2.054
0.110	1.227	0.235	0.722	0.360	0.358	0.485	0.038		0.610	-0.2/9		0.735	-0.628		0.860	-1.080		0.985	-2.170
0.115	1.200	0.240	0.706	0.365	0.345	0.490	0.025		0.615	-0.292		0.740	-0.643		0.865	-1.103		0.990	-2.326
0.120	1.175	0.245	0.690	0.370	0.332	0.495	0.013		0.620	-0.305		0.745	-0.659		0.870	-1.126		0.995	-2.576
0.125	1.150	0.250	0.674	0.375	0.319	0.500	0.000	\langle	0.625	-0.319	>	0.750	-0.674		0.875	-1.150			

What Do We Learn from the Newsvendor?

- Forecasts are always wrong. A demand estimate that only gives the mean is too simple, you also need the standard deviation.
- The optimal order quantity depends on the relative cost of stocking too much and stocking too little.
- The smaller the standard deviation, the closer will be the order to the mean.