An Optimal Investment Problem

An investor hires your firm to manage his portfolio for the next two years. Here is the relevant information:

1. There are only two securities in which to invest: a non-dividend paying stock and the money market. The risk-free rate is 10%.

2. The price of the stock now at time $t = 0$ is $S_0 = 100$. Its price process follows a binomial lattice with $u = 1.25$ and $d = 0.80$. The period length is 1 year. The objective probability of an “up move” is 0.60.

3. The initial investment $X_0 = 1000$; that is, the investor gives you 1000 with which to invest. He is only concerned with his wealth at time $t = 2$, so he will not provide any cash infusions nor will he withdraw any amount at time $t = 1$.

4. Your company may use the allocation of 1000 to build a portfolio of $S$ and $M$ today, and may subsequently rebalance this portfolio at time $t = 1$. (No rebalancing occurs at time $t = 2$.) The dynamic trading strategy your firm uses must be self-financing, i.e., the value of the rebalanced portfolio at any state must equal the value of the portfolio at that state before rebalancing.

5. The investor is risk-averse. His utility function for his wealth $X_2$ at time 2 is given by $U(X_2) = \ln X_2$.

6. Describe your firm’s investment plan over time, the payoffs the investor will receive at each state, and the value of the investor’s maximum expected utility. Be prepared to describe your algorithm or software your firm used to solve this problem.