ISyE 6669 Solution of HW 4

1. (a)

$$Max \sum_{i=1,...,n} c_i x_i$$
$$s.t. \sum_{i=1,...,n} a_i x_i \le b$$
$$x_i \ge 0 \text{ for } i = 1, ..., n$$

(b) There is only one basic variable since there is only one constraint. Only one type of items will be an optimal basic solution.

(c) Since there is only one type of items will be in an optimal basic solution, for item i, we can take at most $\frac{b}{a_i}$ units. And the benefit will be $c_i \times (\frac{b}{a_i})$. Because $\frac{c_1}{a_1} \le \frac{c_2}{a_2} \le \ldots \le \frac{c_n}{a_n}$, we have $\frac{c_1}{a_1}b \le \frac{c_2}{a_2}b \le \ldots \le \frac{c_n}{a_n}b$ $(b \ge 0)$. Hence, $\frac{b}{a_n}$ units of item n is optimal solution.

3. (Primal)

$$Min\ 5x_1 + 2x_2 + x_3$$
$$s.t. x_1 + x_2 + 2x_3 \ge 2$$
$$2x_1 + x_2 \le 6$$
$$x_2 \le -1$$
$$x_1 \ge 0$$
$$x_3 \text{ unrestricted}$$

(Dual)

$$Max\ 2y_1 - 6y_2 + y_3$$
$$s.t. y_1 - 2y_2 \le 5$$
$$y_1 - y_2 - y_3 = 2$$
$$2y_1 = 1$$
$$y_1, y_2, y_3 \ge 0$$

4.

(a) Objective: minimize total production cost

subject to :min minutes at Mill 1 to 4

max minutes at Mill 1 to 4

demand constraints

(b) (ignored)

(c) nonzeros $x^*_{1,1} = 250, x^*_{2,1} = 150, x^*_{3,2} = 150, x^*_{4,2} = 80, x^*_{5,2} = 14, x^*_{5,4} = 176, x^*_{6,3} = 136.667, x^*_{6,4} = 53.333, x^*_{7,3} = 160, x^*_{8,3} = 150$

(d) nonzeros $v^*_4 = 0.1, v^*_7 = -.1833, v^*_9 = 0.1, v^*_{10} = 0.15, v^*_{11} = 0.15, v^*_{12} = 0.5, v^*_{13} = 0.2, v^*_{14} = 0.29, v^*_{15} = 0.4267, v^*_{16} = 0.7833$

(e) Substitute dual variable values in the constraints of part (b), and verify that all constraints are satisfied. Also, compute primal and dual objective function values, and check that they match at 334,867.

(f) Marginal costs will be dual variable values on demand constraints. For .5" solid this is $v^*_9 = 0.1$, for 1" solid $v^*_{10} = 0.15$, for 2" solid $v^*_{11} = 0.15$, for 4" solid $v^*_{12} = 0.5$, for .5" hollow $v^*_{13} = 0.2$, for 1" hollow $v^*_{14} = 0.29$, for 2" hollow $v^*_{15} = 0.4267$, and for 4" hollow $v^*_{16} = 0.7833$.

(g) The one shift minimum is limiting the solution because the dual price v_4 on the lower bound for Mill 4 is positive at value 0.1.

(h) The proposed changes would increase mill capacities from 360 thousand minutes per year to 504 thousand minutes. At Mill 3, 504 is above the range [278,367]. With constraint relaxation helping less and less, the saving will be at least the extension of the dual variable rate to the end of the range, or (-0.1833)(367-360)=1.283 thousand, and at most its extension over the full change or most (-0.1833)(504-360)=26.395 thousand savings; At Mill 4, there would be no change because the corresponding dual variable is $v^*_8 = 0$.

(i) Unit costs would have to be reduced to lower objective ranges $c^*_{1,4} = 0.11$, $c^*_{2,4} = 0.18$, $c^*_{3,4} = 0.21$, $c^*_{5,4} = 0.242$, $c^*_{6,4} = 0.324$, $c^*_{7,4} = 0.507$.

(j) Product 4 at Mill 4 would have dual constraint $1.0v_4 + 1.0v_8 + v_{12} \leq c_{4,4}$, and product 8 would have $1.0v_4 + 1.0v_8 + v_{16} \leq c_{8,4}$. Substituting optimal dual values gives $1.0(0.1) + 1.0(0) + (0.5) \leq c_{4,4}$, or requirement $c_{4,4} < 0.6$ on product 4, and $1.0(0.1) + 1.0(0) + (0.783) \leq c_{8,4}$, or $c_{8,4} < 0.883$ on product 8.