7686 - Adv. Combinatorial Optimization
Homework 1 (due 1/30)

1. (a) Find a maximum matching and minimum vertex cover in graph given in Figure 1.

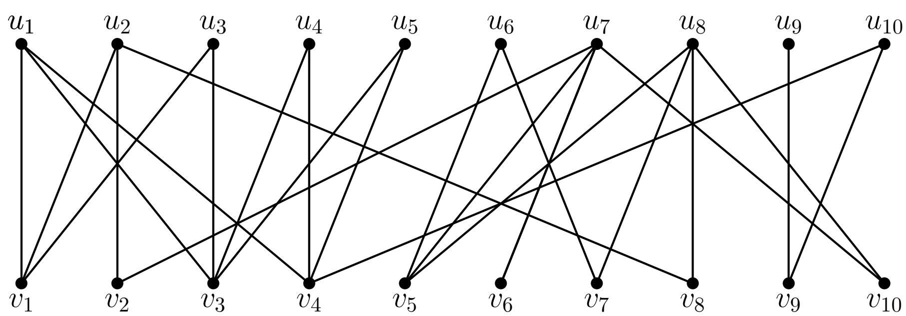

Figure 1: Find Maximum Matching and Minimum Vertex Cover

(b) Find the maximum (s, t)-flow and minimum (s, t)-cut in graph given in Figure 2. The numbers on the arcs denote the capacities.

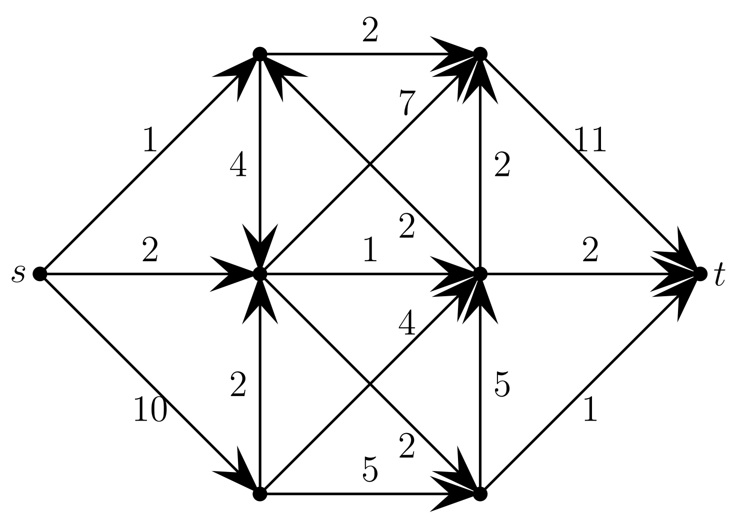

Figure 2: Find Maximum (s, t)-Flow and Minimum (s, t)-cut

2. Five mechanics, stationed in the cities A,B,C,D,E, have to perform jobs in the cities F, G,H, I, J. The jobs must be assigned in such a way to the mechanics that everyone gets one job and that the total distance traveled by them is as small as possible. The distances are given in the table below. Solve the assignment problem with the weighted matching algorithm.

	F	G	H	I	J
A	6	17	10	1	3
B	9	23	21	4	5
C	2	8	5	0	1
D	19	31	19	20	9
E	21	25	22	3	9

3. Let $G = (V, E)$ be a bipartite graph with bipartition $V = U \cup W$ and weight function $w :\to \mathbb{R}_+$. A matching M is perfect if every vertex is incident at one edge. Can you use minimum cost flow problem to find a minimum weight perfect matching? Can you rewrite the algorithm in the original graph itself without using the flow reduction on the lines of the maximum weight matching algorithm discussed in class.

4. An *independent set* in a graph G is a subset W of nodes such that no edge has both endpoint in W. Show that if $G = (V, E)$ is bipartite then maximum independent set is given by

$$\max\{\sum_{v \in V} x_v : x_u + x_v \leq 1 \text{ for all edge } uv \in E, x \geq 0\}.$$

Which min-max relation do you get from the dual? What about general graphs?

5. Let $G = (V, E)$ be a bipartite graph with colour classes U and W. Let $b : V \to Z_+$ be so that $\sum_{v \in U} b(v) = \sum_{v \in W} b(v) := t$. Show that there exists a subset F of E so that each vertex v of G is incident with exactly $b(v)$ of the edges in F, if and only if

$$t + |E(X)| \geq \sum_{v \in X} b(v)$$

for each subset X of V, where $E(X)$ denotes the set of edges with both endpoints in X.