Option to Defer Investment

1. (a) The project’s value next year $P$ is perfectly correlated with the security, i.e., $P = 10S$. Thus, the market should value $P$ today as 1000.

(b) Let $p$ denote the probability that the market will go up. The value for $p$ must satisfy $150p + 66.6(1 - p) = 125$, which implies that $p = 0.7$.

(c) Let $\bar{r}$ denote the expected rate of return. It must satisfy $\frac{150(0.6) + 66.6(0.4)}{1.05} = 100$, which implies that $\bar{r} = 16.6\%$.

2. (a) $NPV = PV - I_0 = 1000 - 1050 = -50 < 0$. Reject project.

(b) Consider the project’s $PV$ next year if the delay option is purchased. When the market is up $PV = 1500 - (1.1)(1050) = 345$; when the market is down the $PV = 0$ since you would not make the investment. The project’s cost of capital is 25%. Accordingly, using Decision-Tree Analysis (DTA) the $NPV = \frac{(0.7)(345) + (0.3)(0) - 175}{1.25} = 18.2 > 0$, which says the delay option should be purchased, and the project undertaken next year if the market goes up.

However, this is not correct given the present setup. Let’s use Real Options Analysis (ROA). The risk-neutral probability of the market going up is 0.46, since $\frac{0.46(150) + 0.54(66.6)}{1.05} = 100$. Therefore, the correct value of the project today is computed as $\frac{0.46(345) + 0.54(0)}{1.05} - 175 = -23.86 < 0$, and so the correct $NPV$ says the delay option and the project should be rejected. How can we verify this? The $PV$ of the project is 151.14 (151.14 − 175 = −23.86). Since $\frac{\Delta P}{\Delta S} = \frac{345 - 0}{150 - 66.6} = 4.14$, the replicating portfolio of $S$ and bond $B$ is $4.14S - 262.86B$, which not surprisingly costs 151.14 to purchase. This portfolio’s value next year will either be 345 if the market goes up or 0 if the market goes down, which exactly matches the value of the project with the delay option.

Using DTA can work, but only if the cost of capital is properly adjusted. Since the expected value of next year’s project value is $(0.7)(345) = 241.5$, if we use a cost of capital of 59.79%, then the $PV$ will be 151.14, as required. It is often difficult to arrive at correct values for the cost of capital, making this approach more difficult to implement.

(c) In what follows we assume that $\theta > 0$ and $(1 + \theta)(1050) < 1500$. The project’s correct $NPV$ is $\frac{0.46[1500 - (1 + \theta)(1050)]}{1.05} - p$, which must be positive if the project with the delay option is to be accepted. The acceptance region is therefore $\{(\theta, p) : 197.14 \geq 460\theta + p\}$.

3. The project will not be undertaken next year if the market goes down. We need to assess the correct project value if the market goes up. If we do not delay 1 more
year, the project’s value is 345, as before. However, we can delay 1 more year. If we choose to do so, the discounted expected project value using the risk-neutral probability is 
\[ 0.46[1.5(1500)-(1.1)^2(1050)] = 429.11. \]
After subtracting the cost of 25 the NPV is 404.11. Since 404.11 > 345 it is optimal to delay the project 1 more year should the market go up next year. The project’s value next year is thus 404.11 if the market goes up and 0 if the market goes down. The correct NPV for this project is then 
\[ 0.46(404.11)1.05 - 175 = 2.04 > 0, \]
and so we should accept the project with the 2-year delay option.

**Option to Contract Operations**

1. (a) The PV event tree for each project has 6 “nodes”, which we shall label, respectively, as \{0, U, D, UD, DU, DD\}. For Project 1 the value for \(u\) is \(e^{0.40} = 1.4918\) and \(d\) is \(e^{-0.40} = 0.6703\), and for Project 2 the value for \(u\) is \(e^{0.20} = 1.2214\) and \(d\) is \(e^{-0.20} = 0.8187\). The PV’s corresponding to each state for Project 1 are \{100, 149.18, 67.03, 222.55, 100, 44.93\}, and the PV’s corresponding to each state for Project 2 are \{100, 122.14, 81.87, 149.18, 100, 67.03\}.

(b) Each project’s NPV is 100 − 80 = 20.

(c) Let \(p_i\) denote the probability for the “up” state for Project \(i\), \(i = 1, 2\). We must have 
\[ 149.18p_1 + 67.03(1 - p_1) = 1.12, \] which implies \(p_1 = 0.547\), and 122.14\(p_2\) + 81.87(1 − \(p_2\)) = 1.12, which implies \(p_2 = 0.748\).

2. (a) We must work backwards through the PV event tree. Assume we have reached year 2 and we have yet to exercise our option to contract. It does not pay to contract in states \(UU\) and \(UD\), and so their respective values remain the same at 222.55 and 100, respectively. In state \(DD\) it does pay to contract since 0.6(44.93)+33 = 59.96 > 44.93, and so the value here would be 59.96. Now let’s move back to year 1, and suppose we have not exercised our option to contract. In state \(U\) it would not pay to contract. Now consider state \(D\). If we choose to contract now, then the value would be 0.6(67.03) + 33 = 73.22. We can, however, choose not to contract, and then the value according to DTA would be 
\[ \frac{0.547(100)+0.453(59.96)}{1.12} = 73.09. \] The best choice is therefore to contract, and the associated PV is 73.22. Now let’s move back to year 0. Its PV according to DTA is 
\[ \frac{0.547(149.18)+0.453(73.22)}{1.12} = 102.47, \] which gives an NPV of 102.47 − 80 = 22.47.

(b) According to DTA the option to contract should be exercised in year 1 if state \(D\) occurs.

(c) According to DTA the option to contract is worth 2.47.

3. We must work backwards through the PV event tree. Assume we have reached year 2 and we have yet to exercise our option to contract. It does not pay to contract in states \(UU\), and so its value remains the same at 222.55. In state \(UD\) it does pay to contract since 0.6(100) + 42 = 102 > 100, and so the value here would be 102. In state
The difference in using ROA is we will use the risk-neutral probabilities and the risk-free rate to perform the discounted expectation for valuation purposes when working backwards through the PV tree. Using ROA requires that an underlying marketable security be identified that can be used to construct the replicating portfolio; here, we take the project without flexibility as such a security. The project and the bond complete the market, i.e., span all possible valuations on the event tree, which then implies risk-neutral valuation will work. For Project 1 the risk-neutral probability $p_1$ must satisfy 

$$
\frac{149.18 p_1 + 67.93 (1-p_1)}{1.05} = 100
$$

which implies $p_1 = 0.462$. We now proceed, as before.

Assume we have reached year 2 and we have yet to exercise our option to contract. The terminal values in year 2 are unaffected here by the change in probabilities and discount rate. The values for states $\{UU, UD, DD\}$ are, respectively, $\{222.55, 100, 59.96\}$. (If we have not contacted operations by year 2 we should contract them if we reach state $DD$.) Now let’s move back to year 1, and suppose we have not exercised our option to contract. In state $U$ it would not pay to contract. Its value will be 

$$
\frac{0.748(149.18) + 0.252(102)}{1.12} = 122.58
$$

Now consider state $D$. If we choose to contract now, then the value would be 

$$
0.6(81.87) + 42 = 91.09
$$

We can, however, choose not to contract, and then the value according to $DTA$ would be 

$$
\frac{0.748(102) + 0.252(82.22)}{1.12} = 85.29
$$

The best choice is therefore to contract, and the associated $PV$ is 91.09. Now let’s move back to year 0. Its $PV$ according to $DTA$ is 

$$
\frac{0.748(122.58) + 0.252(91.09)}{1.12} = 102.36
$$

which gives an NPV of $102.36 - 80 = 22.36$.

4. The difference in using ROA is we will use the risk-neutral probabilities and the risk-free rate to perform the discounted expectation for valuation purposes when working backwards through the PV tree. Using ROA requires that an underlying marketable security be identified that can be used to construct the replicating portfolio; here, we take the project without flexibility as such a security. The project and the bond complete the market, i.e., span all possible valuations on the event tree, which then implies risk-neutral valuation will work. For Project 1 the risk-neutral probability $p_1$ must satisfy 

$$
\frac{149.18 p_1 + 67.93 (1-p_1)}{1.05} = 100
$$

Now consider state $D$. If we choose to contract now, then the value would be 

$$
0.6(79.03) + 33 = 73.22
$$

this has not changed. We can, however, choose not to contract, and then the value according to ROA would be 

$$
\frac{0.462(222.55) + 0.538(100)}{1.05} = 74.72
$$

The best choice is therefore to not contract, and the associated $PV$ is 74.72. (Here is an example of a difference between recommendations using $DTA$ and ROA.) Now let’s move back to year 0. Its $PV$ according to ROA is 

$$
\frac{0.462(149.18) + 0.538(74.72)}{1.05} = 103.92
$$

which gives an NPV of $103.92 - 80 = 23.92$. The correct value of the option to contract is 3.92 for Project 1. We should only contract operations should we reach state $DD$ in year 2.

5. For Project 2 the risk-neutral probability $p_2$ must satisfy 

$$
\frac{122.14 p_2 + 81.87 (1-p_2)}{1.05} = 100
$$

which implies $p_2 = 0.574$. When discounting we use the risk-free rate of 5%. We now proceed, as before.

Assume we have reached year 2 and we have yet to exercise our option to contract. The terminal values in year 2 are unaffected here by the change in probabilities and discount rate. The values for states $\{UU, UD, DD\}$ are, respectively, $\{149.18, 102, 82.22\}$. (If we have not contacted operations by year 2 we should contract them if we reach state $UD$ or $DD$.) Now let’s move back to year 1, and suppose we have not exercised our option to contract. In state $U$ it would not pay to contract. Its value will be
Let the NP V be 1. (a) Now consider state D. If we choose to contract now, then the value would be 0.6(81.87) + 42 = 91.09—this has not changed. We can, however, choose not to contract, and then the value according to ROA would be \( \frac{0.574(122.93) + 0.426(91.09)}{1.05} = 89.12 \). The best choice is therefore to contract, and the associated PV is still 91.09. Now let’s move back to year 0. Its PV according to ROA is \( \frac{0.574(122.93) + 0.426(91.09)}{1.05} = 104.16 \), which gives an NPV of 104.16 − 80 = 24.16. The correct value of the option to contract is 4.16 for Project 1. We should contract operations if we reach state D in year 1.

6. DTA recommends Project 1 since 22.47 > 22.36, whereas ROA recommends Project 2 since 24.16 > 23.92. (They are close.) When to contract changes, too.

Option to Abandon Operations

1. (a) The PV event tree has 10 “nodes”, which we shall label, respectively, as \{0, U, D, UU, UD = DU, DD, UUU, UUD = UDU = DUU, UDD = DUD = DDU, DDD\}. The value for \( u \) is \( e^{0.30} = 1.3499 \) and \( d \) is \( e^{-0.30} = 0.7408 \). The PV’s corresponding to each state are \{100, 134.99, 74.08, 182.21, 100, 54.88, 245.96, 134.99, 74.08, 40.66\}.

(b) NPV is 100 − 108 = −8 < 0, so reject the project without flexibility.

(c) Let \( p \) denote the probability for the “up” state for the project. We must have 134.99\( p \) + 74.08(1 − \( p \)) = 1.15, which implies \( p = 0.672 \).

2. (a) We work backwards through the PV event tree. Suppose it is year 3 and we have yet to execute the option to abandon. This option will not be executed in states UUU and UUD, and so their PV values remain the same. In states UDD and DDD it does pay to execute the option, and the PV values in both states are 90. Now go back to year 2. In state UU it does not pay to execute the option, and its PV value will remain the same. It also does not pay to execute the option in state UD; its PV value is calculated as \( \frac{0.672(134.99) + 0.328(90)}{1.15} = 104.55 \). In state DD it should be clear that the option should be executed, as it is better to receive the 90 now instead of next year. (In both future states from state DD the PV value is 90.) Now go back to year 1. In state U the option will not be executed; its PV value is calculated as \( \frac{0.672(182.21) + 0.328(104.55)}{1.15} = 136.29 \). Now consider state D. If we execute the option now we receive 90. However, we can choose not to execute now; the PV value if we do not execute is calculated as \( \frac{0.672(104.55) + 0.328(90)}{1.15} = 86.76 \). Thus, the option to abandon should be executed in state D, and its PV value is 90. In year 0 the PV value is calculated as \( \frac{0.672(136.29) + 0.328(90)}{1.15} = 105.31 \). The NPV of the project is therefore 105.31 − 108 = −2.69 < 0, and so the project would be rejected.

(b) Abandon the project should state D be reached in year 1.

(c) The change in NPV is −2.69 − (−8) = 5.31, which represents the value of the option to abandon.
3. The difference in using ROA is we will use the risk-neutral probabilities and the risk-free rate to perform the discounted expectation for valuation purposes when working backwards through the PV tree. Using ROA requires that an underlying marketable security be identified that can be used to construct the replicating portfolio; here, we take the project without flexibility as such a security. The project and the bond complete the market, i.e., span all possible valuations on the event tree, which then implies risk-neutral valuation will work. For this project the risk-neutral probability \( p \) must satisfy \\
\[
\frac{134.99p + 74.08(1-p)}{1.05} = 100, \\
\]
which implies \( p = 0.508 \). We now proceed, as before.

We work backwards through the PV event tree. Assume we have reached year 3 and we have yet to exercise our option to abandon. The terminal values in year 3 are unaffected here by the change in probabilities and discount rate. The values for states \( \{UUU, UUD, UDD, DDD\} \) are, respectively, \( \{245.96, 134.99, 90, 90\} \). (If we have not abandoned operations by year 3 we should abandon them if we reach state \( UDD \) or \( DDD \).) Now go back to year 2, and suppose we have not exercised our option to abandon. In state \( UU \) it does not pay to execute the option, and its PV value will remain the same. It also does not pay to execute the option in state \( UD \); its PV value is calculated as \\
\[
\frac{0.508(134.99) + 0.492(90)}{1.05} = 107.48. \\
\]
In state \( DD \) it should be clear that the option should be executed, as it is better to receive the 90 now instead of next year. (In both future states from state \( DD \) the PV value is 90.) Now go back to year 1. In state \( U \) the option will not be executed; its PV value is calculated as \\
\[
\frac{0.508(182.21) + 0.492(107.48)}{1.15} = 138.52. \\
\]
Now consider state \( D \). If we execute the option now we receive 90. However, we can choose not to execute now; the PV value if we do not execute is calculated as \\
\[
\frac{0.508(107.48) + 0.492(90)}{1.05} = 94.17. \\
\]
Thus, the option to abandon should not be executed in state \( D \), and its PV value is 94.17. In year 0 the PV value is calculated as \\
\[
\frac{0.508(138.52) + 0.492(94.17)}{1.05} = 111.14. \\
\]
The NPV of the project is therefore \( 111.14 - 108 = 3.14 > 0 \), and so the project would be accepted.

4. ROA accepts the project; DTA rejects the project. ROA would not abandon the project if state \( D \) is reached in year 1, whereas DTA would.

**Option to Default on Planned Investments or The Installment Option**

1. (a) The PV event tree has 6 “nodes”, which we shall label, respectively, as \( \{0, U, D, UU, UD = DU, DD\} \). The value for \( u \) is \( e^{0.80} = 2.2255 \) and \( d \) is \( e^{-0.80} = 0.4493 \). The PV’s corresponding to each state are \( \{100, 222.55, 44.93, 495.30, 100, 20.19\} \).

(b) \( NPV = 100 - 102 = -2 < 0 \). Reject the project without flexibility.

2. (a) Let \( p \) denote the risk-neutral probability for the “up” state. We must have \\
\[
222.55p + 44.93(1-p) = 1.05, \\
\]
which implies \( p = 0.338 \).

We must work backwards through the PV event tree. Assume we have reached year 2 and we have yet to exercise our option to default on planned investments. In states \( UU \) and \( UD \) it pays to make our final planned investment, and so their
values become, respectively, 462.22 and 66.92. It does not pay to make the final investment in state \(DD\), and so its value becomes 0. Now let’s move back to year 1, and suppose we have not exercised our option to default on planned investments. In state \(U\) it would pay to make the investment. Its (new) \(PV\) would be
\[
\frac{0.338(462.22) + 0.662(66.92)}{1.05} = 21 = 169.98.
\]
Now consider state \(D\). If we choose to continue with the project now, then the value would be
\[
\frac{0.338(66.92) + 0.662(0)}{1.05} = 21 = 0.54.
\]
Since this value is positive, we should continue in state \(D\). Now let’s move back to year 0. Its \(PV\) is
\[
\frac{0.338(169.98) + 0.662(0.54)}{1.05} = 55.06,
\]
which gives an \(NPV\) of 55.06 − 52 = 3.06. As the \(NPV\) is positive, we accept the project. The value of the default option is
\[
3.06 - (-2) = 5.06.
\]
If state \(DD\) is reached, then the final investment should not be made.

(b) We must work backwards through the \(PV\) event tree. Assume we have reached year 2 and we have yet to exercise our option to default on planned investments. In states \(UU\) and \(UD\) it pays to make our final planned investment, and so their values become, respectively, 440.17 and 44.87. It does not pay to make the final investment in state \(DD\), and so its value becomes 0. Now let’s move back to year 1, and suppose we have not exercised our option to default on planned investments. In state \(U\) it would pay to make the investment. Its (new) \(PV\) would be
\[
\frac{0.338(440.17) + 0.662(44.87)}{1.05} = 31.5 = 138.48.
\]
Now consider state \(D\). If we choose to continue with the project now, then the value would be
\[
\frac{0.338(44.87) + 0.662(0)}{1.05} = 31.5 = -16.61.
\]
Since this value is negative, we should not make the planned investment in state \(D\). Now let’s move back to year 0. Its \(PV\) is
\[
\frac{0.338(138.48) + 0.662(0)}{1.05} = 44.58,
\]
which gives an \(NPV\) of 44.58 − 22 = 22.58. As the \(NPV\) is positive, we accept the project. The value of the default option is 22.58 − (-2) = 24.58. If state \(D\) is reached, then the project should be terminated.

Option on Equity and Valuation of Debt

1. (a) The Firm Value event tree has 10 “nodes”, which we shall label, respectively, as \(\{U, D, UU, UD = DU, DD, UUU, UUD = UDU = DUU, UDD = DU' D = DDU, DDD\}\). The value for \(u\) is \(e^{0.20} = 1.2214\) and \(d\) is \(e^{-0.20} = 0.8187\). The Firm Values corresponding to each state are
\[
\{100, 122.14, 81.87, 149.18, 100, 67.03, 182.21, 122.14, 81.87, 54.88\}.
\]
(b) Let \(p\) denote the risk-neutral probability for the “up” state. We must have 122.14\(p\) + 81.87(1 − \(p\)) = 1.05, which implies \(p = 0.574\).

We work backwards through the Firm Value event tree to construct the Equity Value event tree. In year 3 if the firm value exceeds the face value of debt, then the debt holders are repaid; otherwise, the firm defaults on its debt. Thus, in year 3 the equity value of the firm for states \(\{UUU, UUD, UDD, DDD\}\) are, respectively, \(\{92.21, 32.14, 0, 0\}\). To value the equity at previous nodes, we use discounted risk-neutral expectation. For example, the equity value in state \(UU\) is calculated as
Since the value of the firm $V$ equals $S + B$, where $B$ denotes the value of debt, we have that the current value of debt $B = 100 - 26.75 = 73.25$. The current yield on this debt is therefore $(90/73.25)^{1/3} - 1 = 0.071$ or 7.1%.

(d) The option to acquire the equity of the XYZ firm for 60 is “in the money” only when state $UUU$ is reached; therefore, the value of this option today is simply
\[
(0.574)^3(92.21-60) = 5.26.
\]

2. The Firm Value event tree has 10 “nodes”, which we shall label, respectively, as \{0, $U, D, U, U, U, U, U, U, D, D, U, U, U, U, D, D, U, D, 0\}. To value the equity at previous nodes, we use discounted risk-neutral expectation. For example, the equity value in state $UU$ is calculated as $\frac{0.574(92.21)+0.426(32.14)}{1.05} = 63.45$. The equity values at the 6 nodes are, respectively, \{17.57, 0, 41.81, 9.60, 26.75\}. Thus, the current equity value $S$ at time 0 is 26.75.

Since the value of the firm $V$ equals $S + B$, where $B$ denotes the value of debt, we have that the current value of debt $B = 100 - 26.75 = 73.25$. The current yield on this debt is therefore $(90/73.25)^{1/3} - 1 = 0.071$ or 7.1%.

We work backwards through the Firm Value event tree to construct the Equity Value event tree. In year 3 the equity value of the firm for states \{UUU, UUD, UDD, DDD\} are, respectively, \{514.96, 92.21, 0, 0\}. To value the equity at previous nodes, we use discounted risk-neutral expectation. For example, the equity value in state $UU$ is calculated as $\frac{0.574(514.96)+0.426(92.21)}{1.05} = 246.45$. The equity values in states $UD, DD, U, D, 0$ are, respectively, \{34.60, 0, 112.45, 12.98, 49.69\}. Thus, the current equity value $S$ at time 0 is 49.69. (It’s higher when the volatility is higher.)

Since the value of the firm $V$ equals $S + B$, where $B$ denotes the value of debt, we have that the current value of debt $B = 100 - 49.69 = 50.31$. (It’s lower when the volatility is higher.) The current yield on this debt is therefore $(90/50.31)^{1/3} - 1 = 0.214$ or 21.4%.

The option to acquire the equity of the XYZ firm for 60 is “in the money” only when states $UUU$ or $UUD$ are reached; therefore, the value of this option today is simply
\[
\frac{(0.394)^3(514.96-60)+(3)(0.394)^2(0.606)(92.21-60)}{1.05} = 31.89.
\] (The call option value is higher with higher volatility.)

Option to Switch Use

1. To determine the $PV$ event tree we first must determine the Cash Flow event tree. The values at the 6 nodes \{0, $U, D, U, U, U, U, U, D, D, U, U, U, U, D, D, U, D, 0\} are \{100, 140, 71.42, 196, 100, 51.02\}. Now we are in position to compute the present values of future discounted expected cash flows. The $PV$’s at states \{UU, UDD, DD\} in year 2 correspond to their net cash flows, namely, they are \{196, 100, 51.02\}, respectively. Now consider state $U$ in year 1. Its value is calculated as $\frac{0.5(196)+0.5(100)}{1.09} + 140 = 275.78$. Similarly, the value in state $D$ in year
2 is calculated as \( \frac{0.5(100) + 0.5(51.02)}{1.09} + 71.42 = 140.70 \). In year 0 the PV is calculated as \( \frac{0.5(275.78) + 0.5(140.70)}{1.09} + 100 = 291.05 \).

2. To determine the PV event tree we first must determine the Cash Flow event tree. The values at the 6 nodes \{0, U, D, UD, DU, DD\} are \{100, 110, 90.91, 121, 100, 82.64\}. It is tempting at this point to proceed as we did for technology X; however, the cash flow stream for Y is clearly less volatile than that for X so the cost of capital should be lower.

In lieu of estimating the cost of capital for Y, since X has been in use, we will use the PV event tree as our marketed security (along with the bond) upon which to build a replicating portfolio. Accordingly, the risk-neutral probability \( p \) satisfies \( \frac{275.78p + 140.70(1-p)}{1.05} = 191.05 \), which implies \( p = 0.443 \). It will be the same at every node. (This value for the risk-neutral probability is definitely not the same as the one that would be calculated using the Cash Flow event tree directly, which would give a probability of \( \frac{105-71.42}{140-71.42} = 0.4896 \).) Now we are in position to properly value the present values of future discounted expected cash flows for technology Y.

The PV’s at states \{UU, UD, DD\} in year 2 correspond to their net cash flows, namely, they are \{121, 100, 82.64\}, respectively. Now consider state U in year 1. Its value is calculated as \( \frac{0.443(121) + 0.557(100)}{1.05} + 110 = 214.10 \). Similarly, the value in state D in year 2 is calculated as \( \frac{0.443(214.10) + 0.557(82.64)}{1.05} + 90.91 = 176.94 \). In year 0 the PV is calculated as \( \frac{0.443(214.10) + 0.557(176.94)}{1.05} + 100 = 284.19 \).

To determine the appropriate cost of capital \( k_Y \) for technology Y we use the objective probabilities; that is, \( k_Y \) satisfies \( 0.5(214.10) + 0.5(176.94) = (184.19)(1 + k_Y) \), which implies \( k_Y = 6.15\% \). It will be the same at any node along the tree. (If one uses the risk-neutral probabilities, the cost of capital would be simply 5%.)

3. Determine the PV event tree corresponding to technology Z assuming the changeover costs are zero. (Assume the PV event trees for X and Y correspond to the same states of nature.)

With no changeover costs it is clearly optimal to use the technology with the highest net cash flow each period depending on the state of nature. In year 2 the PV’s corresponding to states \{UU, UD, DD\} are, respectively, \{196, 100, 82.64\}. Now consider state U in year 1. Its value is calculated as \( \frac{0.443(196) + 0.557(100)}{1.05} + 140 = 275.78 \). The PV value for state D in year 1 is calculated as \( \frac{0.443(275.78) + 0.557(176.94)}{1.05} + 90.91 = 176.94 \). In year 0 the PV value is calculated as \( \frac{0.443(275.78) + 0.557(176.94)}{1.05} + 100 = 310.22 \).

4. Determine the PV event tree corresponding to technology Z with the changeover costs. (Assume the PV event trees for X and Y correspond to the same states of nature.)

When there are changeover costs the calculations become more involved, as it is necessary to know what technology currently in operation in order to decide if it is optimal to switch use. Here is how we do it. Let \( V_X(\cdot) \) (resp. \( V_Y(\cdot) \) denote the value function in year \( t \) given we enter year \( t \) using technology X (resp. Y). In state UU in year 2 it pays to
switch from $Y$ to $X$, and in state $DD$ in year 2 it pays to switch from $X$ to $Y$; otherwise, it does not pay to switch use. Thus, $V_{2X}(UU) = 196; V_{2X}(UD) = 100; V_{2X}(DD) = 67.64$, and $V_{2Y}(UU) = 186; V_{2Y}(UD) = 100; V_{2Y}(DD) = 82.64$. Now consider year 1. In each state $U$ or $D$ we must decide whether to switch assuming that we entered each state using either technology $X$ or $Y$. That is, we must perform 4 separate analyses, as follows:

\[
V_{1X}(U) = \max\{140 + \frac{0.443V_{2X}(UU) + 0.557V_{2X}(UD)}{1.05}, (110 - 15) + \frac{0.443V_{2Y}(UU) + 0.557V_{2Y}(UD)}{1.05}\} = 275.78;
\]

\[
V_{1Y}(U) = \max\{110 + \frac{0.443V_{2Y}(UU) + 0.557V_{2Y}(UD)}{1.05}, (140 - 10) + \frac{0.443V_{2X}(UU) + 0.557V_{2X}(UD)}{1.05}\} = 265.78;^*
\]

\[
V_{1X}(D) = \max\{71.42 + \frac{0.443V_{2X}(DU) + 0.557V_{2X}(DD)}{1.05}, (90.91 - 15) + \frac{0.443V_{2Y}(DU) + 0.557V_{2Y}(DD)}{1.05}\} = 161.94;^*
\]

\[
V_{1Y}(D) = \max\{90.91 + \frac{0.443V_{2Y}(DU) + 0.557V_{2Y}(DD)}{1.05}, (71.42 - 10) + \frac{0.443V_{2X}(DU) + 0.557V_{2X}(DD)}{1.05}\} = 176.94.\]

(The asterisk symbol denotes the decision to switch-use.) Now in year 0 we compute:

\[
V_{0X} = \max\{100 + \frac{0.443V_{1X}(U) + 0.557V_{1X}(D)}{1.05}, (100 - 15) + \frac{0.443V_{1Y}(U) + 0.557V_{1Y}(D)}{1.05}\} = 302.26;\]

\[
V_{0Y} = \max\{100 + \frac{0.443V_{1Y}(U) + 0.557V_{1Y}(D)}{1.05}, (100 - 10) + \frac{0.443V_{1X}(U) + 0.557V_{1X}(D)}{1.05}\} = 306.00.\]

At time 0 the $PV$ of technology $Z$ is 302.26 if its initial state is technology $X$ or 306.00 if its initial state is technology $Y$. Obviously, if one has a choice, it would be preferable to begin in technology $Y$.

5. The $NPV$ for technology $X$ is 191.05, the $NPV$ for technology $Y$ is 184.19, and the $NPV$ for technology $Z$ is either 192.26 or 196.00. In any case, technology $X$ is preferred to technology $Y$, and technology $Z$ is preferred to $X$ regardless of its initial state.