

# Linear Inequalities

## Homework 0

Please use this homework to test your preparedness for the course. Please refer to chapter 1 of the textbook or basic references for a quick refresher.

1. Define the following
  - (a) Convex set, convex hull, convex cone, convex function, polytope, polyhedron, Ball in  $n$ -dimensions, Ellipsoid.
  - (b) Linear independence, Affine combination of vectors, Linear combination of vectors, Vector space, orthogonal vector space.
  - (c) Linear Subspace, determinant, inverse matrix, Gaussian Elimination, Row rank, Column rank, Rank of a matrix, Singular Matrix, Null space of a matrix, Orthogonal space, Non-singular matrix, Positive Definite Matrix, Positive Semi-definite matrix.
  - (d) Graph, Tree, Forest, connected graph, subgraph, planar graph, path, Hamiltonian path, Hamiltonian circuit, clique, matching, independent set, bipartite graph, coloring.
  - (e) Given a parameter  $n$ , what is  $O(n)$ ,  $o(n)$ ,  $\Omega(n)$ ,  $\omega(n)$  (Big-Oh, Small-Oh, Big-Omega, small-Omega).
  - (f) Class  $P$ , Class  $NP$ , Class  $NP$ -complete, class  $co-NP$ .
2. Show the following.
  - (a) For any matrix  $M$ , the row rank of  $M$  equals the column rank of  $M$ .
  - (b) Give an algorithm to check whether a graph is connected. Give an algorithm to find a shortest path between two vertices  $s$  and  $t$  in a graph.
  - (c) Given a matrix  $A$ , how does one compute the inverse of  $A$ , determinant of  $A$  and rank of  $A$ ?