Some cut-generating functions for second-order conic sets

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1
Introduction and Motivation

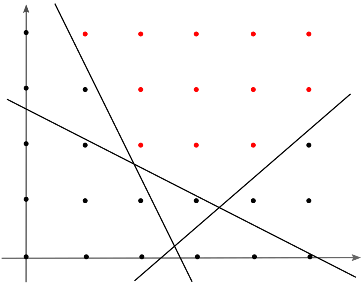
1.1
Conic integer programs

Standard integer program

Consider the following integer program:

$$\begin{array}{ll} \min & c^\top x \\ \text{s.t.} & Ax \geq b \\ & x \in \mathbb{Z}^n_+ \end{array} \quad \Leftrightarrow \quad \begin{array}{ll} \min & c^\top x \\ \text{s.t.} & Ax - b \geq 0 \\ & x \in \mathbb{Z}^n_+ \end{array}$$

where $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$ and $c \in \mathbb{R}^n$.



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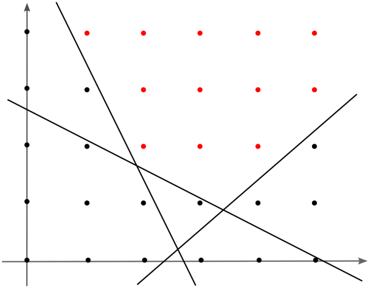

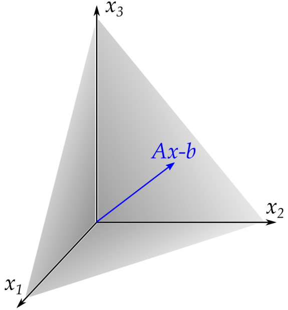

~~Standard~~ conic integer program

Consider the following conic integer program:

$$\begin{aligned} \inf \quad & c^\top x \\ \text{s.t.} \quad & Ax - b \in K \\ & x \in \mathbb{Z}^n_+, \end{aligned}$$

$K \subseteq \mathbb{R}^m$ is a *regular cone*: closed, convex, pointed and full dimensional.

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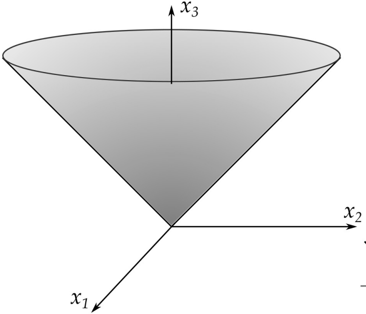
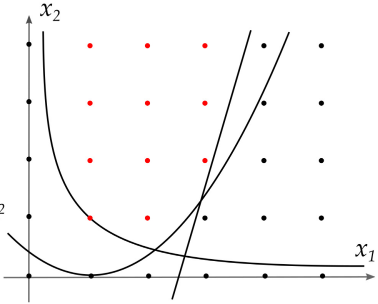
For instance, K is the second-order cone:





1.2

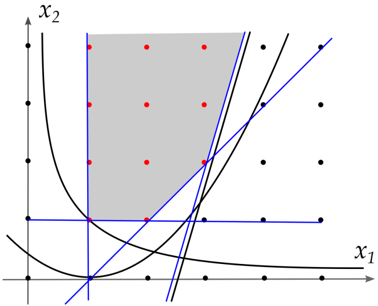
Cutting planes: Cut-generating functions

Cutting planes approach

Cutting planes are linear inequalities valid for:

$$\operatorname{conv}\{x \in \mathbb{Z}_+^n \mid Ax \succeq_K b\},$$

in order to improve the dual bound over the standard convex relaxation.



How can we generate these cuts in a systematic fashion?

Cuts via cut-generating functions

Cut-generating functions

Let m be the number of rows of the A matrix. Consider $f : \mathbb{R}^m \to \mathbb{R}$ such that

1. $f(u) + f(v) \geq f(u + v)$ for all $u, v \in \mathbb{R}^m$ (subadditive);
2. $u \succeq_K v \;\Rightarrow\; f(u) \geq f(v)$ (non-decreasing w.r.t. K);
3. $f(0) = 0$.

Denote the set of functions satisfying (1.), (2.), (3.) as $\mathcal{F}_K$.

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A valid inequality for the convex hull of the set of feasible solutions to

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$$\sum_{j=1}^{n} f(A^j)x_j \geq f(b).$$

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Proof: $\hat{x}$ is feasible $\Longrightarrow \sum_{j=1}^{n} f(A^j)\hat{x}_j \underbrace{\geq}_{(1.),(3.)} f\left(\sum_{j=1}^{n} A^j \hat{x}_j\right) \underbrace{\geq}_{(2.)} f(b).$

How good are cut-generating functions?

Question:

$$\overline{\text{conv}}\left\{x \in \mathbb{Z}^n_+ \mid Ax \succeq_K b\right\} \stackrel{?}{=} \bigcap_{f \in \mathcal{F}_K} \{x \in \mathbb{R}^n_+ \mid f(A^j)x_j \geq f(b)\} \qquad (1)$$

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Table: Strong Duality

K	Conditions	
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K	Conditions	
$\mathbb{R}^m_+$ (standard IP)	A is rational data matrix	Johnson (1973, 1979), Jeroslow (1978, 1979)
K is arbitrary regular cone	'Discrete Slater' condition: $\exists \hat{x} \in \mathbb{Z}^n_+$ s.t. $A\hat{x} - b \in \text{int}(K)$	Morán, D., Vielma (2012)

More structure known on $\mathcal{F}_K$ when $K = \mathbb{R}^m_+$

1. We only need a subset $C^m \subseteq \mathcal{F}_{\mathbb{R}^m_+}$ (Blair, Jeroslow (1982)):

$$\text{conv}\left\{x \in \mathbb{Z}^n_+ \mid Ax \succeq_{\mathbb{R}^m_+} b\right\} = \bigcap_{f \in \cancel{\mathcal{F}_{\mathbb{R}^m_+}}\, C^m} \{x \in \mathbb{R}^n_+ \mid f(A^j)x_j \geq f(b)\},$$

where C^m is a set of functions:

- C^m contains (non-decreasing) linear functions.
- $f, g \in C^m$, then $\alpha f + \beta g \in C^m$, where $\alpha, \beta \geq 0$.
- $f \in CG^m$, then $\lceil f \rceil \in C^M$.

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2. The simplest function in this family are the famous Chvátal-Gomory cuts:

$$f_\lambda(u) = \lceil \lambda u \rceil,$$

where $\lambda \in \mathbb{R}^m_+$.

Question: Can we identify structured sub-family of CGFs that already provide the integer hull for more general K?

2
Linear composition functions (LCF)

2.1
Linear composition functions (LCF): Definition

Linear composition functions (LCF)

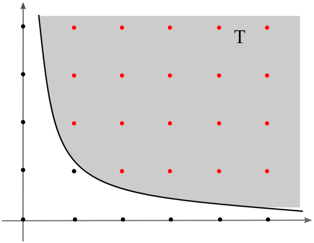
(Dual cone $K^* := \{y \mid \langle x, y \rangle \geq 0 \forall x \in K\}$.)

Linear composition functions (LCF)

For $w^1, w^2, \cdots, w^p \in K^*$ define $f : \mathbb{R}^m \to \mathbb{R}$ as

$$f(v) = g((w^1)^\top v, (w^2)^\top v, \ldots, (w^p)^\top v),$$

where $g \in \mathcal{F}_{\mathbb{R}^p_+}$. Then, $f \subseteq \mathcal{F}_K$.



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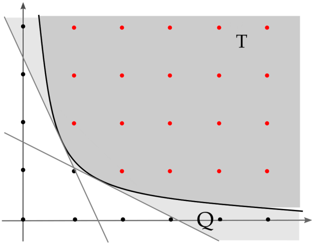

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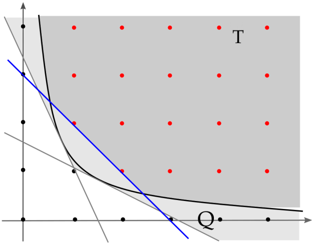


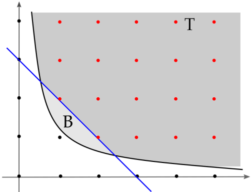
2.2
Linear composition functions and bounded cut-off region

Which cuts can we obtain using LCFs

Theorem

Suppose $T = \{x \in \mathbb{R}^n \mid Ax \succeq_K b\}$ has non-empty interior. Let $\pi^\top x \geq \pi_0$ be a valid inequality for the integer hull of T. Assume $B := \{x \in T \mid \pi^\top x \leq \pi_0\}$ is **bounded**. *Then, there exist $w^1, w^2, \ldots, w^p \in K^*$, such that*

1. *$\pi^\top x \geq \pi_0$ is a valid inequality for the integer hull of $Q = \{x \in \mathbb{R}^n \mid (w^i)^\top Ax \geq (w^i)^\top b,\ i \in [p]\}$, with*
2. *$(w^i)^\top A$ rational and*
3. *$p \leq 2^n$.*



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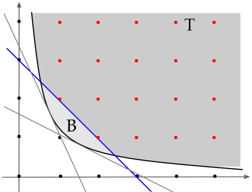


Proof Sketch

1. Because B is bounded, it is possible to find dual multipliers $w^1, \ldots, w^q$ such that

$$G := \underbrace{\{x \,|\, (w^i)^\top Ax \geq (w^i)^\top b\}}_{\text{Outer approx of T}} \cap \underbrace{\{x \,|\, \pi^\top x \leq \pi_0\}}_{\text{Cut-off region}} \text{ is bounded.}$$

2. If $\text{int}(G) \cap \mathbb{Z}^n \neq \emptyset$, add additional finite separating hyperplanes and update G.
3. Standard parity argument to say that we need only 2^n these inequalities to obtain Q.

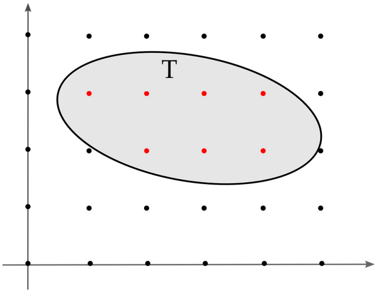
Lemma

Let C be a closed convex set. If $\text{int}\left((\text{rec.cone}(C))^*\right) \neq \emptyset$ and $z \notin C$, then there exists $\pi \in \mathbb{Q}^n$ such that

$$\pi^\top z < \pi_0 \leq \pi^\top x \;\forall x \in C.$$

Linear composition functions are sufficient for compact sets

In particular, if T is compact and has non-empty interior, then linear composition functions describe the integer hull of T, using no more than 2^n dual multipliers at a time.



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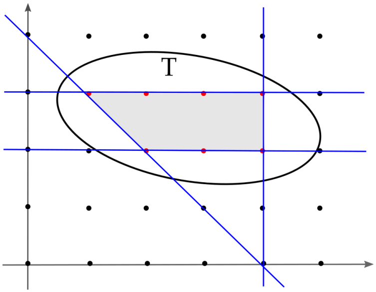
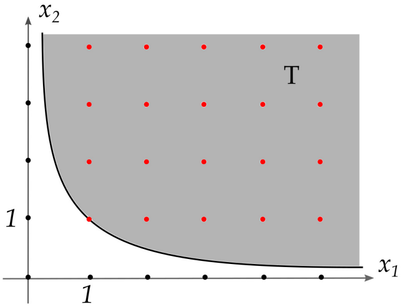


2.2
Linear composition functions are not enough

Epigraph of a standard hyperbola

Let

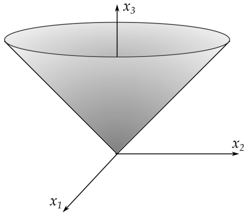
$$T = \{(x_1, x_2) \in \mathbb{R}^2_+ : x_1 x_2 \geq 1\}.$$



This set is conic representable.

Second-order cone

$$\mathcal{L}^m := \left\{ x \in \mathbb{R}^m \mid \sqrt{x_1^2 + x_2^2 + \cdots + x_{m-1}^2} \leq x_m \right\}$$



If $K = \mathcal{L}^3$, then

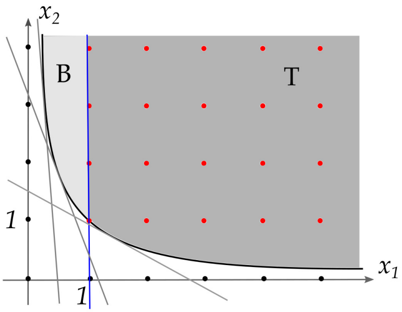
$$T = \{(x_1, x_2) \in \mathbb{R}^2_+ : x_1 x_2 \geq 1\} = \{x \in \mathbb{Z}^2_+ : Ax \succeq_K b\},$$

where

$$A = \begin{bmatrix} 0 & 0 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}.$$

Epigraph of a standard hyperbola (cont.)

Clearly, $x_1 \geq 1$ is valid inequality for the integer hull of T.



However, every linear outer approximation for T contains integer points of the form $(0, k)$.

Thus, linear composition functions will never produce the cut $x_1 \geq 1$!

3
A new class of cuts

3.1
A new class of cuts: Description

The new CGF

Theorem

Consider the function $f_\gamma : \mathbb{R}^m \to \mathbb{R}$ defined as:

$$f_\gamma(v) = \begin{cases} \gamma^\top v + 1 & \text{if } v_j \neq 0 \text{ and } \gamma^\top v \in \mathbb{Z}, \\ \lceil \gamma^\top v \rceil & \text{otherwise,} \end{cases}$$

where $\gamma \in \Gamma_j \cup \text{interior}\,(\mathcal{L}^m)$ with $j \in \{1, 2, \cdots, m-1\}$ and

$$\Gamma_j := \{\gamma \in \mathbb{R}^m \mid \gamma_m \geq \sum_{i=1}^{m-1} |\gamma_i|,\ \gamma_m > |\gamma_j|\}.$$

Then, $f_\gamma \in \mathcal{F}_{\mathcal{L}^m}$.

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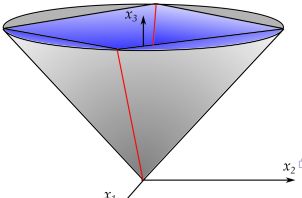


Example

Consider again the hyperbola $T = \{x \in \mathbb{Z}^2_+ : Ax \succeq_K b\}$, where

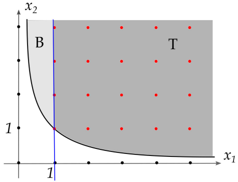
$$A = \begin{bmatrix} 0 & 0 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}.$$

Choose $j = 1$ and $\gamma = (0, 0.5, 0.5)$. Then,

$$f_\gamma(v) = \begin{cases} 0.5v_2 + 0.5v_3 + 1 & \text{if } v_1 \neq 0 \text{ and } 0.5v_2 + 0.5v_3 \in \mathbb{Z}, \\ \lceil 0.5v_2 + 0.5v_3 \rceil & \text{otherwise.} \end{cases}$$

Thus, $f_\gamma(A^1) = 1$, $f_\gamma(A^2) = 0$, $f_\gamma(b) = 1$, which yields

$$f_\gamma(A^1)x_1 + f_\gamma(A^2)x_2 \geq f_\gamma(b) \;\Leftrightarrow\; x_1 \geq 1.$$



Subadditivity

Proposition

For all $j \in \{1, 2, \cdots, m-1\}$ *and* $\gamma \in \Gamma_j \cup \text{interior}(\mathcal{L}^m)$,

$$f_\gamma(v) = \begin{cases} \gamma^\top v + 1 & \text{if } v_j \neq 0 \text{ and } \gamma^\top v \in \mathbb{Z}, \\ \lceil \gamma^\top v \rceil & \text{otherwise,} \end{cases}$$

is subbaditive, i.e., $u, v \in \mathbb{R}^m \Rightarrow f_\gamma(u+v) \leq f_\gamma(u) + f_\gamma(v)$.

Proof: If u or v fits the first clause, then we have

$$f_\gamma(u+v) \leq \left\lceil \gamma^\top (u+v) \right\rceil + 1 \leq \left\lceil \gamma^\top u \right\rceil + \left\lceil \gamma^\top v \right\rceil + 1 \leq f_\gamma(u) + f_\gamma(v).$$

Suppose u and v do not satisfy the first clause. Two cases:
1. $u + v$ does not fit in the first clause: it follows from subadditivity of $\lceil \cdot \rceil$;

Subadditivity (cont)

2. $u+v$ satisfies the first clause:

$$u_j + v_j \neq 0, \quad \gamma^\top(u+v) = \gamma^\top u + \gamma^\top v \in \mathbb{Z}. \tag{a}$$

Then, $u_j \neq 0$ or $v_j \neq 0$. WLOG assume $u_j \neq 0$. Hence

$$\gamma^\top u \notin \mathbb{Z}, \tag{b}$$

since u doesn't fit in the first clause. It follows from (a) and (b)

$$\gamma^\top v \notin \mathbb{Z}. \tag{c}$$

Combining (a), (b) and (c) we conclude

$$f_\gamma(u) + f_\gamma(v) = \lceil \gamma^\top u \rceil + \lceil \gamma^\top v \rceil = \gamma^\top u + \gamma^\top v + 1 = f_\gamma(u+v).$$

□

Non-decreasing w.r.t. $\mathcal{L}^m$

Proposition

For all $j \in \{1, 2, \cdots, m-1\}$ and $\gamma \in \Gamma_j \cup \operatorname{interior}(\mathcal{L}^m)$,

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is non-decreasing w.r.t. $\mathcal{L}^m$, i.e., $u \succeq_{\mathcal{L}^m} v \;\Rightarrow\; f_\gamma(v) \leq f_\gamma(u)$.

Proof: Let $w \in \mathcal{L}^m$ and $j \in [m-1]$. If $\gamma \in \mathcal{L}^m$, then $\gamma^\top w \geq 0$. If, in addition, $\gamma \in \Gamma_j \cup \operatorname{interior}(\mathcal{L}^m)$ and $w_j \neq 0$, then one can prove that $\gamma^\top w > 0$.
Suppose $u \succeq_{\mathcal{L}^m} v$. For $w := u - v$ we find

$$\gamma^\top u \geq \gamma^\top v, \qquad (\star)$$

which holds strictly whenever $u_j - v_j \neq 0$. Two cases:

Non-decreasing w.r.t. $\mathcal{L}^m$ (cont)

1. u fits in the first clause: using ($\star$) we obtain

$$f_\gamma(v) \leq \gamma^\top v + 1 \leq \gamma^\top u + 1 = f_\gamma(u).$$

2. u does not fit in the first clause:

(i) v does not fit in the first clause: using ($\star$) we obtain

$$f_\gamma(v) = \lceil \gamma^\top v \rceil \leq \lceil \gamma^\top u \rceil = f_\gamma(u).$$

(ii) v fits in the first clause: so $v_j \neq 0$ and $\gamma^\top v \in \mathbb{Z}$.

- If $u_j = 0$, then $u_j - v_j \neq 0$ and hence ($\star$) holds strictly. Thus,

$$f_\gamma(v) = \gamma^\top v + 1 \leq \lceil \gamma^\top u \rceil = f_\gamma(u).$$

- If $u_j \neq 0$, then $\gamma^\top u \notin \mathbb{Z}$ (since u does not satisfy the first clause), and using ($\star$) we obtain

$$\gamma^\top v \leq \gamma^\top u < \lceil \gamma^\top u \rceil \Rightarrow f_\gamma(v) = \gamma^\top v + 1 \leq \lceil \gamma^\top u \rceil = f_\gamma(u).$$

f_γ does not belong to $\mathcal{F}_{\mathcal{R}^m_+}$

Note that f_γ is not necessarily non-decreasing with respect to $\mathbb{R}^3_+$.

Indeed, let $j = 1$ and $\gamma = (0, \rho, \rho)$ where ρ is a positive scalar. Then

$$f_\gamma(v_1, v_2, v_3) = \begin{cases} \rho(v_2 + v_3) + 1 & \text{if } v_1 \neq 0 \text{ and } \rho(v_2 + v_3) \in \mathbb{Z}, \\ \lceil \rho(v_2 + v_3) \rceil & \text{otherwise.} \end{cases}$$

Consider the vectors $u = (0, 0, 1/\rho)$ and $v = (-1, 0, 1/\rho)$. Then $u \geq_{\mathbb{R}^3_+} v$, however

$$f_\gamma(u) = 1 < 2 = f_\gamma(v).$$

3.

f_γ in $\mathbb{R}^2$

Conic sections in $\mathbb{R}^2$

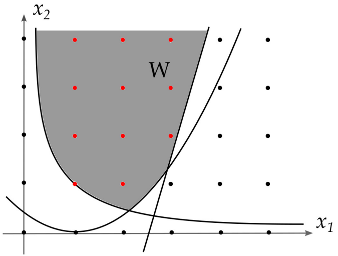
Consider

$$W = \bigcap_{i=1}^{m} W^i,$$

where

$$W^i = \{x \in \mathbb{R}^2 \mid A^i x \succeq_{\mathcal{L}^{m_i}} b^i\},$$

where $A^i \in \mathbb{R}^{m_i \times 2}$, $b^i \in \mathbb{R}^{m_i}$ and $\mathcal{L}^{m_i}$ is the second-order cone in $\mathbb{R}^{m_i}$. (W^i is a parabola, ellipse, branch of hyperbola, half-space)

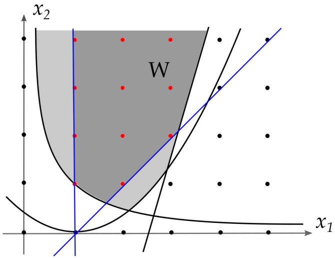


Theorem

Assume interior $W \neq \emptyset$ *and each constraint* $A^i x \succeq_{\mathcal{L}^{m_i}} b^i$ *in the description of* W *is either a half-space or a single conic section. Then the following statements hold:*

(*i*) *If* $W \cap \mathbb{Z}^2 = \emptyset$, *then this fact can be certified with the application of at most two inequalities generated from linear composition functions or some* f_γ;

(*ii*) *Assume* interior$(W) \cap \mathbb{Z}^2 \neq \emptyset$. *Every face* $\pi^\top x \geq \pi_0$ *of the integer hull of* W, *where* $\pi \in \mathbb{Z}^2$ *is non-zero, can be obtained with exactly one composition function or one* f_γ.



Conclusions

- Given a conic set with non-empty interior and a valid inequality for its integer hull, if the set cut-off is bounded, then the valid inequality can be obtained via composition functions;
- If the conic set is compact, then its integer hull can be described by composition functions;
- For sets that are second-order conic representable, we introduced a new interesting family of cut-generating functions, f_γ;
- In $\mathbb{R}^2$, the family f_γ combined with linear composition functions are enough to describe the integer hull of the underlying conic set, under minor assumptions.

Research questions

1. Is the closure wrt f_γ *locally polyhedral*?
2. Is the rank wrt f_γ finite?
3. Is there any sort of natural generalization of f_γ to other cones?

Thank you!