

**ISyE 6669 Homework #5, Due 11/29/2002**

- Every year, the United States Congress must approve a budget for the country. In order to be approved, the budget must get a majority of the votes in the Senate, a majority of votes in the House, and the signature of the President.

Most of the budget is easy to agree on – everyone supports things like basic operating expenses for the government, diplomats in foreign countries, and the armed forces; emergency relief funds in case of earthquakes, hurricanes, etc.; maintenance of the interstate highway system; and so on.

Other aspects of the budget, however, are less easy to agree on. For example, should there be a tax cut? Should the government spend money on defense research? Should they use some of the budget surplus to solidify social security? To pay off our debt to the United Nations? To give aid to farmers unable to make a living due to low food prices?

Because of monetary restrictions, the budget can have money allocated for at most 4 of the 5 issues.

One Senator has asked his staff to categorize all of Congress (Senators and members of the House), as well as the President, by their views on the five controversial issues. The Senator’s staff has compiled the following table.

	Issues					Number of Votes
	Tax Cut	Defense Research	Social Security	United Nations	Farm Aid	
<b>Senate (Total = 100)</b>						
Senate Group 1	Y	Y	N	N	Y	32
Senate Group 2	Y	Y	D	N	D	16
Senate Group 3	N	N	Y	Y	Y	30
Senate Group 4	D	Y	N	D	Y	22
<b>House of Representatives (Total = 435)</b>						
House Group 1	Y	Y	N	N	Y	140
House Group 2	Y	Y	D	N	D	75
House Group 3	N	N	Y	Y	Y	155
House Group 4	D	Y	N	D	Y	65
<b>President (Total = 1)</b>						
President	N	D	Y	Y	Y	1

**Table. Stance on Budget Issues: “Y” = yes, “N” = no, “D” = don’t care**

In order for a particular group to vote for a budget proposal, the proposal must allocate money in agreement with the group's views on *more than half* of the issues that the group cares about. For example, the Senate Group 2 only cares about 3 of the issues. Therefore, any budget proposal that agrees with their views on at least two out of the three issues they care about (allocating money for a Tax Cut and Defense Research, but not for the United Nations) will get their 16 votes in the Senate.

In order to be approved, a budget must get a majority of the votes (at least 51 out of 100) in the Senate, a majority of the votes (at least 218 out of 435) in the House of Representatives, and the approval of the President.

Formulate an integer program that the Senator could use to devise a budget proposal that would be approved by all three (Senate, House, and President) while funding as many budgetary issues as possible.

2. A soccer team is looking to sign free agents during the off-season. They have just hired a new coach, who is trying to decide whether the team should become an offense-oriented team or a defense-oriented team. If he decides to make the team offense-oriented, then the team should sign at least three times as many strikers as fullbacks. If he decides to make the team defense-oriented, then the team should sign at least two times as many fullbacks as strikers. The team can sign a total of up to 6 free agents.

Let  $x_s$  be the number of strikers the team signs, and let  $x_f$  be the number of fullbacks the team signs. Both  $x_s$  and  $x_f$  are integer variables. Let  $y$  be a binary variable that is 1 if the team is offense-oriented and 0 if it is defense-oriented.

Pick the *strongest* set of constraints that accurately describe all of the team's restrictions.

$$\begin{aligned} \text{(i)} \quad & x_s \geq 3x_f - 24(1 - y) \\ & x_f \geq 2x_s - 16y \\ & x_s + x_f \leq 6 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad & x_s \geq 3x_f - 24(1 - y) \\ & x_f \geq 2x_s - 16y \\ & x_s + x_f \leq 6 \\ & x_f \leq 1 + 5(1 - y) \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad & x_s \geq 3x_f - 18(1 - y) \\ & x_f \geq 2x_s - 12y \\ & x_s + x_f \leq 6 \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad & x_s \geq 3x_f - 18(1 - y) \\ & x_f \geq 2x_s - 12y \\ & x_s + x_f \leq 6 \\ & x_f \leq 1 + 5(1 - y) \end{aligned}$$

3. In the following branch-and-bound tree, answer the following questions:
- Which nodes do you still need to branch from? Why?
  - Which nodes do you not need to branch from? Why?
  - What is the gap between the best solution and the best bound found so far?
  - In what order were the three integer solutions found in the branch-and-bound process? Explain why your answer must be correct.

