An Automated Intensity-Modulated Radiation Therapy Planning System

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Abstract

We design and implement an intensity-modulated radiation therapy (IMRT) plan generation technology that effectively and efficiently optimizes beam geometry as well as beam intensities. Our approach is based on an existing linear programming based fluence map optimization model that approximates dose volume requirements using conditional value-at-risk (C-VaR) constraints. We show how the parameters of the C-VaR constraints can be used to control various metrics of treatment plan quality. Next, we develop an automated search strategy for parameter tuning. Finally, beam angle selection is integrated with fluence map optimization. The beam angle selection scheme employs a bi-criteria scoring of beam angle geometries and a selection mechanism to choose from among the set of non-dominated geometries. The overall technology is automated and generates several high-quality treatment plans satisfying dose prescription requirements in a single invocation and without human guidance. The technology has been tested on various real patient cases with uniform success.

Key words: IMRT; cancer radiation therapy; beam selection; linear programming

1. Introduction

External beam radiation therapy is used to treat over 500,000 cancer patients annually in the United States. This therapy uses multiple beams of radiation from different directions to cross-fire at a cancerous tumor volume in order to kill the cancer cells, thereby shrinking the tumor. Intensity-modulated radiation therapy (IMRT) is an advanced mode of high-precision external radiotherapy that utilizes computer-controlled mega-voltage x-ray accelerators to deliver precise radiation doses to targeted tissues. Rather than being treated with a large uniform beam, in IMRT the patient is treated by a series of beam shapes. Each shape is modeled as a collection of pencil beams (called
beamlets). For many types of cancer, such as prostate cancer and head and neck cancer, the use of intensity modulation allows a highly concentrated treatment of the tumor volume, while limiting the radiation dose to adjacent healthy tissue (see Veldeman et al. 2008 for comparisons of IMRT and non-IMRT treatments on different tumor sites).

Constructing an IMRT treatment plan that radiates the cancerous tumor volume (called target) without impacting adjacent normal structures (called organs at risk) is challenging. The planning is concerned with selecting a beam geometry and beamlet intensities to produce the best dose distribution that can be delivered efficiently. Because of the many possible beam geometries and the range of intensities, there is an infinite number of treatment plans, and consistently and efficiently generating high-quality treatment plans is complicated. A significant challenge is that there is no single metric to assess the quality of a treatment plan; therefore, trade-offs have to be made. Typically, a radiation oncologist specifies a set of dose-related requirements (a prescription) that has to be satisfied in any acceptable treatment plan. A number of measures have been developed to assess quality of acceptable treatment plans, e.g. the coverage of a target volume by a prescription dose, the conformity of a prescription dose around a target volume, and the highest and the lowest doses received by a target volume (Lee et al. 2006). The oncologist also considers dose-volume histograms (DVHs) depicting the dose distributions over the structures (both target volumes and organs at risk) associated with the treatment. The quality measures and DVHs are used to choose among acceptable treatment plans. Further complicating the evaluation of a treatment plan is the fact that the requirements and their relative importance are subjective, as are the underlying trade-offs they are trying to capture.

A variety of optimization based approaches have been developed for the different aspects (determining beam geometries, intensities, and delivery options) of the IMRT planning process. These approaches are iterative in nature and necessitate human evaluation and guidance. This makes the process time-consuming and costly. In this paper we propose an automated treatment plan generation technology that can efficiently provide multiple high quality plans for physician analysis and selection. Our technology addresses both the beam geometry and intensity aspects of the planning process. The final stage of the IMRT planning problem in which the calculated beamlet intensities are converted to a series of beam shapes for efficient delivery is not addressed in this paper. For a more detailed description, review, history, and physical basis of IMRT, see Bortfeld (2006), Boyer et al. (2001), Shepard et al. (1999) and Webb (2003).

Our proposed approach is based on an existing model (due to Romeijn et al. 2006) for optimizing
beamlet intensities (also called the fluence map) that uses conditional value-at-risk constraints (C-VaR constraints) to approximate constraints on the dose volume distribution. The use of C-VaR constraints has significant computational advantages as they can be handled using linear programming (LP) models, which are efficiently solved. However, the parameters controlling the C-VaR constraints have to be chosen carefully to get an accurate approximation of the dose-volume constraints.

We first show how the parameters of the C-VaR constraints can be used to control the coverage and conformity measures of plan quality. This is achieved by introducing C-VaR constraints on the target tumor volume as well as virtual critical structures. Virtual critical structures surround target volumes and are implemented to control the dose deposits specifically at the boundary of the target volume (Bahr et al. 1968, Price et al. 2003, Lee et al. 2006). Next we propose and implement a bi-criteria parameter tuning strategy for automatically generating multiple high quality treatment plans. Key to this approach is the possibility of efficiently solving the underlying LP based fluence map optimization (FMO) problem. Finally we use the above mentioned FMO model within a heuristic for selecting beam geometries. At the heart of the scheme is a multi-attribute beam scoring mechanism based on a treatment plan constructed for a beam geometry involving a large number of beam angles.

In summary, we develop a treatment plan generation technology that optimizes both beam geometry and beamlet intensities. The technology is automated and generates several high-quality treatment plans satisfying the provided requirements in a single invocation and without human guidance. The technology has been tested on various real patient cases with success. Solution times range from a few minutes to a quarter of an hour which are clinically acceptable.

The remainder of the paper is organized as follows. In Section 2, we provide details on the IMRT treatment planning problem and the various evaluation metrics considered in this paper. In Section 3, we describe the core components of our IMRT treatment plan generation technology. Finally, in Section 4, we present the results of an extensive computational study.

2. Problem Description

An IMRT treatment plan has to specify a beam geometry (a set of beam angles) and for each beam angle a fluence map (a set of beamlet intensities; a beam can be thought of as consisting of a number of beamlets that can be controlled individually). Typically, a small number of equi-spaced
coplanar beam angles are used. Coplanar beam angles are obtained by rotating the gantry while
keeping the treatment couch in a fixed position parallel to the gantry axis of rotation. There are
practical reasons for limiting the number of beam angles (between 5 and 8) as it reduces patient
positioning times, chances for patient positioning errors, and delivery time.

For modeling purposes, the structures (target and critical structures) are discretized into cubes
called voxels (e.g., cubes of $5 \times 5 \times 5$ mm) and the dose delivered to each voxel per intensity of
each beamlet is calculated (see Ahnesjo 1989, Mackie et al. 1985, and Mohan et al. 1986 for dose
calculation methods). The total dose received by a voxel is the sum of doses deposited from each
beamlet. The goal is to build a treatment plan that creates a dose distribution that will destroy,
or at least damage, target cells while sparing healthy tissue by choosing proper beam angles and
beamlet intensities.

In order to guide the construction of a treatment plan, a radiation oncologist specifies a set of
requirements that have to be satisfied in any acceptable treatment plan. These requirements are
in the form of a minimum prescription dose for target structure voxels and a maximum tolerance
dose for nearby critical structure voxels. A prescription dose is the dose level necessary to destroy
or damage target cells, while a tolerance dose is the level above which complications for healthy
tissues may occur.

In addition to the set of dose requirements defining acceptable treatment plans, clinical oncol-
egists use a set of evaluation metrics to assess the quality of a treatment plan. In this paper we
consider the following metrics: cold spot, hot spot, coverage and conformity. These metrics are
defined next. The following notation is used throughout. Let $N$ denote the set of beamlets, $S$ the
set of structures, $V_s$ the number of voxels in a structure $s \in S$, and $z_{js}$ the dose received by voxel
$j$ of structure $s$. (Of course $z_{js}$ depends on the treatment plan.) For simplicity of exposition we
assume that there is only one target structure $\tau \in S$.

The cold spot of target structure $\tau$ is defined as the ratio of the minimum dose received by any
of the voxels of the structure to the prescription dose of structure $\tau$, i.e.,

$$ \text{cold spot}(\tau) = \frac{\min\{z_{j\tau} : j = 1, \ldots, V_\tau\}}{PD_\tau}, $$

(1)

where $PD_\tau$ is the prescription dose for target structure. Similarly, the hot spot of target structure
is defined as the ratio of the maximum dose received by any of the voxels of the structure to the
prescription dose, i.e.,

$$ \text{hot spot}(\tau) = \frac{\max\{z_{j\tau} : j = 1, \ldots, V_\tau\}}{PD_\tau}. $$

(2)
Ideally, every voxel in a target structure receives exactly the prescription dose. The cold spot metric and hot spot metric measure the deviation from this ideal situation by examining the voxel receiving the smallest dose and the voxel receiving the largest dose.

The coverage of target structure $\tau$ is the proportion of voxels receiving a dose greater than or equal to the prescription dose $PD_\tau$, i.e.,

$$\text{coverage}(\tau) = \frac{\sum_{j=1}^{V_\tau} \mathbb{I}_+(z_{j\tau} - PD_\tau)}{V_\tau},$$

where $\mathbb{I}_+$ is the indicator function for the non-negative real line, i.e., $\mathbb{I}_+(a)$ is equal to 1 if $a \geq 0$ and 0 otherwise. Note that $0 \leq \text{coverage}(\tau) \leq 1$ and that values closer to 1 are preferable. The conformity of target structure $\tau$ is the ratio of the number of voxels in the structure and its surrounding tissue receiving a dose greater than or equal to the prescription dose $PD_\tau$ to the number of voxels in the structure itself receiving greater than or equal to the prescription dose, i.e.,

$$\text{conformity}(\tau) = \frac{\sum_{s \in S} \sum_{j=1}^{V_s} \mathbb{I}_+(z_{j\tau} - PD_\tau)}{\sum_{j=1}^{V_\tau} \mathbb{I}_+(z_{j\tau} - PD_\tau)}.$$

Note that $1 \leq \text{conformity}(\tau)$ and that values closer to 1 are preferable. The coverage and conformity metrics are illustrated in Figure 1. In schematic (a), the set of voxels receiving a dose greater than or equal to the prescription dose is exactly the set of voxels in target structure $\tau$ and thus $\text{coverage}(\tau) = \text{conformity}(\tau) = 1$; this is the ideal solution. In schematic (b) the set of voxels receiving a dose greater than or equal to the prescription dose is about twice the size of the set of voxels in target structure $\tau$ (and includes all the voxels of $\tau$) and thus $\text{coverage}(\tau) = 1$ and $\text{conformity}(\tau) = 1.96$. In schematic (c) the set of voxels receiving a dose greater than or equal to the prescription dose is less than half of the size of the target structure $\tau$ (but located inside the target structure) and thus $\text{coverage}(\tau) = 0.36$ and $\text{conformity}(\tau) = 1$. Finally, in schematic (d) the set of voxels receiving a dose greater than or equal to the prescription dose has the same size as that of the target structure but is offset from the target. In this case $\text{coverage}(\tau) = 0.39$ and $\text{conformity}(\tau) = 2.56$. Due to the fact that coverage and conformity consider the target as a whole, these metrics are typically of higher importance than the cold spot and hot spot metrics. (See Lee et al. 2003, 2006 for earlier use of these metrics for plan evaluation.)

The goal of our proposed approach is to determine fluence map and beam geometry consistent with treatment plans that satisfy the clinical dose requirements and have ideal values for the
evaluation metrics. In the next section, we present an optimization based approach towards this end.

3. Methodology

We first describe the C-VaR based FMO model of Romeijn et al. (2006) and show how the C-VaR parameters can be used to alter coverage and conformity. Next, we describe a bi-criteria search strategy for generating multiple high quality treatment plans. Finally, beam angle selection is integrated with fluence map optimization. The beam angle selection scheme employs a bi-criteria scoring of beam angle geometries and a selection mechanism to choose from among the set of non-dominated geometries.

3.1. Fluence Map Optimization Model

Let $D_{ijs}$ be the dose received by voxel $j$ of structure $s$ per unit intensity of beamlet $i$. Let $x_i$ be the intensity of beamlet $i$, i.e., a decision variable, then the dose $z_{js}$ received by voxel $j$ of structure $s$, is

$$z_{js} = \sum_{i \in N} D_{ijs} x_i \quad \forall j = 1, ..., V_s; \; s \in S. \quad (5)$$
Full volume constraints

Let $L_s$ and $U_s$ be the prescribed lower and upper dose limits for structure $s$, respectively. The full-volume constraints for dose limits are

\[ L_s \leq z_{js} \leq U_s \quad \forall j = 1, \ldots, V_s; \ s \in S. \tag{6} \]

Prescribed requirements on cold spot and hot spot can be enforced by the bounds $L_\tau$ and $U_\tau$ in the full volume constraints (6).

Partial volume constraints

Partial-volume constraints specify dose limits that have to be satisfied by a specified fraction of the voxels of a structure. For target structure $\tau$ these constraints are formulated as the C-VaR constraint:

\[ c_\tau - \frac{1}{(1-\alpha_\tau).V_\tau} \sum_{j=1}^{V_\tau} (c_\tau - z_{j\tau})^+ \geq L_\tau^\alpha \tag{7} \]

where $(a)^+$ is equal to $a$ if $a \geq 0$ and 0 otherwise. This C-VaR constraint enforces that the average dose in the $(1-\alpha_\tau)$-fraction of voxels of target structure $\tau$ receiving the lowest amount of dose is greater than or equal to $L_\tau^\alpha$. If satisfied, at least $\alpha_\tau \times 100$ per cent of the voxels receive a dose greater than or equal to $L_\tau^\alpha$. For example, with $\alpha_\tau = 0.9$ and $L_\tau^\alpha = 30$ at least 90% of the voxels will receive a dose greater than 30 Gy. Here $c_\tau$ is a free variable associated with the dose-volume constraint of structure $\tau$.

Similarly, the partial-volume constraints for critical structures are

\[ c_s + \frac{1}{(1-\alpha_s).V_s} \sum_{j=1}^{V_s} (z_{js} - c_s)^+ \leq U_s^\alpha \quad s \in S. \tag{8} \]

For critical structure $s$ the average dose in the subset of size $(1-\alpha_s)$ receiving the highest amount of dose is required to be less than or equal to $U_s^\alpha$. When satisfied at least $\alpha_s$ percent of the voxels will receive a dose less than or equal to $U_s^\alpha$.

Due to the indicator function in (3), requirements on coverage are difficult to model using convex constraints. Instead, we use partial volume constraints to enforce the coverage requirements using the following result.

**Proposition 1** Given $\alpha_s \in (0, 1)$, if there exists a $c_s$ such that

\[ c_s - \frac{1}{(1-\alpha_s).V_s} \sum_{j=1}^{V_s} (c_s - z_{js})^+ \geq PD_s \tag{9} \]
then

\[
\text{coverage}(s) \geq \alpha_s. \tag{10}
\]

A proof of this proposition is given in Section 1 of the Online Supplement to this paper available at http://joc.pubs.informs.org. Thus coverage(\(\tau\)) can be increased by increasing \(\alpha_\tau\) in the partial volume constraints (7) with \(L_\tau^\alpha = PD_\tau\). Of course, \(\alpha_\tau = 1\) is preferable, but such an aggressive value may lead to infeasibility. Note that if \(\alpha_\tau = 1\) the partial volume constraint reduces to a full volume constraint.

Conformity requirements cannot be directly enforced by full or partial volume constraints on the dose on the target structure. This is because the conformity metric considers dose deposited outside the target. In order to keep track of the intended dose for target \(\tau\) that is deposited outside the target, we use a virtual critical structure around the target structure \(\tau\). This is typically a 30 mm band around the target structure \(\tau\), denoted by structure \(\beta \in S\). Bahr et al. (1968), Price et al. (2003) and Lee et al. (2006) showed on clinical cases that limiting or minimizing the dose on virtual critical structures around the target structures can improve the conformity of the plans. Girinsky et al. (2006) also used virtual critical structures to avoid high doses around the tumor volume.

Assuming that all the dose intended for the target structure \(\tau\) is deposited in the voxels in structures \(\tau\) and \(\beta\), the only structure in \(S\) other than \(\tau\) that can have more than the prescribed dose \(PD_\tau\) will be structure \(\beta\). Then from (3) and (4)

\[
\text{conformity}(\tau) = 1 + \frac{\sum_{j=1}^{V_\beta} I_+(z_{j\beta} - PD_\tau)}{\sum_{j=1}^{V_\tau} I_+(z_{j\tau} - PD_\tau)}
= 1 + \frac{\text{coverage}(\beta) \cdot V_\beta}{\text{coverage}(\tau) \cdot V_\tau},
\]

We know that by enforcing the partial volume constraint (7) on structure \(\tau\) with \(L_\tau^\alpha = PD_\tau\) for \(\alpha_\tau \in (0, 1)\), we ensure \(\text{coverage}(\tau) \geq \alpha_\tau\). Similarly, it can be shown that by enforcing the partial volume constraint (8) on structure \(\beta\) with \(U_\beta^\alpha = PD_\tau\) for \(\alpha_\beta \in (0, 1)\), we ensure \(\text{coverage}(\beta) \leq (1 - \alpha_\beta)\). Thus

\[
\text{conformity}(\tau) \leq 1 + \frac{(1 - \alpha_\beta) \cdot V_\beta}{\alpha_\tau \cdot V_\tau}. \tag{11}
\]
Therefore we can reduce conformity ($\tau$) by increasing $\alpha_\tau$ and $\alpha_\beta$. Setting these parameters at their maximum value of 1 typically leads to problem infeasibility, and so a careful selection is important. This is further discussed in Section 3.2.

Including the partial-volume constraints our FMO model is

$$\min \sum_{s \in S \setminus \{\tau\}} \frac{1}{V_s} \sum_{j=1}^{V_s} z_{js} - \frac{1}{V_\tau} \sum_{j=1}^{V_\tau} z_{j\tau}$$

s.t.

$$z_{js} = \sum_{i \in N} D_{ij} x_i \forall j = 1, ..., V_s; s \in S$$

$$L_s \leq z_{js} \leq U_s \forall j = 1, ..., V_s; s \in S$$

$$c_s + \frac{1}{(1 - \alpha_s) V_s} \sum_{j=1}^{V_s} (z_{js} - c_s)^+ \leq U_s^o \quad s \in S \setminus \{\tau\}$$

$$c_\tau - \frac{1}{(1 - \alpha_\tau) V_\tau} \sum_{j=1}^{V_\tau} (c_\tau - z_{j\tau})^+ \geq L_\tau^o$$

$$x_i \geq 0 \quad i = 1, ..., N$$

$$z_{js} \geq 0 \quad j = 1, ..., V_s; s \in S$$

$$c_s \text{ free} \quad s \in S.$$  

The objective function (12) attempts to decrease the average dose on the critical structures while increasing the average dose on the target structure. We can integrate various other objective functions, but we leave this as a future research subject (see Kessler et al. 2005; Yang and Xing 2004 for objective functions used in IMRT formulations).

3.2. Parameter Search

As mentioned before, the coverage and conformity metrics for the target structures depend on the values of the parameters $\alpha_\beta$ and $\alpha_\tau$. In this section we describe a search technique to identify appropriate values of these parameters that improve treatment quality.

High values of $\alpha_\beta$ and $\alpha_\tau$ are preferable for good coverage and conformity for the target $\tau$. However, this may lead to infeasibility. Let

$$F_\tau = \{(\alpha_\beta, \alpha_\tau) \in (0, 1)^2 : \text{ The FMO model (12)-(19) is feasible}\}.$$ 

Note that $F_\tau$ is a monotone set, i.e., if $(\alpha_\beta^*, \alpha_\tau^*) \in F_\tau$, $\alpha_\tau \leq \alpha_\tau^*$ and $\alpha_\beta \leq \alpha_\beta^*$, then $(\alpha_\tau, \alpha_\beta) \in F_\tau$. Assuming $F_\tau$ is non-empty we would like to maximize $\alpha_\tau$ and $\alpha_\beta$ over $F_\tau$. Such solutions will lie on the upper boundary of $F_\tau$. Our search technique starts with an initial solution and goes through
several search phases to identify solutions on the upper boundary of $F_\tau$. Figure 2 illustrates the search phases, which are detailed next.

Initial values of the parameters $\alpha_\tau$ and $\alpha_\beta$ are chosen based on a minimum coverage requirement $MinCov$ and a maximum allowed conformity value $MaxConf$. In particular

$$\alpha_\tau^0 = MinCov \times \gamma$$

$$\alpha_\beta^0 = (1 - \frac{MinCov(MaxConf - 1)V_\tau}{V_\beta}) \times \gamma$$

where $0 < \gamma < 1$ is scaling factor to account for the fact that the bounds (10) and (11) are conservative. In our implementation for case studies we used $\gamma = 0.9$.

We solve the FMO model (12)-(19) with the initial parameter values $(\alpha_\beta^0, \alpha_\tau^0)$. If the problem is infeasible, i.e., $(\alpha_\beta^0, \alpha_\tau^0) \notin F_\tau$, then we decrease both parameters by $\lambda$, where $0 < \lambda < 1$, and resolve the FMO model. This is continued until a feasible set of parameters is found. The monotone structure of the set $F_\tau$ guarantees that if $F_\tau$ is non-empty, then Phase 0 will produce a feasible set of parameters.

Once we have initial feasible parameters, we execute Phase 1, where we increase both $\alpha_\beta$ and $\alpha_\tau$ by $\lambda$ and resolve the FMO model. This is repeated until we reach an infeasible set of parameters. The last set of feasible parameters, denoted by $(\alpha_\beta^1, \alpha_\tau^1)$, then is a solution near the upper boundary of $F_\tau$.

Next we execute Phase 2 where, starting from $(\alpha_\beta^1, \alpha_\tau^1)$, we increase $\alpha_\tau^1$ by $\lambda$ until we reach an infeasible set of parameters. The motivation here is that, since increasing $\alpha_\tau$ improves both coverage and conformity, we want to obtain additional solutions with good coverage and conformity values. The last feasible solution from Phase 2 is denoted by $(\alpha_\beta^2, \alpha_\tau^2)$.

In order to find additional candidate solutions on the boundary of $F_\tau$, we execute Phase 3. Here we reduce $\alpha_\beta$ by $\lambda$, and then start Phase 2 to increase $\alpha_\tau$. Note that in the solutions produced in this phase the conformity values may be worse than those from Phase 2, however coverage will not worsen.

Finally, if Phases 2 and 3 are not able to produce feasible parameters greater than $(\alpha_\beta^1, \alpha_\tau^1)$, then we execute Phase 4. Here we increase $\alpha_\beta$ by $\lambda$ in order to find additional solutions near the upper boundary of $F_\tau$.

The step size $\lambda$ is an important component of the search process. From initial experimentations, we chose this to be $\lambda = 0.01$. 

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Figure 2: Parameter Search for $\alpha_r$ and $\alpha_\beta$
3.3. Beam Angle Selection

Careful selection of beam angles has long been recognized as an important part of creating IMRT plans (Stein et al. 1997). See Ehrgott et al. (2008) for a rigorous presentation of the beam selection problem. Constructing an IMRT plan for all possible beam angle configurations and selecting the best is computationally prohibitive due to the number of possible configurations. Therefore, most algorithms assign scores to the candidate beam angles and use these scores to evaluate possible beam angle configurations. (See Bedford and Webb 2007 for a classification of other beam selection algorithms proposed in the literature.) Scoring may be based on geometric and dosimetric information as in Pugachev and Lei (2001); or based on optimization of the fluence for each beam as in D’Souza et al. (2004). The drawback of these scoring-based approach is that the interaction between different beam angles is ignored. We introduce a bi-criteria scoring approach to limit our choices to a relatively small number of non-dominated configurations. We then optimize the fluence only for these configurations to assess interaction effects. We compare the efficacy of our beam selection methodology with scoring-based approach of Pugachev and Lei (2001) in Section 4.

We start by using the FMO model assuming that all candidate beam angles (say $M$) can be used. Let $x_i^*$ be the intensity of beamlet $i$ in beam (angle) $B$ (where $B \in \{1, \ldots, M\}$) and $z_j^{*\tau}$ be the dose delivered to voxel $j$ of the target structure $\tau$ in this treatment plan. Each beam $B$ is then assigned the following two scores:

**Total Dose Delivered to the Target Structure.** The total dose delivered to the target structure by beam $B$ is computed as

$$DPTV_B = \sum_{i \in B} \sum_{j=1}^{V_\tau} D_{ij\tau} x_i^*.$$  \hspace{1cm} (22)

Preferring beams with a higher value of $DPTV_B$ favors beams that deliver higher dose to the target. The disadvantage of this measure is that it does not directly consider the dose received by target voxels in the lower tail of the dose distribution, i.e., the cold spots.

**Weighted Sum of Dose Delivered to the Low Dose Region of the Target Structure.** We define the low dose region $C_\tau$ of the target structure $\tau$ as the set of voxels that receive a dose less than 10% above the minimum dose observed in the target structure, i.e.,

$$C_\tau = \{j = 1, \ldots, V_\tau : z_j^{*\tau} \leq 1.10 \min_j (z_j^{*\tau})\}.$$  \hspace{1cm} (23)

The weighted sum of dose delivered to the low dose region of the target structure from beam
angle $B$ is

$$WPTV_B = \sum_{i \in B} \sum_{j \in C} D_{ijr} x_i^* \frac{1}{\max \{ x_j^*, \epsilon \}},$$

where $\epsilon > 0$ is included to avoid division by zero. Note that we distinguish voxels in the low dose region by assigning weights inversely proportional to the actual dose received by these voxels. Preferring beams with higher value of $WPTV_B$ favors beams that deliver more dose to the low dose region of the target. The advantage of this measure is that it helps identify beams that deliver dose to what are commonly difficult regions to treat.

The score of a configuration with $L$ angles selected from $M$ candidates, i.e., a set of beam angles $C \subset \{1, \ldots, M\}$ such that $|C| = L$, is the sum of individual beam scores, i.e.,

$$DPTV(C) = \sum_{B \in C} DPTV_B,$$

and

$$WPTV(C) = \sum_{B \in C} WPTV_B.$$

We identify a set $K$ of non-dominated configurations, i.e.,

$$K = \{ C : \exists D \text{ s.t. } DPTV(D) \geq DPTV(C) \text{ and } WPTV(D) > WPTV(C) \} \cup \{ C : \exists D \text{ s.t. } DPTV(D) > DPTV(C) \text{ and } WPTV(D) \geq WPTV(C) \}.$$

The set $K$ can be determined by enumerating all $\binom{M}{L}$ beam angle configurations. When the set of all configurations is very large, a more computationally effective approach is to solve a series of cardinality constrained knapsack problems to identify the non-dominated configurations. This scheme is illustrated in Algorithm 1.

We would ideally generate an optimized treatment plan for each of the non-dominated configurations and select the one with the highest quality as evaluated in this paper. However this is computationally intensive, and instead we perform a “greedy” search. We start with an arbitrary configuration $C \in K$ and calculate $(\alpha_2^2, \alpha_2^\tau)$. Next, we search for a configuration $\hat{C} \in K$ for which $(\alpha_2^2, \alpha_2^\tau + \lambda)$ is feasible. If no such configuration $\hat{C}$ exists, then configuration $C$ is selected. On the other hand, if such a configuration $\hat{C}$ exists, we iterate and execute Phase 1 and Phase 2 of the parameter search for $\hat{C}$ starting from $(\alpha_2^2, \alpha_2^\tau)$.

4. Computational Study

In this section, we present the results of a computational study of three real-life cases: a pediatric brain case, a head and neck case, and a prostate case. The computational study focuses on the
Algorithm 1 Finding non-dominated beam angle configurations

Require: \(DPTV_i\) and \(WPTV_i\) scores of \(M\) candidate angles to choose \(L\) angles. Let \(\epsilon > 0\) be small.

\[\text{RHS} \leftarrow 0\]

repeat

solve the cardinality constrained knapsack problem:

\[
\max \sum_{i=1}^{M} DPTV_i y_i
\]

s.t. \[
\sum_{i=1}^{M} y_i = L
\]

\[
\sum_{i=1}^{M} WPTV_i y_i \geq \text{RHS}
\]

\(y_i \in \{0, 1\}\)

if IP is feasible then

a non-dominated configuration is found

\[\text{RHS} \leftarrow \sum_{i=1}^{M} WPTV_i y_i + \epsilon\]

end if

until IP is infeasible

value of the techniques and solution schemes introduced above: (1) the development of an automated, systematic parameter search scheme for treatment plan construction models using C-VaR constraints to model partial dose volume constraints, and (2) the development of an efficient and effective beam angle selection scheme.

4.1. Case Descriptions

The three cases represent different parts of the body, and thus a variety of challenges in terms of treatment plan generation. So these cases demonstrate the robustness of our approach; no tuning is necessary for the specific cases.

Information concerning the prescription requirements for the target structures as well as for the critical structures in terms of full volume and partial volume dose constraints for the three cases can be found in Tables 1, 2, and 3. Partial dose volume constraints for the target structure (referred to as PTV) and the virtual critical structure (VCS) are computed using the formulas
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<tr>
<td></td>
<td>Left cochlea</td>
<td>21</td>
<td>-</td>
<td>0</td>
<td>1,260</td>
</tr>
<tr>
<td></td>
<td>Pituitary</td>
<td>11</td>
<td>-</td>
<td>0</td>
<td>2,160</td>
</tr>
<tr>
<td></td>
<td>Left eye</td>
<td>158</td>
<td>-</td>
<td>0</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>Right eye</td>
<td>163</td>
<td>-</td>
<td>0</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>VCS</td>
<td>6,581</td>
<td>-</td>
<td>0</td>
<td>3,300</td>
</tr>
<tr>
<td>Partial volume</td>
<td>PTV</td>
<td>1,620</td>
<td>95</td>
<td>3,060</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>VCS</td>
<td>6,581</td>
<td>95</td>
<td>-</td>
<td>3,060</td>
</tr>
</tbody>
</table>

Table 1: Brain Case - Structures and Constraints

presented in Section 3 using a minimum coverage requirement of 0.95 and a maximum conformity requirement of 1.2. Observe that there are no partial dose-volume constraints for critical structures in the pediatric brain case.

To be able to analyze and judge the value of the different techniques and solution schemes, we start with base settings in each of the three cases. The base settings have 8 equi-spaced beams in the pediatric brain case, 8 equi-spaced beams in the head and neck case, and 6 equi-spaced beams in the prostate case. Furthermore, the full and partial dose volume constraints as specified in Tables 1, 2, and 3 are used.

4.2. Value of the Parameter Search

As the treatment plan construction model only indirectly attempts to optimize coverage and conformity, we may be able to improve coverage and conformity by judiciously adjusting the dose-volume constraint parameters of the target structure and the virtual critical structure. We focus on the improvement in coverage and conformity that can be achieved through the parameter search. We compare the base settings (with full and partial dose volume constraints for the virtual critical structure included) with and without parameter search. The results are presented in Table 4. We observe that there are improvements for coverage and conformity for the brain and the prostate case. However, for the head-and-neck case the slight improvement in coverage is offset by a deteri-
<table>
<thead>
<tr>
<th>Constraint Type</th>
<th>Structure</th>
<th>#Voxels</th>
<th>Percentage</th>
<th>L (cGy)</th>
<th>U (cGy)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full volume</td>
<td>PTV</td>
<td>2,647</td>
<td>-</td>
<td>0</td>
<td>6,120</td>
</tr>
<tr>
<td></td>
<td>Brainstem</td>
<td>1,075</td>
<td>-</td>
<td>0</td>
<td>5,400</td>
</tr>
<tr>
<td></td>
<td>Spinal cord</td>
<td>387</td>
<td>-</td>
<td>0</td>
<td>4,500</td>
</tr>
<tr>
<td></td>
<td>Globe_LT</td>
<td>388</td>
<td>-</td>
<td>0</td>
<td>2,000</td>
</tr>
<tr>
<td></td>
<td>Globe_RT</td>
<td>355</td>
<td>-</td>
<td>0</td>
<td>2,000</td>
</tr>
<tr>
<td></td>
<td>Optic_chiasm</td>
<td>13</td>
<td>-</td>
<td>0</td>
<td>5,400</td>
</tr>
<tr>
<td></td>
<td>Optic_nerve_LT</td>
<td>26</td>
<td>-</td>
<td>0</td>
<td>5,400</td>
</tr>
<tr>
<td></td>
<td>Optic_nerve_RT</td>
<td>10</td>
<td>-</td>
<td>0</td>
<td>5,400</td>
</tr>
<tr>
<td></td>
<td>Parotid_LT</td>
<td>811</td>
<td>-</td>
<td>0</td>
<td>5,400</td>
</tr>
<tr>
<td></td>
<td>Parotid_RT</td>
<td>832</td>
<td>-</td>
<td>0</td>
<td>5,400</td>
</tr>
<tr>
<td></td>
<td>VCS</td>
<td>7,634</td>
<td>-</td>
<td>0</td>
<td>6,120</td>
</tr>
<tr>
<td>Partial volume</td>
<td>PTV</td>
<td>2,647</td>
<td>95</td>
<td>5,100</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Parotid_LT</td>
<td>811</td>
<td>50</td>
<td>-</td>
<td>2,600</td>
</tr>
<tr>
<td></td>
<td>Parotid_RT</td>
<td>832</td>
<td>50</td>
<td>-</td>
<td>2,600</td>
</tr>
<tr>
<td></td>
<td>VCS</td>
<td>7,634</td>
<td>93</td>
<td>-</td>
<td>5,100</td>
</tr>
</tbody>
</table>

Table 2: Head & Neck Case - Structures and Constraints

oration in conformity. The head and neck case also shows a significant cold spot. This is due to the fact that the target structure is up against the Globe_RT structure which has a full volume dose restriction of 2000 cGy. We will examine this issue in more detail in Section 4.6. The parameter search for the brain case and for the head and neck case end with a Phase 3 iteration, in which the virtual critical structure constraint is relaxed in hopes of finding a treatment plan with higher coverage. As a result, the value of conformity increases slightly in the last iteration. More details are provided in Figure 3 where we plot the change in the evaluation metrics during the course of the parameter search.

### 4.3. Beam Angle Selection

A core ingredient of our angle selection scheme is identifying non-dominated beam configurations with respect to $DPTV_B$ and $WPTV_B$. In Figures 4, 5, and 6, we plot the scores for all 8-beam configurations for the brain and head and neck cases and all 6-beam configurations for the prostate.
<table>
<thead>
<tr>
<th>Constraint Type</th>
<th>Structure</th>
<th>#Voxels</th>
<th>Percentage</th>
<th>L (cGy)</th>
<th>U (cGy)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full volume</td>
<td>PTV</td>
<td>239</td>
<td>-</td>
<td>0</td>
<td>8694</td>
</tr>
<tr>
<td></td>
<td>Rectum</td>
<td>1,267</td>
<td>-</td>
<td>0</td>
<td>8694</td>
</tr>
<tr>
<td></td>
<td>Bladder</td>
<td>1,513</td>
<td>-</td>
<td>0</td>
<td>8694</td>
</tr>
<tr>
<td></td>
<td>VCS</td>
<td>2,544</td>
<td>-</td>
<td>0</td>
<td>8694</td>
</tr>
<tr>
<td>Partial volume</td>
<td>PTV</td>
<td>239</td>
<td>95</td>
<td>7,560</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Rectum</td>
<td>2,544</td>
<td>70</td>
<td>-</td>
<td>7,560</td>
</tr>
<tr>
<td></td>
<td>Bladder</td>
<td>1,513</td>
<td>50</td>
<td>-</td>
<td>4,500</td>
</tr>
<tr>
<td></td>
<td>VCS</td>
<td>2,544</td>
<td>98</td>
<td>-</td>
<td>7,560</td>
</tr>
</tbody>
</table>

Table 3: Prostate Case - Structures and Constraints

case selected from 18 equi-spaced candidate beams and highlight the configurations that are non-dominated. The figures show that few non-dominated configurations exist. In Table 5, we provide additional information to support the observation that very few non-dominated configurations exist. For configurations of different sizes, we report the total number of configurations as well as the number of non-dominated configurations.

Next, we explore the effect of carefully selecting beam angles, and thus of integrating all the techniques and solution schemes developed. The results are presented in Table 6. The results are mostly self-explanatory and clearly demonstrate the value of the various techniques and solution schemes. However, these evaluation metrics only tell part of the story. Clinicians examine dose volume histograms and dose distributions to evaluate treatments, which are shown in Figures 7, 8, and 9 for the brain case, the head and neck case, and the prostate case, respectively. Each of the figures displays dose volume histograms for three treatment plans: (1) a plan with equi-spaced beams obtained by solving the model, but without parameter search, (2) a plan with equi-spaced beams obtained by solving the model incorporating parameter search, and (3) a plan with optimized beam angles obtained by solving the model incorporating parameter search. The dose volume histograms show that improving coverage and conformity (i.e., by performing a parameter search) can have a negative impact on the dose volume histograms of the critical structures (increased doses delivered), but that by carefully selecting the beam angles these undesirable effects can be mostly negated. This is observed especially well in the head and neck case. We show dose distribution
images in Figures 1, 2 and 3 in the Online Supplement.

The above results indicate that our beam angle selection scheme produces beam configurations that allow the construction of high-quality treatment plans. Next, we compare the proposed beam configurations to those produced by an optimization approach based on an integer programming formulation. We also compare our scheme to a scoring-based beam selection algorithm.

Comparison with Mixed-Integer Programming Approach

We consider the MIP model for beam selection from Lee et al. (2003, 2006), Yang et al. (2006), and Lim et al. (2008). We experimented with selecting 5 beam angles out of 8 equi-spaced candidate beam angles for the brain and the head and neck case; and selecting 6 beam angles out of 18 equi-spaced beam angles for the prostate case. (Due to the computational requirements of the optimization approach this comparison can only be performed for settings with a relatively small number of candidate beam angles for the brain and the head and neck cases.) For these cases, our selection scheme produced the optimal configuration, i.e., the same configuration that the optimization approach produced. It is informative to look at the difference in required computation time; see Table 7. It is clear that the optimization approach will become computationally prohibitive when 10 or more candidate beam angles are used.

Comparison with a Scoring-Based Beam Selection Algorithm

To further validate our approach, we compare our results to the results obtained by selecting the beam angles according to the pseudo Beam’s-Eye-View (pBEV) scores introduced by Pugachev and Lei (2001). The steps for the pBEV calculation of a given beam angle are as follows:

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Configuration Size | # Non-Dominated Configurations | # Configurations
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Brain</td>
<td>Head &amp; Neck</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>13</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>12</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 5: Number of Non-dominated Configurations for Various Configuration Sizes

1. Assign each beamlet an initial intensity value that delivers at least the prescription dose to every target voxel it is crossing;

2. For each critical structure voxel crossed by the beamlet, calculate the factor by which the initial beamlet intensity has to be multiplied to ensure the tolerance dose is not exceeded.

3. Find the minimum factor among all critical structures and adjust the initial beamlet intensity.

4. Perform a forward dose calculation using the beam intensity profile obtained;

5. Compute the score for chosen beam angle $i$ according to an empiric score function as follows:

$$S_i = \frac{1}{V_s} \sum_{j=1}^{V_s} \left( \frac{d_{ij}}{PD_s} \right)^2$$

where $d_{ij}$ is the dose delivered to voxel $j$ from beam angle $i$, $V_s$ is the number of voxels in the target, and $PD_s$ is the target prescription dose.

We select the final configuration simply by picking the highest scoring beams. (Pugachev and Lei 2001 manually pick the beams in the case examples when some high-scoring angles are too close to each other to create a separation.)

We compute beam angle configurations for the cases using pBEV scores and our proposed scores, and then perform fluence map optimization. Multiple solutions are produced for each case.
Table 6: Evaluation Metrics for Different Schemes

<table>
<thead>
<tr>
<th>Case</th>
<th>Evaluation measure</th>
<th>Equi-spaced</th>
<th>Equi-spaced &amp; VCS</th>
<th>Equi-spaced &amp; VCS &amp; Parameter search</th>
<th>Optimized angles &amp; VCS &amp; Parameter search</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brain</td>
<td>Coverage</td>
<td>0.954</td>
<td>0.957</td>
<td>0.984</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>Conformity</td>
<td>1.533</td>
<td>1.274</td>
<td>1.212</td>
<td>1.125</td>
</tr>
<tr>
<td></td>
<td>Coldspot</td>
<td>0.669</td>
<td>0.704</td>
<td>0.822</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>Hotspot</td>
<td>1.078</td>
<td>1.078</td>
<td>1.078</td>
<td>1.078</td>
</tr>
<tr>
<td>Head &amp; Neck</td>
<td>Coverage</td>
<td>0.953</td>
<td>0.956</td>
<td>0.959</td>
<td>0.974</td>
</tr>
<tr>
<td></td>
<td>Conformity</td>
<td>1.742</td>
<td>1.230</td>
<td>1.261</td>
<td>1.232</td>
</tr>
<tr>
<td></td>
<td>Coldspot</td>
<td>0.328</td>
<td>0.311</td>
<td>0.313</td>
<td>0.310</td>
</tr>
<tr>
<td></td>
<td>Hotspot</td>
<td>1.200</td>
<td>1.200</td>
<td>1.200</td>
<td>1.200</td>
</tr>
<tr>
<td>Prostate</td>
<td>Coverage</td>
<td>0.933</td>
<td>0.950</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>Conformity</td>
<td>1.601</td>
<td>1.264</td>
<td>1.046</td>
<td>1.033</td>
</tr>
<tr>
<td></td>
<td>Coldspot</td>
<td>0.853</td>
<td>0.829</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>Hotspot</td>
<td>1.150</td>
<td>1.150</td>
<td>1.150</td>
<td>1.150</td>
</tr>
</tbody>
</table>

by varying the number of beam angles. The configuration sizes are 6, 7, 8 and 9 for both brain and head and neck cases, and 4, 5, 6 for the prostate case.

The average values of the evaluation metrics (over the different configuration sizes) are presented in Table 8. The values of coverage and conformity are better in all cases with our proposed scheme, especially in the brain case. (See Tables 1, 2 and 3 in the Online Supplement for details of the comparison over different configuration sizes.)

The main difference between our scoring scheme and the pBEV scoring scheme is that we score beams based on their exact contributions to the delivered doses to target structures in an optimized solution rather than just using their geometric properties and prescription requirements. Another important difference is that we consider multiple beam configurations in the parameter search phase, which allows us to adjust dynamically in response to observed interaction effects between beams. In the pBEV algorithm there is only one configuration that is chosen based on initial parameter values.
### Table 7: Comparison with Mixed Integer Programming Approach

<table>
<thead>
<tr>
<th></th>
<th>Time (secs.)</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Proposed Scheme</td>
<td>MIP</td>
</tr>
<tr>
<td>Brain</td>
<td>348</td>
<td>5,676</td>
</tr>
<tr>
<td>Head &amp; Neck</td>
<td>363</td>
<td>16,988</td>
</tr>
<tr>
<td>Prostate</td>
<td>52</td>
<td>540</td>
</tr>
</tbody>
</table>

#### 4.4. Beam Configuration Size

The results presented so far assumed that the desired number of beam angles was decided in advance. We have seen that carefully selecting beam angles improves treatment plans. This suggests that it may be possible to get high-quality treatment plans with fewer beam angles. Figure 10 presents the values of the evaluation criteria for different sizes of beam configurations.

As expected, there are diminishing returns when we continue to increase the size of the configuration. This is especially clear for the prostate case where treatment plans of almost equal quality are produced for configurations of size 4 and up. Furthermore, treatment plans with good quality can already be achieved with configurations of relatively small size (which has many practical advantages). If we compare the results presented in Table 6 to those presented in Figure 10, we see that carefully selecting beam angles allows us to construct treatment plans of equal or better quality than those that can be obtained with equi-spaced beams with fewer beams.

#### 4.5. Solution Times

As mentioned above, we chose to develop only linear programming based technology, because linear programs can be solved efficiently. Several methodologies exist for solving linear programs, e.g., primal simplex, dual simplex, and interior point methods, and we examine the solution times for the different solution methods. We used the linear programming solver of XPRESS (Xpress-Optimizer 2007). Table 9 presents the instance characteristics for the head and neck case and the brain case (for 8 equi-spaced beams) and the prostate case (for 18 equi-spaced beams). All our experiments were run on a 2.66GHz Pentium Core 2 Duo processor with 3GB of RAM under Windows XP Operating System. The differences are staggering; for this class of linear programs, interior point methods are by far superior as noted by Holder (2003). All remaining computational experiments therefore use XPRESS’s barrier algorithm to solve linear programs.
<table>
<thead>
<tr>
<th>Case</th>
<th>Evaluation metric</th>
<th>pBEV</th>
<th>Proposed Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brain</td>
<td>Coverage</td>
<td>0.974</td>
<td>0.991</td>
</tr>
<tr>
<td></td>
<td>Conformity</td>
<td>1.209</td>
<td>1.146</td>
</tr>
<tr>
<td></td>
<td>Coldspot</td>
<td>0.949</td>
<td>0.989</td>
</tr>
<tr>
<td></td>
<td>Hotspot</td>
<td>1.078</td>
<td>1.078</td>
</tr>
<tr>
<td>Head &amp; Neck</td>
<td>Coverage</td>
<td>0.966</td>
<td>0.971</td>
</tr>
<tr>
<td></td>
<td>Conformity</td>
<td>1.219</td>
<td>1.214</td>
</tr>
<tr>
<td></td>
<td>Coldspot</td>
<td>0.344</td>
<td>0.311</td>
</tr>
<tr>
<td></td>
<td>Hotspot</td>
<td>1.200</td>
<td>1.200</td>
</tr>
<tr>
<td>Prostate</td>
<td>Coverage</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>Conformity</td>
<td>1.045</td>
<td>1.033</td>
</tr>
<tr>
<td></td>
<td>Coldspot</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>Hotspot</td>
<td>1.150</td>
<td>1.150</td>
</tr>
</tbody>
</table>

Table 8: Comparison of Evaluation Metrics for Beam Angles Selected with pBEV and Our Scheme

The first step of our scheme to select beam angles is to solve the problem assuming that all candidate beam angles are used. In our computation experiments this meant solving the problem using 18 candidate beam angles (equi-spaced). The solution is used to get the beam angle scores $W_{PTV}$ and $D_{PTV}$. Table 10 presents the instance characteristics and the run times for the three cases. Once the beam angle scores are computed they are used to identify non-dominated configurations. Then treatment plans are generated for these configurations (using a virtual critical structure and parameter search). In Table 11, we present the solution times for this component of the scheme for different beam configuration sizes. The maximum run time is about 20 minutes and observed for head and neck case to choose 9 beams. Our proposed scheme was able to produce high quality solutions for the prostate case in less than 2 minutes.

4.6. Cold Spot Issue in the Head & Neck Case

In the head and neck case, we observed a significant cold spot in the treatment plan produced by our algorithm. Further analysis revealed that the cold spot results because the target structure ($PTV$) is up against the $Globe_{RT}$ structure. (This can be seen in Figure 2 in the Online Supplement.) As
Voxels | Beamlets | #cons | #vars | primal simplex | dual simplex | barrier
---|---|---|---|---|---|---
Brain | 8,668 | 1,010 | 17,881 | 16,871 | 1,548 | 1,395 | 48
Head & Neck | 14,178 | 1,599 | 27,705 | 26,106 | 31,118 | 4,912 | 58
Prostate | 5,563 | 1,146 | 12,398 | 13,544 | 377 | 35 | 24

Table 9: Run Time (seconds) for Various Linear Programming Algorithms

<table>
<thead>
<tr>
<th></th>
<th>Voxels</th>
<th>Beamlets</th>
<th>#cons</th>
<th>#vars</th>
<th>time (secs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brain</td>
<td>8,668</td>
<td>2,306</td>
<td>16,871</td>
<td>19,177</td>
<td>123</td>
</tr>
<tr>
<td>Head &amp; Neck</td>
<td>14,178</td>
<td>3,650</td>
<td>26,106</td>
<td>29,756</td>
<td>278</td>
</tr>
<tr>
<td>Prostate</td>
<td>5,563</td>
<td>1,146</td>
<td>12,398</td>
<td>13,544</td>
<td>24</td>
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</table>

Table 10: Run Time for 18 Equi-Spaced Beams for Scoring

the maximum tolerance dose for \textit{Globe\_RT} is 2,000 cGy and the target dose for the \textit{PTV} is 5,100 cGy, it is not surprising that a cold spot results. The specified maximum tolerance dose of 2,000 cGy for \textit{Globe\_RT} is aggressive and upon seeing the generated treatment plan a clinician will likely increase the maximum tolerance dose. To explore what happens when the maximum tolerance dose for \textit{Globe\_RT} is increased, we present the evaluation metrics for the treatment plans generated for different maximum tolerance dose levels in Figure 11. We see that with a maximum tolerance dose of about 4,000 cGy the cold spot is eliminated.

<table>
<thead>
<tr>
<th></th>
<th>2 beams</th>
<th>3 beams</th>
<th>4 beams</th>
<th>5 beams</th>
<th>6 beams</th>
<th>7 beams</th>
<th>8 beams</th>
<th>9 beams</th>
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<tbody>
<tr>
<td>Brain</td>
<td>145</td>
<td>199</td>
<td>263</td>
<td>514</td>
<td>478</td>
<td>689</td>
<td>798</td>
<td>445</td>
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<tr>
<td>Head &amp; Neck</td>
<td>147</td>
<td>141</td>
<td>231</td>
<td>311</td>
<td>365</td>
<td>646</td>
<td>863</td>
<td>1,003</td>
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<td>Prostate</td>
<td>28</td>
<td>12</td>
<td>14</td>
<td>23</td>
<td>28</td>
<td>35</td>
<td>50</td>
<td>53</td>
</tr>
</tbody>
</table>

Table 11: Run Time (seconds) after Beam Scoring for different configuration sizes
Acknowledgement

The authors thank the AE and two anonymous referees for very constructive suggestions on an earlier version of the paper.

References


Figure 3: Change in Evaluation Metrics with Parameter Search: (a) Brain Case 8 Equi-spaced Beams, (b) Head & Neck Case 8 Equi-spaced Beams, (c) Prostate Case 6 Equi-spaced Beams
Figure 4: Brain Case - Dominated and Non-dominated Configurations

Figure 5: Head & Neck Case - Dominated and Non-dominated Configurations
Figure 6: Prostate Case - Dominated and Non-dominated Configurations
Figure 7: Brain Case Dose-Volume-Histograms: (a) Basic 8 Equi-spaced Beams Solution, (b) Final 8 Equi-Spaced Beams Solution, (c) Final 8 Selected Beams Solution
Figure 8: Head & Neck Case Dose-Volume-Histograms: (a) Basic 8 Equi-spaced Beams Solution, (b) Final 8 Equi-Spaced Beams Solution, (c) Final 8 Selected Beams Solution
Figure 9: Prostate Case Dose-Volume-Histograms: (a) Basic 6 Equi-spaced Beams Solution, (b) Final 6 Equi-Spaced Beams Solution, (c) Final 6 Selected Beams Solution
Figure 10: Final Results Returned by the Algorithm with Different Number of Beams Selected: (a) Brain Case, (b) Head & Neck, (c) Prostate
Figure 11: Head and Neck Case - Dose limit relaxation to eliminate cold spot