A Framework for Systematic Warehousing Design

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Introduction

The goal is the development of a systematic methodology to rapidly design warehousing systems. At the current time, most of warehousing design is either based on ad-hoc insight and experience of the warehouse designer or on detailed simulation of the equipment and material flows through the warehouse. However, the current business climate does not allow for the several weeks it takes to develop such a detailed simulation model. Third party logistics service providers (TPL) are routinely faced with a two-week cycle from the start of the project to the signing of a binding contract. With these deadlines, the highly detailed simulations by powerful contemporary simulation packages such as Witness, Arena, and AutoMod require too much time to develop, even with the built-in material handling constructs. There exists an urgent need for a design methodology that requires less detailed data, is less complex, and requires a shorter time-to-design. Typically the software programs that implement this type of design methodology are called rapid prototyping tools.

The warehouse design has to satisfy a number of high-level objectives and constraints. Most high-level warehousing constraints are elastic, in other words, they can be violated with a certain penalty.

We will develop a warehouse design formulation, whose overall structure is a multiperiod, multicommodity, capacitated network flow problem, where capacities are determined by binary configuration variables. There are costs associated both with the continuous flow and storage variables and the binary configuration variables. The formulation hence belongs to the class of mixed-integer linear programming problems. Such problems are known to be difficult to solve for large problem instances.

Formulation Development

Fundamental Time Units

Warehouse design and operations require the definition of two fundamental time units. The major time unit corresponds to the major planning period in the planning horizon. Typically, the major time unit is a year in a planning horizon of three to five years. The major time periods are also called epochs and are indexed by *e* and range from *1* to *E*. The minor time unit corresponds to the shortest period for which separate data on the material flows, resource costs, or capacities are available. The minor time unit may be as small as a two-hour pick wave. The major time periods are indexed by *t* and range from *1* to *T*. Typically, the minor time unit is a day or a week. It is assumed that the minor time periods repeat themselves in an integer number of identical cycles during a major time period.

$freq_t$ The frequency is the number of times a minor time unit is replicated per major time unit.

cdf_t The time discount factor with which the costs in a minor time period are reduced to the current time.

For example, the warehouse may experience vastly different material flows depending on the day of the week, where most deliveries occur on Monday and Tuesday and most shipments occur on Thursday and Friday. The appropriate minor time period for this situation would be one day. The major time period could be a year. If the material flows repeat themselves every week, the frequency in this case would be 52. Warehouse operations with a weekly cycle and operating seven days a week over a planning horizon of two years are illustrated in Figure 1.

Year 1 *e = 1*							Year 2 *e = 2*						
Mon *t=1*	Tue *t=2*	Wed *t=3*	Thu *t=4*	Fri *t=5*	Sat *t=6*	Sun *t=7*	Mon *t=8*	Tue *t=9*	Wed *t=10*	Thu *t=11*	Fri *t=12*	Sat *t=13*	Sun *t=14*

Figure 1. Epoch and Time Period Illustration for Daily Operations with a Weekly Cycle

Another example is the case of a seasonal warehouse with major sales periods during the summer and fall quarters and inventory accumulation during the winter and spring quarters. The appropriate minor time period in this case would be a quarter or a month. The major time period could be a year. Since each minor time period occurs only once per major time period, the frequency would be one. Sales growth can be easily accommodated since the winter quarter of year one would be a different minor time period than any other quarter in the planning horizon and would have its individual data values. Seasonal warehouse operations with a minor time period of a quarter and a planning horizon of two years are illustrated in Figure 2.

Year 1 *e = 1*				Year 2 *e = 2*			
Winter 2001 *t=1*	Spring 2001 *t=2*	Sum. 2001 *t=3*	Fall 2001 *t=4*	Winter 2002 *t=5*	Spring 2002 *t=6*	Sum. 2002 *t=7*	Fall 2002 *t=8*

Figure 2. Epoch and Time Period Illustration for Quarterly Warehouse Operations

Objective Function

The objective function is typically related to the economic performance of the warehousing system. The most common objective is the minimization of the sum of the time-discounted costs associated with operating the warehouse. Both one-time investment costs, annual or major time-unit fixed costs, and variable costs can be included in the objective function.

Functional Areas

The warehouse is modeled as a collection of components, also called departments or functional areas. These components are connected by a function flow network, which models the flow of materials from arrival at to departure from the warehousing system. The functional flow network is illustrated in Figure 3. The individual components are subject to constraints. The function flow network also requires conservation of flow constraints between the individual components. An example of a functional area would reserve storage.

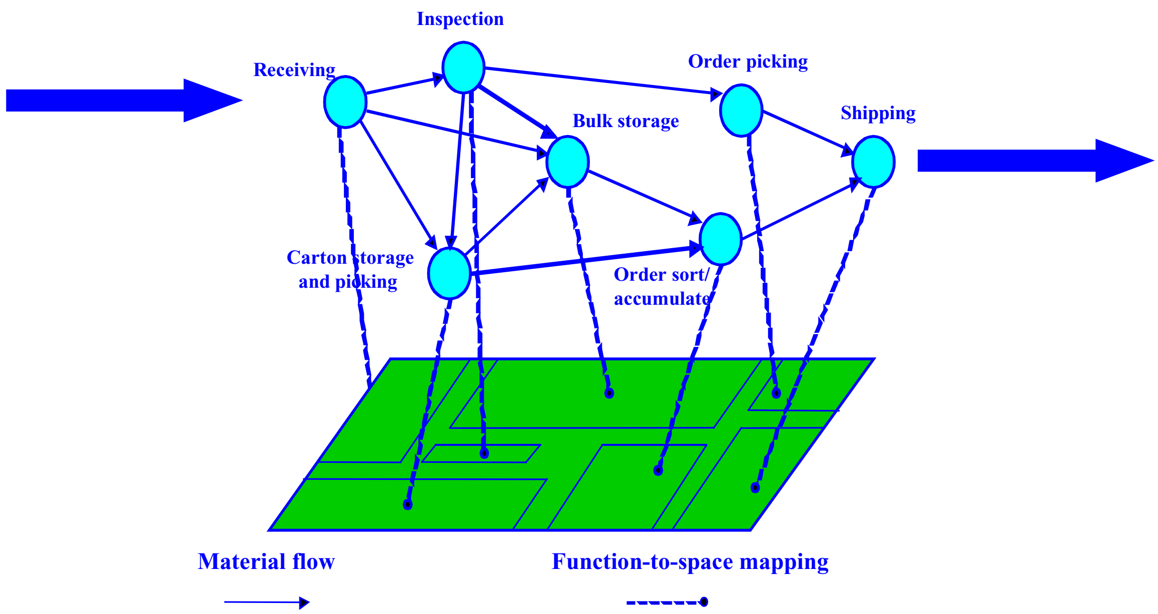

Figure 3. Warehouse Functional Flow Network Illustration

Commodity flows

Different product or commodity flows arrive at the warehouse, pass through the functional areas of the warehouse, and finally depart the warehousing system. The number of commodity flows modeled in the warehouse should be kept as small as possible to maintain control of the model size, data, and algorithm requirements, but it should be large enough to capture the major activities in the warehouse. Pareto analysis may be used to group the large number of SKUs in a small number of commodities with similar characteristics, such as flow path, physical characteristics, and schedules. A different commodity flow is also defined for the three major unit load sizes being stored and handled in the warehouse: pallet (also called unit load), case (also called box or carton), and item (also called piece).

x_{ijpt} Amount of flow of commodity p flowing from area i to functional area j during minor time period t.

v_{jlpt} Amount of flow of commodity p flowing through technology l of functional area j during minor time period t.

Observe that all material flows, stores, flow and storage capacities are defined in function of the minor time periods in the system.

Technologies

Each module or functional area that is implemented in the warehouse must use one or more technologies to execute its function. For example, the principal technologies for pallet-sized unit load storage are floor stacking, single deep rack, double deep rack, and deep lane storage. The main performance requirements for a pallet storage area are storage capacity and maximum input and output flows. The main resource requirements associated with a pallet storage area are floor space, cubic space, labor, material handling equipment acquisition and operating costs, and investment and operating capital. In the next paragraphs the variables and equations to model these characteristics will be developed.

L_j	The set of candidate technologies that can be used for functional area *j*
y_{jl}	Equals one if technology *l* of L_j is used in functional area *j*, otherwise it equals zero.
y_{j0}	Equals one if functional area j is not used and hence does not use any technology. Equivalently, $\left(1-y_{j0}\right)$ is used to indicate that a functional area is implemented.

The constraints that each functional area, if it is implemented, must at least use one technology are then expressed as:

$$\begin{aligned} & y_{jl} \le \left(1-y_{j0}\right) && \forall j, \forall l \\ & \sum_{l \in L_j} y_{jl} \ge \left(1-y_{j0}\right) && \forall j \end{aligned} \tag{1}$$

Technologies may have the following costs associated with them

$TechInvestCost_{jl}$	One-time investment cost, assumed to occur at time zero.
$TechFixedCost_{jlt}$	Fixed cost occurring during time period *t*
$TechFlowCost_{jlpt}$	Marginal cost for one unit of flow of commodity *p* to flow through functional area *j* using technology *l* during time period *t*

The maximum flow for an individual commodity through a technology is denoted by

$TechFlowCap_{jlpt}$ Flow capacity for commodity p through technology l of functional area j during time period t

The corresponding terms in the objective function are then

$$\sum_{j=1}^{N}\sum_{l\in L_j} TechInvestCost_j \cdot y_{jl} \tag{2}$$

$$\sum_{j=1}^{N}\sum_{l\in L_j}\sum_{t=1}^{T} cdf_t \cdot freq_t \cdot TechFixedCost_{jlt} \cdot y_{jl} \tag{3}$$

$$\sum_{j=1}^{N}\sum_{l\in L_j}\sum_{t=1}^{T} cdf_t \cdot freq_t \cdot TechFlowCost_{jlpt} \cdot v_{jlpt} \tag{4}$$

The flow capacity constraint is combined with the consistency or linkage constraint to

$$v_{jlpt} \le TechFlowCap_{jlpt} \cdot y_{jl} \tag{5}$$

The flow capacity of technology has to be determined in advance. It is assumed that each technology is sufficiently configured before it is entered into the model so that the flow capacity, the storage capacity, and the cost coefficients can be determined. For example, an AS/RS system with four or fives aisles would be modeled as two different technologies, one with four aisles and one with five aisles, because of the significant impact of the number of aisles on the throughput capacity and cost of the AS/RS.

Resources

If joint capacity restrictions exist that impact more than a single commodity, resources model these capacity restrictions. Resources are indexed by r and range from 1 to R. Typical resources are labor hours by labor grade, equipment hours by equipment type, space such as two-dimensional floor space and three-dimensional cubic space, and investment budget. By definition resources have a maximum availability during each time period.

$ResCap_{rt}$ Availability of resource r during time period t

The investment budget capacity constraint typically is modeled for time period zero. Each commodity that uses the resource consumes the resource at a constant rate and occurs a marginal cost

$ResProdReq_{rjlpt}$	Marginal consumption of resource *r* by one unit of flow or storage of commodity *p* in functional area *j* using technology *l* during time period *t*
$ResProdCost_{rjlpt}$	Marginal cost for resource *r* by one unit of flow or storage of commodity *p* in functional area *j* using technology *l* during time period *t*

The total resource cost and individual resource requirement in each time period are then computed with

$$\sum_{r=1}^{R}\sum_{j=1}^{N}\sum_{l\in L_j}\sum_{t=1}^{T} cdf_t \cdot freq_t \cdot ResProdCost_{rjlpt} \cdot v_{jlpt} \qquad (6)$$

$$\sum_{j=1}^{N}\sum_{l\in L_j}\sum_{p=1}^{P} ResProdReq_{rjlpt} \cdot v_{jlpt} \le ResCap_{rt} \qquad \forall r, \forall t \qquad (7)$$

A one-time investment cost and fixed cost per time period can be introduced for each technology by adding a binary variable indicating the resource is used at all during a time period or not.

Throughput Requirements

So far the upper bound on product flows (5) and available resources (7) equations have modeled the individual commodity and joint commodity capacity constraints. The next set of constraints assures that mandatory throughput requirements are met.

$FlowReq_{jpt}$	Minimum flow requirements for commodity *p* out of functional area *j* during time period *t*

$$\sum_{l\in L_j} v_{jlpt} \ge FlowReq_{jpt} \qquad \forall j, \forall p, \forall t \qquad (8)$$

This type of flow requirement constraint may not be present for any functional area except the departure area or shipping department. This ensures that the warehousing system can ship out the required number of pallets, cases, and items.

Generalized Conservation of Flow

There are three possible types of generalized conservation of flow constraints that must be enforced in order to yield a feasible warehouse design. The first type is the traditional balance of flow by commodity and time period of input and output flows in a functional area. The second type is the balance of flow across time periods, which incorporates the storage and inventory effects. The third type is the balance of flow among different commodities when commodities are transformed into another commodity. The prime example is a case picking functional area where all input flows are in pallets, but all output flows are in cartons or cases. The types are cumulative, in other words, type three constraints incorporate all effects of type one and type two. The goal is to keep the model as simple as possible, so if type one constraints suffice, type two and type three constraints will not be included in the model for that functional area.

Type One Constraints

Type one constraints ensure balance of input and output flows and consistency with the throughput flow for a functional area. It should be noted that type one constraints do not incorporate any relationship between material flows in different time periods.

$$\begin{aligned} \sum_{i=1}^{N} x_{ijpt} &= \sum_{l \in L_j} v_{jlpt} \qquad \forall j, \forall p, \forall t \\ \sum_{k=1}^{N} x_{jkpt} &= \sum_{l \in L_j} v_{jlpt} \qquad \forall j, \forall p, \forall t \end{aligned} \tag{9}$$

Type Two Constraints

Type two constraints incorporate in addition the initial and final inventory of the commodity in a functional area. Type two constraints model the relationships between material flows in different time periods.

u_{jlpt} Amount of flow of commodity p flowing into technology l of functional area j during time period t

v_{jlpt} Amount of flow of commodity p flowing out of technology l of functional area j during time period t

w_{jlpt} Amount of flow of commodity *p* stored in technology *l* of functional area *j* at the end of time period *t*

$$\begin{aligned}
&\sum_{i=1}^{N} x_{ijpt} = \sum_{l \in L_j} u_{jlpt} && \forall j, \forall p, \forall t \\
&\sum_{l \in L_j} u_{jlpt} + \sum_{l \in L_j} w_{jlp(t-1)} = \sum_{l \in L_j} v_{jlpt} + \sum_{l \in L_j} w_{jlpt} && \forall j, \forall p, \forall t \qquad (10) \\
&\sum_{k=1}^{N} x_{jkpt} = \sum_{l \in L_j} v_{jlpt} && \forall j, \forall p, \forall t
\end{aligned}$$

$TechStoreCost_{jlpt}$ Marginal cost for one unit of flow of commodity *p* to be stored in functional area *j* using technology *l* during time period *t*

The terms in the objective function related to storage cost are then

$$\sum_{j=1}^{N} \sum_{l \in L_j} \sum_{t=1}^{T} cdf_t \cdot freq_t \cdot TechStoreCost_{jlpt} \cdot w_{jlpt} \qquad (11)$$

The storage capacity constraint is combined with the consistency or linkage constraint to

$$w_{jlpt} \le TechStoreCap_{jlpt} \cdot y_{jl} \qquad (12)$$

Note that joint commodity storage capacity restrictions are modeled using resources. Examples of joint commodity storage capacities are cubic space or rack locations. The total resource cost and individual resource requirement in each time period associated with storing products are then computed with

$$\sum_{r=1}^{R} \sum_{j=1}^{N} \sum_{l \in L_j} \sum_{t=1}^{T} cdf_t \cdot freq_t \cdot ResProdCost_{rjlpt} \cdot w_{jlpt} \qquad (13)$$

$$\sum_{j=1}^{N} \sum_{l \in L_j} \sum_{p=1}^{P} ResProdReq_{rjlpt} \cdot w_{jlpt} \le ResCap_{rt} \qquad \forall r, \forall t \qquad (14)$$

The inventory cost associated with holding products in storage is given by

$$\sum_{p=1}^{P} \sum_{j=1}^{N} \sum_{l \in L_j} \sum_{t=1}^{T} cdf_t \cdot freq_t \cdot h_t \cdot ProdValue_{pt} \cdot w_{jlpt} \qquad (15)$$

$ProdValue_{pt}$ Value of a unit of commodity *p* during time period *t*

h_t Inventory holding cost rate, expressed in monetary units per monetary units per time period

For example, a value of 0.25 indicates the typical inventory holding cost rate of 25 cents per dollar of product value for holding the product one minor time period in inventory.

Type Three Constraints

Conservation of flow constraints of type three model the transformation of one commodity into another commodity in a functional area. This type of conservation constraints is typically associated with manufacturing and supply chains, but it occurs also in warehousing. The prime example of such transformation operations are case picking operations, where the products enter the area on pallets and leave the functional area as individual cases. Here two different commodities are required to model the different physical characteristics of the material flows, even though all products in the warehouse may follow the same flow path and be otherwise identical.

$COMP_p$ Set of commodities that can be converted into commodity *p*

$ASSY_p$ Set of commodities that commodity *p* can be converted into

$ProdTransform_{pq}$ Number of units of commodity *q* that correspond to one unit of commodity *p*

∂_{jlpqt} Number of units of commodity *p* converted into commodity *q* in functional area *j* using technology *l* during time period *t*

For example, assume that an average pallet contains 96 cases and that *p* indicates the pallet and *q* indicates the cases, then $ProdTransform_{pq}$ = 96 and $ProdTransform_{qp}$ = 0.010417.

$$
\begin{aligned}
&\sum_{i=1}^{N} x_{ijpt} = \sum_{l \in L_j} u_{jlpt} && \forall j, \forall p, \forall t \\
&\sum_{l \in L_j} u_{jlpt} + \sum_{l \in L_j} w_{jlp(t-1)} = \sum_{l \in L_j} v_{jlpt} + \sum_{l \in L_j} w_{jlpt} && \forall j, \forall p, \forall t \\
&\sum_{k=1}^{N} x_{jkpt} = \sum_{l \in L_j} v_{jlpt} + \\
&\qquad \sum_{q \in COMP_p} ProdTransform_{qp} \cdot \partial_{jlqpt} - \sum_{q \in ASSY_p} \partial_{jlpqt} && \forall j, \forall p, \forall t
\end{aligned}
\tag{16}
$$

The above set of constraints assumes that the transformation occurs on the outgoing flows from the functional area, in other words, there is no inventory of transformed (finished) commodity, only of the component commodities.

$TransformCap_{jlpqt}$	Transformation capacity for conversion of commodity p into commodity q in functional area j using technology l during time period t

The transformation capacity constraint is combined with the consistency or linkage constraint to

$$\partial_{jlpqt} \leq TransformCap_{jlpqt} \cdot y_{jl} \tag{17}$$

Finally, there may be a cost associated with the transformation from commodity p to commodity q.

$TransformCost_{jlpqt}$	Transformation cost of one unit of commodity p into commodity q in functional area j using technology l during time period t

The transformation costs are included in the objective function by

$$\sum_{p=1}^{P} \sum_{q=1}^{P} \sum_{j=1}^{N} \sum_{l \in L_j} \sum_{t=1}^{T} cdf_t \cdot freq_t \cdot TransformCost_{jlpqt} \cdot \partial_{jlpqt} \tag{18}$$

While each of the constraints or objective function components has a simple structure, the overall formulation represents a mixed-integer programming formulation. It is very important to keep the number of commodities to a minimum to constrain the overall size of the formulation. The greatest challenge however is to determine and validate the data required to populate this formulation.

Our next efforts will focus on constructing the formulation for a warehouse example out of the literature. This will be followed by computational experiment.

Iterative Warehouse Design Algorithm

The above formulation is posed before the layout of the warehouse is known, since at this time the required areas for the functional areas are unknown because their technologies have not yet been defined. This implies that the formulation cannot contain costs and resources used associated with inter-departmental moves and transportation. To determine the warehouse configuration and layout, we propose the following iterative algorithm.

1. Solve the ***Capacitated Material Flow*** (CMF) formulation developed above. For each functional area the solution determines the one or more technologies used, the required area for each functional area, and the material flows between the various functional areas.
2. Based on the required areas for each functional department and the material flows solve the ***Warehouse Block Layout*** (WBL) formulation. Traditional techniques for determining the block layout can be used. The resulting layout solution determines the location of each functional area and the distances between the various functional areas.
3. Compute the required transportation resources and cost and add them to the objective function of CMF to obtain the overall cost of warehousing operations.
4. If so desired, the cost parameters for the CMF formulation can be updated and CMF and WBL solved iteratively until both solutions converge.

$TransportCost_{ijpt}$	Transportation cost to transport one unit of commodity p from functional area i to functional area j during time period t

This cost is computed based on the solution of WBL and then used as a parameter in CMF. The terms in the objective function related to interdepartmental material flows in the CMF formulation are then

$$\sum_{p=1}^{P}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T} cdf_t \cdot freq_t \cdot TransportCost_{ijpt} \cdot x_{ijpt} \qquad (19)$$

Each commodity that uses a transportation resource consumes the resource at a constant rate and occurs a marginal cost

$ResProdReq_{rijpt}$	Marginal consumption of resource r by one unit of flow of commodity p from functional area i to functional area j during time period t

$ResProdCost_{rijpt}$ Marginal cost for resource *r* by one unit of flow or storage of commodity *p* from functional area *i* to functional area *j* during time period *t*

Note that again joint capacities and costs are modeled using resources. The total resource cost and individual resource requirements in each time period associated with moving products between departments are then computed with

$$\sum_{r=1}^{R}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T} cdf_t \cdot freq_t \cdot ResProdCost_{rijpt} \cdot x_{ijpt} \tag{20}$$

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{p=1}^{P} ResProdReq_{rijpt} \cdot x_{ijpt} \leq ResCap_{rt} \qquad \forall r, \forall t \tag{21}$$

Again the cost and resource consumption parameters are computed based on the solution of the WBL formulation and then used as parameters in the CMF formulation. Note that equations (6) and (7) model flows inside a functional area and that equations (20) and (21) model flows between two functional areas.

Finally, mandatory flow ratios and any other linear relationships between flows can be easily added to the CMF formulation. For example, the case that 20 % of the pallets leaving the receiving department (*i*) go to cross docking (*j*) and 80 % of the pallets go to pallet storage (*k*) is modeled as:

$$\begin{aligned} 0.2 \cdot \sum_{l \in L_i} v_{ilpt} &= \sum_{l \in L_j} v_{jlpt} \\ 0.8 \cdot \sum_{l \in L_i} v_{ilpt} &= \sum_{l \in L_k} v_{klpt} \end{aligned} \qquad \forall p, \forall t \tag{22}$$

To solve the above formulation, the following technologies are currently used. All the data parameters and solution variables are stored in an object-oriented database. To solve the mixed-integer formulations, we use the MIP module of CPLEX, a commercial linear programming solver. A custom program has been developed to extract the data from the database and populate the formulation and to extract the values of the decision variables of the solution and insert them back into the database.