Linear Inequalities
Homework 1 (Due 9/14)

You are free to discuss the homework with one other person but write your solutions on your own.

1. Use Gaussian elimination to solve the following system of linear equations.

   (a)

$$\begin{aligned} 2x_2 + x_3 &= -8 \\ x_1 - 2x_2 - 3x_3 &= 0 \\ -x_1 + x_2 + 2x_3 &= 3 \end{aligned}$$

   (b)

$$\begin{aligned} x_1 - 2x_2 - 6x_3 &= 12 \\ 2x_1 + 4x_2 + 12x_3 &= -17 \\ x_1 - 4x_2 - 12x_3 &= 22 \end{aligned}$$

2. Assume all data is rational in the following.

   (a) Give a polynomial time algorithm that given a system of equations $Ax = b$, finds a representation for *all* solutions.

   (b) Give a polynomial time algorithm that given a system of diophantine equations $Ax = b, x \in \mathbb{Z}^n$, finds a representation for *all* solutions.

3. (a) Determine the complete set of integral solutions of the following diophantine equation system:

$$\begin{aligned} 6x_1 - 6x_2 + 9x_3 &= -3 \\ 3x_1 + 2x_2 + 2x_3 &= 6 \end{aligned}$$

   (b) Consider the lattice $L(A) := \{Ax : x \in \mathbb{Z}^3\}$ generated by the columns of

$$A = \begin{pmatrix} 4 & 2 & 2 \\ 1 & 3 & 5 \end{pmatrix}$$

   Find two vectors that generate the lattice $L(A)$. Can two columns of A generate the lattice $L(A)$.

4. Show that the additive group generated by $\sqrt{2}$ and 1 is not a lattice.