Online Supplement to "A Distribution-Free Tabular CUSUM Chart for Autocorrelated Data"

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The purpose of this supplement is to provide standard errors of estimated average run lengths (ARLs) in Tables 3 $\sim$ 6 of the main paper. Tables S-1 and S-2 provides standard errors of ARLs in Tables 3 and 4 when AR(1) processes are tested. Table S-3 shows standard errors of ARLs in Table 5 when M/M/1 waiting time processes are tested. Finally, standard errors of ARLs in Table 6 for an AR(2) process are given in Table S-4.

Table S-1. Standard errors of two-sided ARLs in Table 3 for an AR(1) process with small or medium φ_Y and $\sigma_Y^2 = 1$

φ_Y	Shift	J&B	New CuSum	DFTC	R&W	
	$(\mu - \mu_0)/\sigma_Y$				$m = 1$	
0	0	80.509	111.543	137.776	142.584	
	0.25	1.315	1.132	1.119	93.753	
	0.5	0.470	0.400	0.290	39.235	
	0.75	0.258	0.215	0.142	16.945	
	1	0.171	0.141	0.090	7.357	
	1.5	0.091	0.076	0.047	1.688	
	2	0.059	0.049	0.030	0.480	
	2.5	0.043	0.037	0.021	0.167	
	3	0.032	0.027	0.016	0.069	
	4	0.022	0.018	0.011	0.018	
					$m_1 = 4$	$m^* = 15$
0.25	0	82.378	115.720	150.249	140.703	141.602
	0.25	1.942	1.678	1.742	59.871	25.996
	0.5	0.686	0.589	0.463	16.610	4.111
	0.75	0.375	0.317	0.230	5.283	1.046
	1	0.249	0.205	0.138	1.902	0.362
	1.5	0.133	0.113	0.074	0.372	0.070
	2	0.088	0.072	0.047	0.111	0.008
	2.5	0.063	0.052	0.032	0.044	0
	3	0.047	0.040	0.024	0.018	0
	4	0.031	0.026	0.016	0.002	0
					$m_1 = 8$	$m^* = 27$
0.5	0	83.712	114.626	157.699	140.175	141.650
	0.25	3.065	2.585	3.007	59.359	29.369
	0.5	1.069	0.919	0.788	16.380	5.195
	0.75	0.599	0.505	0.394	5.182	1.392
	1	0.386	0.323	0.244	1.947	0.505
	1.5	0.211	0.172	0.125	0.391	0.092
	2	0.138	0.114	0.080	0.135	0.009
	2.5	0.096	0.081	0.057	0.052	0
	3	0.073	0.062	0.042	0.018	0
	4	0.048	0.040	0.026	0	0

Table S-2. Standard errors of two-sided ARLs in Table 4 for an AR(1) process with high φ_Y and $\sigma_Y^2 = 1$

φ_Y	Shift	J&B	New CuSum	DFTC		R&W	
	$(\mu - \mu_0)/\sigma_Y$			Unbatched	$m = 3$	$m_1 = 19$	$m^* = 43$
0.7	0	85.835	119.634	160.341	159.422	147.274	137.796
	0.25	4.722	4.051	5.257	4.903	57.322	38.840
	0.5	1.694	1.486	1.403	1.361	16.015	7.640
	0.75	0.940	0.784	0.686	0.683	5.152	2.307
	1	0.623	0.512	0.429	0.438	1.963	0.861
	1.5	0.338	0.278	0.221	0.227	0.477	0.193
	2	0.214	0.185	0.137	0.142	0.152	0.021
	2.5	0.153	0.128	0.098	0.100	0.052	0
	3	0.119	0.098	0.072	0.077	0.012	0
	4	0.076	0.064	0.046	0.049	0.00	0
				Unbatched	$m = 7$	$m_1 = 58$	$m^* = 97$
0.9	0	88.289	119.169	178.699	145.525	141.081	136.174
	0.25	11.324	10.587	13.876	12.922	70.698	57.476
	0.5	4.296	3.706	3.947	3.913	22.983	16.765
	0.75	2.345	1.983	1.984	2.010	8.782	5.937
	1	1.524	1.277	1.260	1.250	3.823	2.588
	1.5	0.833	0.696	0.648	0.655	1.098	0.675
	2	0.537	0.449	0.402	0.407	0.402	0.172
	2.5	0.382	0.320	0.291	0.278	0.126	0.034
	3	0.290	0.238	0.211	0.216	0.020	0
	4	0.186	0.156	0.132	0.135	0	0
				Unbatched	$m = 15$	$m_1 = 118$	$m^* = 157$
0.95	0	92.818	120.077	169.276	144.699	138.755	141.566
	0.25	19.246	17.745	24.186	22.827	78.792	69.844
	0.5	7.187	6.231	7.522	7.006	28.195	23.042
	0.75	3.971	3.451	3.698	3.595	12.009	9.670
	1	2.603	2.208	2.353	2.213	5.730	4.678
	1.5	1.413	1.210	1.205	1.216	1.754	1.322
	2	0.930	0.778	0.733	0.756	0.609	0.434
	2.5	0.645	0.545	0.521	0.518	0.182	0.063
	3	0.508	0.405	0.387	0.387	0.047	0
	4	0.314	0.255	0.244	0.249	0	0
				Unbatched	$m = 74$	$m_1 = 596$	$m^* = 463$
0.99	0	105.345	138.614	181.207	138.926	128.964	138.470
	0.25	54.977	55.870	71.035	63.434	93.787	96.416
	0.5	24.152	21.536	26.877	25.131	44.310	48.687
	0.75	13.305	11.818	14.360	13.307	22.795	25.239
	1	8.852	7.668	8.845	8.316	12.075	13.952
	1.5	4.882	4.052	4.521	4.381	4.308	5.405
	2	3.091	2.534	2.842	2.775	1.535	2.096
	2.5	2.168	1.760	1.949	1.950	0.412	0.781
	3	1.605	1.330	1.449	1.438	0	0.093
	4	1.038	0.809	0.884	0.905	0	0

Table S-3. Standard errors of two-sided ARLs in Table 5 for $M/M/1$ queue waiting times

τ	Shift	J&B	New CuSum	DFTC		R&W	
	$(\mu-\mu_0)/\sigma_Y$			Unbatched	$m=2$	$m_1=11$	$m=300$
0.3	0	91.475	119.986	119.372	130.784	10.074	130.163
	0.25	3.606	2.999	4.185	3.904		6.611
	0.5	1.270	1.048	0.968	0.959		0.104
	0.75	0.685	0.562	0.454	0.451		0
	1	0.436	0.365	0.272	0.274		0
	1.5	0.236	0.196	0.139	0.143		0
	2	0.151	0.130	0.088	0.090		0
	2.5	0.111	0.091	0.061	0.063		0
	3	0.083	0.069	0.045	0.046		0
	4	0.055	0.044	0.029	0.030		0
				Unbatched	$m=10$	$m=55$	$m=400$
0.6	0	96.130	124.292	201.417	182.401	35.706	138.757
	0.25	10.936	9.214	14.527	13.302		31.657
	0.5	3.845	3.141	3.597	3.471		5.916
	0.75	2.037	1.645	1.646	1.660		0.422
	1	1.273	1.062	0.977	1.023		0
	1.5	0.710	0.575	0.502	0.517		0
	2	0.458	0.365	0.302	0.320		0
	2.5	0.320	0.258	0.213	0.227		0
	3	0.240	0.193	0.159	0.174		0
	4	0.157	0.125	0.102	0.114		0

Table S-4. Standard errors of two-sided ARLs in Table 6 for the AR(2) process with $\varphi_1 = 1.8$ and $\varphi_2 = -0.9$

Shift	J&B	New CuSum	DFTC		R&W
$(\mu-\mu_0)/\sigma_Y$			Unbatched	$m=14$	$m=6$
0	83.042	112.577	96.259	108.724	146.563
0.25	2.146	1.837	1.944	1.732	90.871
0.5	0.788	0.675	0.511	0.566	39.338
0.75	0.427	0.369	0.268	0.299	16.757
1	0.276	0.239	0.166	0.201	7.811
1.5	0.156	0.131	0.094	0.118	1.978
2	0.105	0.086	0.062	0.081	0.592
2.5	0.074	0.064	0.045	0.094	0.225
3	0.057	0.049	0.035	0.061	0.094
4	0.038	0.033	0.026	0.018	0.031