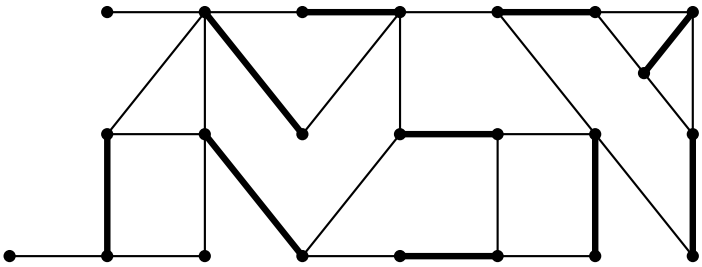
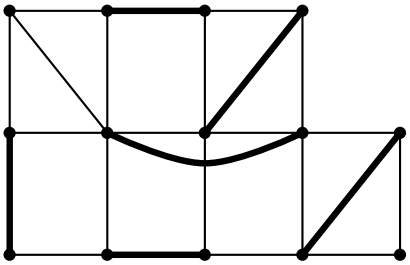
7686 - Adv. Combinatorial Optimization
Homework 2 (due Feb 13)

Note: You can discuss the homework with other students. But you have to write your own solutions. Also write the name of everyone you discussed with.

1. Apply the matching augmenting algorithm to the matchings in the following graphs.





2. Let $D = (V, A)$ be a simple directed graph and let $s, t \in V$. Let α be the minimum length of an $s - t$ path. Show that the maximum number of pairwise arc-disjoint $s - t$ paths is at most $\left(\frac{|V|}{\alpha}\right)^2$.

3. Let $D = (V, A)$ be a directed graph and $\mathcal{C}$ denote the set of cycles in D. Let $\chi(C) \in \mathbb{R}^A$ denote the indicator vector of cycle C. Show that non-negative circulations form the cone generated by $\{\chi(C) : C \in \mathcal{C}\}$.

4. Let G be a graph with vertices $v_1, \ldots, v_n$. Give an algorithm that, given a sequence $d_1, ..., d_n$, decides in polynomial time if G admits an orientation such that $\delta^{out}(v_i) = d_i$ for all $1 \leq i \leq n$.

5. Two persons are playing the following game on a graph. One after another the players choose vertices (one per turn) $v_1, v_2, v_3, \ldots$ so that v_i is adjacent to v_{i1} for all $i \geq 0$. The

last player which is able to choose a vertex wins. Prove that the first player has a winning strategy if and only if the graph has no perfect matching.