ISyE 6669: HW 4

Handed out: 10/4/2002. Due: 10/16/2002.

1. A camper is considering taking n types of items in his knapsack for a camping trip. An item i weighs a_i lbs and earns a benefit to the camper of c_i units ($i = 1, \ldots, n$). His knapsack can hold at most b lbs.

   (a) Assuming that the camper can take fractional numbers of an item, formulate a linear program to maximize his benefit.

   (b) How many different type of items will be in an optimal basic solution of the above LP? Why?

   (c) Suppose that the following holds:

   $$\frac{c_1}{a_1} \leq \frac{c_2}{a_2} \leq \ldots \leq \frac{c_n}{a_n}.$$

   Show that it is optimal to take $\frac{b}{a_n}$ units of item n.

2. Napier Chemicals produces three types of chemicals: A, B, and C. The chemicals are produced by running two machines. Machine 1 cost \$4000 per hour to run, and yields 300 gallons of chemical A, 100 gallons of chemical B, and 100 gallons of chemical C. Machine 2 costs \$1000 per hour to run, and yields 100 gallons of chemical A, 100 gallons of chemical B, and 200 gallons of chemical C. Each day, Napier must produce at least 3000 gallons of chemical A, at least 500 gallons of B, and at least 2000 gallons of C.

   (a) Formulate a linear programming that Napier can use to find the cheapest way of producing the required daily amounts of each chemical.

   (b) Graphically determine the minimum cost that Napier will have to incur to produce the required chemicals. (Hint: if your formulation of part (a) has more than two variables, you might need a different formulation in order to do part (b).)

   (c) Show the first three iterations of the Simplex method on the LP in part (a). Clearly state your choice of the initial basic variables and show all work.

3. Write down the dual of the following linear

   $$\begin{aligned} \min \quad & 5x_1 + 2x_2 + x_3 \\ \text{s.t.} \quad & x_1 + x_2 + 2x_2 \geq 2 \\ & 2x_1 + x_2 \leq 6 \\ & x_1 \geq 0 \\ & x_2 \leq -1 \\ & x_3 \text{ unrestricted.} \end{aligned}$$

4. Problem 7-25 of the text book. Attempt this problem after the class discussion on interpreting dual solutions.