Designing Robust Supply Chains

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Overview

- Supply chain strategic design problem
- Robustness and flexibility
- Hierarchical stochastic design procedure
- Computational experience
- Conclusions

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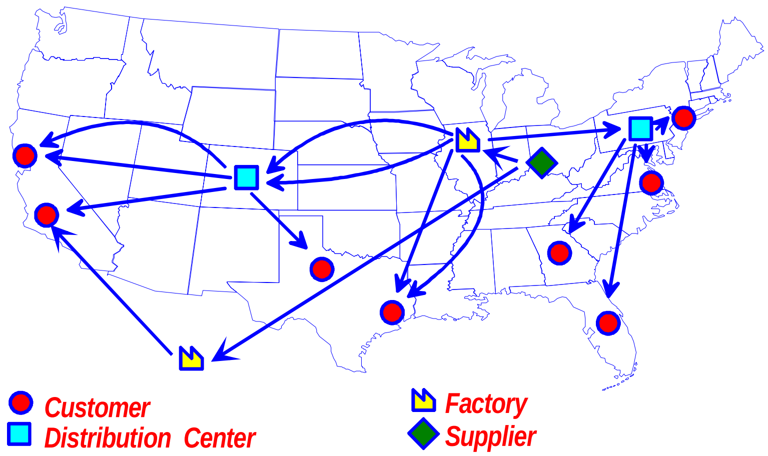
Supply Chain Network



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Supply Chain Strategic Design Problem

- Multicommodity, multiperiod, multi-echelon, multi-country capacitated network flow problem (nodes, arcs)
- Decision variables
  - Binary status variables for facility, technology, and machines
  - Continuous flow & storage variables
- Objective function, constraint matrix, right-hand side all can be stochastic

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Robustness and Flexibility

- Relative robustness, Kouvelis (1997)

$$\max_{s \in S} \left\{ \frac{z_s(x_R) - z_s^*(x_s^*)}{z_s^*(x_s^*)} \right\}$$

- Flexibility, Beamon (1998)
  - Unused capacity in a configuration
- Stability (?) for multi-period planning

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Hierarchical Stochastic Design Algorithm

- Determine a limited number M of feasible supply chain configurations
  - Each based on N scenarios
- For each configuration
  - Sample Ní scenarios
  - Solve linear network flow problems
  - Compute expected value and variance
- Select best configuration
  - Weighted objective or efficiency frontier

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Hierarchical Two-stage Formulation

$$\begin{aligned} Min \quad & cx + E[Q(x,\xi)] \\ s.t. \quad & Hx \le g \\ & x \in \{0,1\} \end{aligned} \qquad Q(x,\xi) = \begin{aligned} Min \quad & dy \\ s.t. \quad & Fy \le h - Ex \\ & y \ge 0 \end{aligned}$$

$$\begin{aligned} Min \quad & cx + \sum_s^N p_s d_s y_s \\ s.t. \quad & E_s x + F_s y_s \le h_s \\ & Hx \le g \\ & x \in \{0,1\}, y \ge 0 \end{aligned}$$

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Stochastic Strategic Supply Chain Solution Algorithm Challenges

- Very uncertain data
  - Requires large number of scenarios
  - Sample Average Approximation (SAA)
- Very large MIP formulation, but with significant structure
  - Primal Benders L-Shape decomposition converges too slowly
  - Acceleration techniques for Benders

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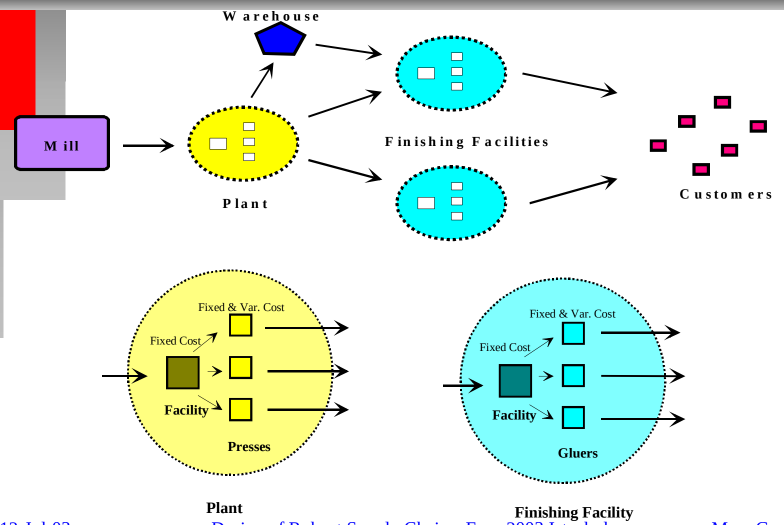
Two Stage Supply Chain With Machine Resources

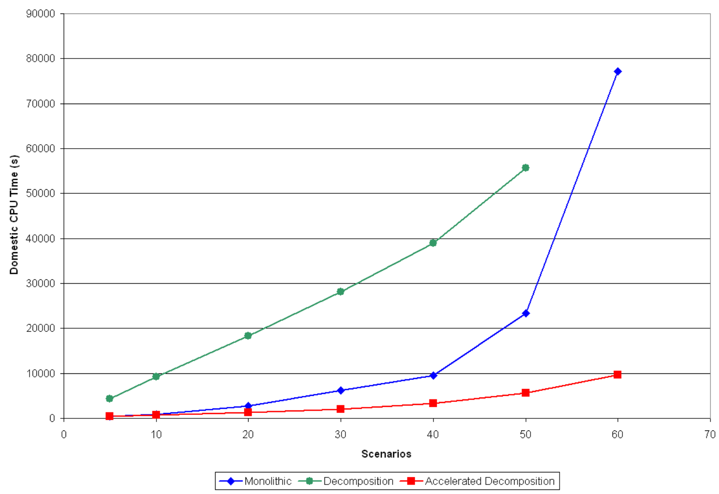


Domestic Case Formulation Characteristics

Problem Statistics	***N*=1**	***N*=20**	***N*=40**	***N*=60**
Constraints	7,822	156,440	312,880	469,320
- Inequality constraints	3,498	69,960	139,920	209,880
- Equality constraints	4,324	86,480	172,960	259,440
Variables	21,052	418,380	836,620	1,254,860
- Continuous variables	20,912	418,240	836,480	1,254,720
- Integer (binary) variables	140	140	140	140

N = Number of scenarios in master problem

Comparison of Run Times for Solution Algorithms (Domestic)



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Stochastic Solutions Robustness for Domestic Case

N=20, M=20, Ní=1000

Configuration	**MPP**	**SS-1**	**SS-2**	**SS-3**
Average obj. value (in million $)	116.77	111.03	111.03	111.05
Max	173.30	122.57	122.08	122.11
Min	99.02	100.38	100.14	100.10
Range	74.28	22.19	21.93	22.01
Standard Deviation	0.34	0.11	0.11	0.11
Absolute Gap	5.91	0.16	0.17	0.18
Relative Gap	0.066574	0.001454	0.001508	0.001619

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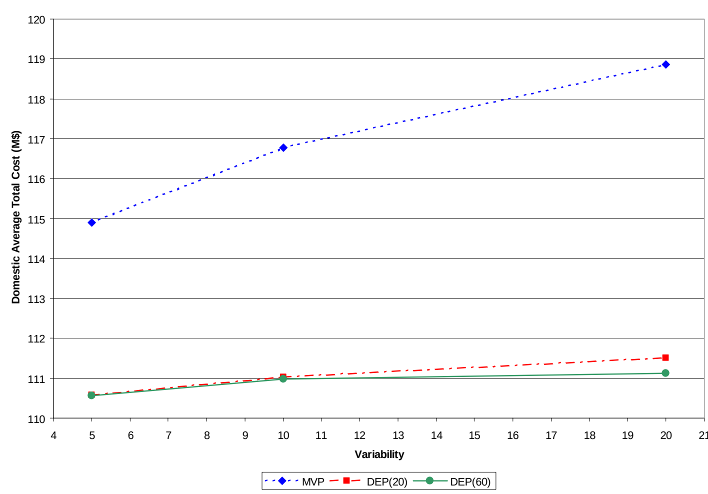
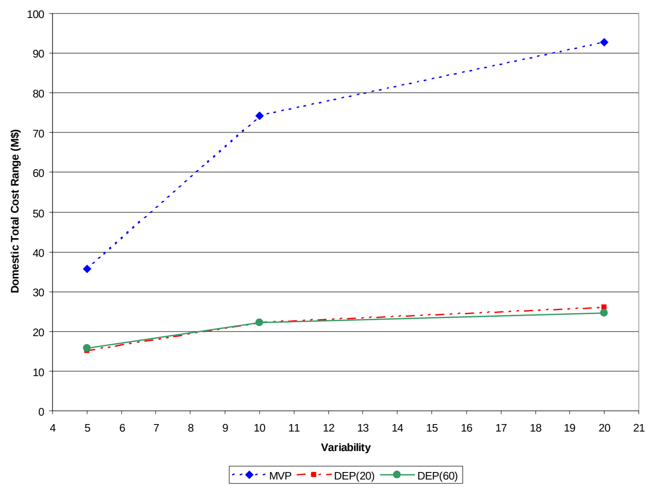
Impact of Variability

Problems	Std. Deviation for customer demand	Std. deviation for all other parameters
Medium variability problem	30%	10%
Low variability problem	15%	5%
High variability problem	40%	20%

Robustness of Average of DEP Solutions for Domestic Case



Robustness of Range of DEP Solutions for Domestic Case



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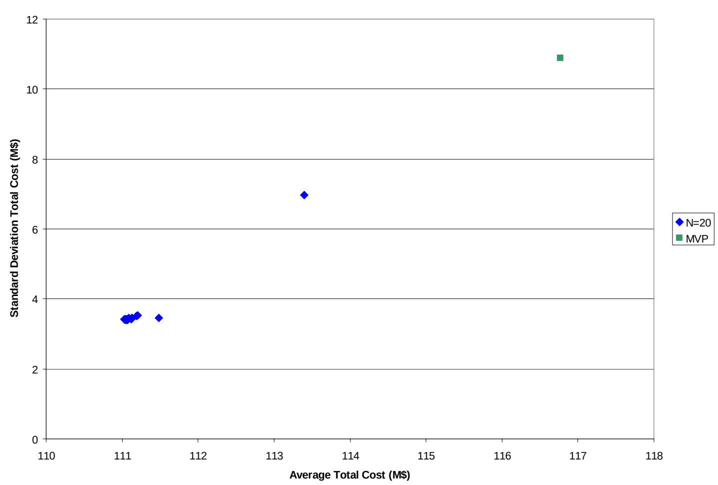
Efficiency Graph For Many Scenarios and Replications



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Global Case Problem Characteristics

Problem Statistics	*N*=1	*N*=10	*N*=20	*N*=60
Constraints	1,467	14,670	29,340	88,020
- Inequality constraints	402	4,020	8,040	24,120
- Equality constraints	1,065	10,650	21,300	63,900
Variables	6,894	68,310	136,550	409,510
- Continuous variables	6,824	68,240	136,480	409,440
- Integer (binary) variables	70	70	70	70

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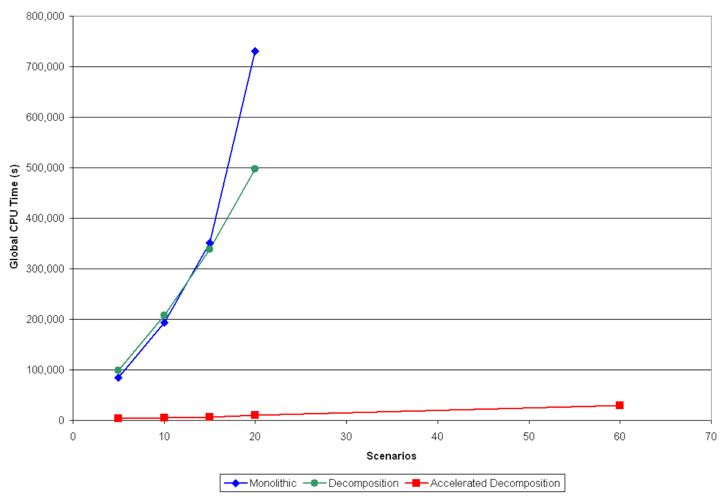
Comparison of Run Times for Solution Algorithms



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Stochastic Solutions Robustness for the Global Case

N=60, M=10, Ní=1000

	MPP	SS
Average NCF	51.021	54.095
Std. Deviation	0.127	0.119
Max NCF	66.996	68.063
Min NCF	31.355	46.531
Range	35.641	21.533
Absolute Gap	3.166	0.092
Relative Gap	0.058425	0.001694

(in million $, except the relative gap)

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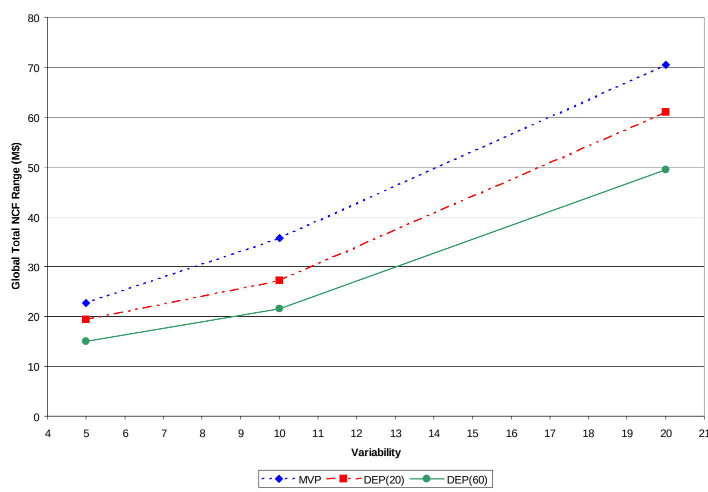
Robustness of Range of DEP Solutions for Global Case



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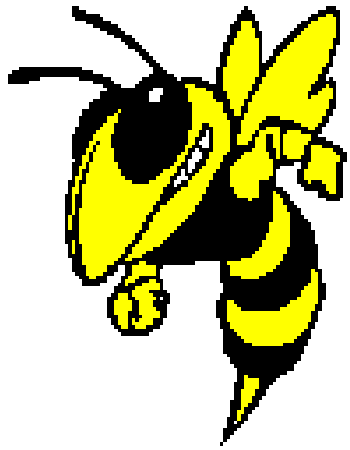
Computational Experiment Conclusions: Robustness

- Designing for average parameters yields a dominated configuration
  - Worse mean and standard deviation
  - Difference larger for more variable data
- More scenarios yield more robust solution
- Statistics stable after 500 samples
- Report multi-criteria solutions diagram

Conclusions

- Good definitions and measures for flexibility and robustness are lacking
- Current methodology is deterministic design and sensitivity or few-scenario analysis
- Many-scenario solutions are more robust
- Only accelerated hierarchical design algorithm fast enough

Thank You
Can I Answer Any Questions?



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