Logistics Systems Design: Inventory Systems

1. Introduction
2. Forecasting
3. Transportation Systems
4. Transportation Models
5. Inventory Systems
6. Supply Chain Systems

Inventory Systems Overview

❶ Introduction

❷ Independent Demand Systems

❸ Dependent Demand Systems

Inventory Systems Overview

❶ Introduction

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Inventory

- Material Being Held in Storage
- Buffer Between and Decoupling of Two Subsequent Processes

Historical Perspective on Inventory

- Before 20th Century Desirable Wealth
  - Maximized
- Early 20th Century Wasteful
  - Minimized
- Currently an Expensive But Necessary Evil
  - Tradeoff or Balance

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Inventory Objective and Cost

- Provide Some Specified Service Level at Minimum Cost
- Common Service Level Definition = Proportion of Long Range Demand Delivered From Inventory (P2 or β)
- Cost = 25 % of Value per Year and Rising

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Customer Service Level Definitions

- Cycle service level α (P1) long range probability of no stock-out (count)
- Fill rate β (P2) long range fraction of demand delivered from inventory (size)
- γ (P3) ratio of cumulative unsatisfied demand over total average demand (size and time),
- ready rate alternative definition

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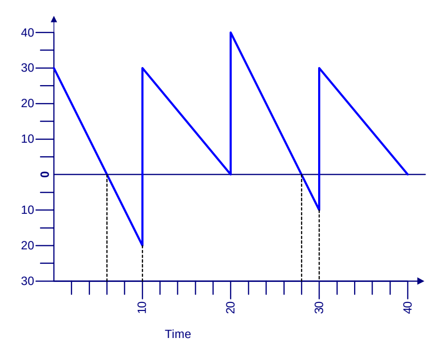
Customer Service Levels Example Demand Graph



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Customer Service Levels Example Computations

$$d = \frac{\sum_{t=1}^{T} d_t}{T} = \frac{160}{40} = 4$$

$$\alpha = 1 - \frac{2}{4} = 0.50$$

$$\beta = 1 - \frac{20+10}{160} = 0.81$$

$$\gamma = 1 - \frac{(5+10+15+20+5+10)/40}{4} = 0.59$$

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Inventory Performance

- Stock Turnover (Turns per Year)
  - Cost of Annual Sales Divided by Average Value of Inventory
- Average Time in Inventory (Years)
  - Inverse of Stock Turnover

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Inventory Justification

- Economies of Scale
  - Production
  - Transportation
- Response Time Constraints
  - Emergency
  - Seasonal
- Required Aging Processes
- Hedging Against Price Changes

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Inventory Justification

- Amplified by Uncertainty
  - Demand
  - Production yield and lead time
  - Transportation time
  - Sales price and exchange rates

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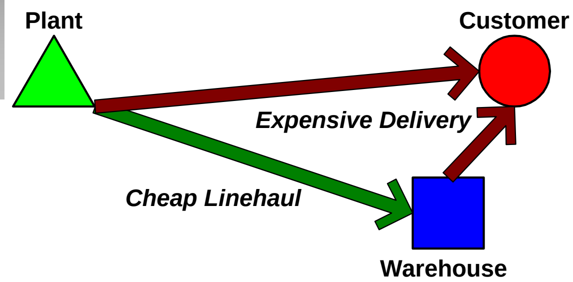
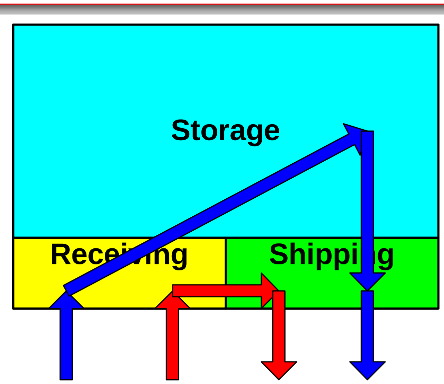
Inventory Tradeoffs

- Tradeoff with Transportation Costs and Schedules
- Tradeoff with Production or Purchasing Costs and Schedules
- Tradeoff with Lost Profit and Lost Sales

Direct Shipping Cost Tradeoff



Cross Docking Material Handling Tradeoff



Inventory Types

- Pipeline Inventory
- Cycle Inventory
- Safety Inventory
- Seasonal Inventory
- Aging Inventory
- Speculative Inventory

Inventory Materials

- Raw Materials
  - Incoming to Organization
- Work in Process
  - Internal to Organization
- Finished Goods
  - Outgoing from Organization

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Inventory Questions

- What to Store
- Where to Store
- When to Place an Order
- How Much to Order

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Inventory Costs

- Unit Cost or Unit Value ($/Unit)
- Reorder Cost ($/Order)
- Holding Cost ($/Unit-Year)
  - Holding cost rate ($/$-Year)
- Shortage Cost ($/Unit)
  - Backorder cost
  - Lost sales cost

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Inventory Policy Classes

- Fixed Order Quantity Systems
  - Adjust order time
  - Small irregular demand
- Fixed Order Frequency Systems
  - Adjust order quantity
  - High or regular demand
- Direct Demand Satisfaction
  - Adjust quantity and time
  - Push or Pull

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Pull Inventory Policies

- Each Stocking Point Considered Independent
- Replenishment Based on Local Conditions
- Precise Local Inventory Control
- Ignores Impact on Other Elements of the Supply Chain
- Retail Systems

Push Inventory Systems

- Stocking Point Viewed as Component in the Supply Chain
- Replenishment Based on Forecasted Requirements
- Central Inventory Control
- Inventory Levels are Set Collectively
- Depends Strongly on Forecast Quality
- Production Systems

Inventory Systems Overview

1. Introduction
2. Independent Demand Systems
3. Dependent Demand Systems

Independent Demand Systems

- Deterministic or Known Demand
- Stochastic or Uncertain Demand

Pipeline Inventory

- In-Transit or Aging & Curing Inventory
- Notation
  - PI = Pipeline inventory (units)
  - D = Total demand (units / year)
  - TT = Transit time (years)

$$PI = D \cdot TT$$

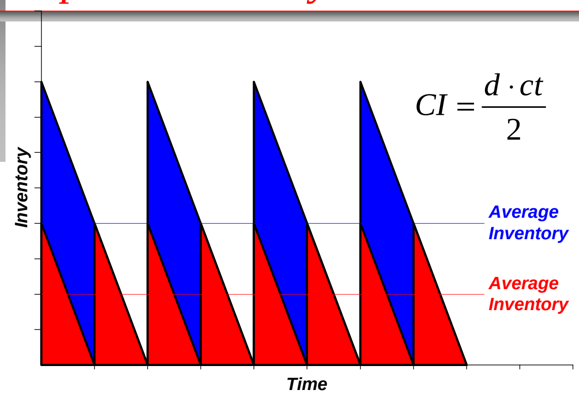
Cycle Inventory

- Transition Between Regular Input and Output Quantities (Batch Sizes)
- Notation
  - CI = Cycle inventory (units)
  - d = Periodic demand (units / period)
  - CT = Cycle time (years)
  - ct = Cycle time (periods)

$$CI = \frac{D \cdot CT}{2} = \frac{d \cdot ct}{2}$$

Cycle Inventory Versus Replenishment Cycle Time

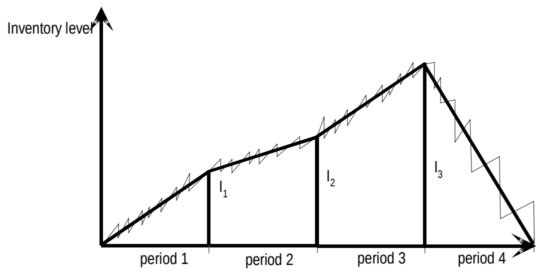
$$CI = \frac{d \cdot ct}{2}$$



Seasonal Inventory

- Reduced production capacity by inventory accumulation in low demand periods
- Notation
  - h_k holding cost rate for period k ($/$.t)
  - I_k inventory at end of period k
  - v product value
  - SIC seasonal inventory cost

Seasonal Inventory With Constant Rates



Seasonal Inventory Equations

- Constant demand and production rate per period

$$SIC = v \sum_{t=1}^{N} \frac{h_t \left(I_{t-1} + I_t \right)}{2}$$

- Equal periods and cyclical system

$$SIC = v \cdot h \sum_{t=1}^{N} I_t$$

Repetitive Order Quantities With Instantaneous Resupply

$$TC(Q) = IC + OC = \frac{hc \cdot Q}{2} + \frac{D}{Q} oc \qquad Q^* = \sqrt{\frac{2 \cdot D \cdot oc}{hc}}$$

$$TC(Q^*) = \frac{hc}{2} \sqrt{\frac{2 \cdot D \cdot oc}{hc}} + \frac{D \cdot oc}{\sqrt{\frac{2 \cdot D \cdot oc}{hc}}}$$

$$= \sqrt{2 \cdot D \cdot oc \cdot hc}$$

Inventory Policies Comparison Example: Data

Microsoft Excel - Inventory EOQ Variants Harvey ...

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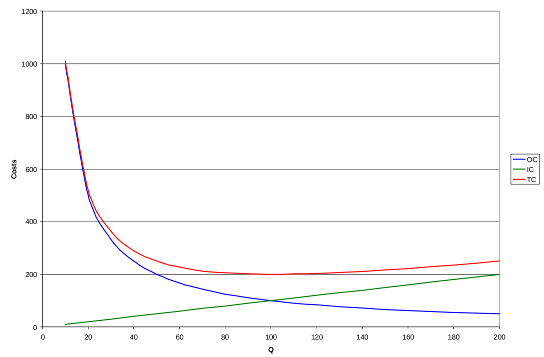
	A	B	C	D
13	Purchase Price	p	10	$/unit
14	Lead Time	LT	0.5	year
15	Holding Cost Rate	hcr	0.2	/year
16	Shortage Cost	sc	25	$/unit
17	Ordering Cost	oc	50	$/order
18	Demand Rate	d	200	units/year
19	SD Demand Lead Time	sdlt	25	units
20	Probability No-Stockout	F(R)	0.98	
21	Fill Rate	fr	0.98	
22	SD Lead Time	slt	0.125	years
23				
24	Mean Demand Lead Time	dlt	100	units
25	SD Demand	sd	35.4	units

Book / Overhead / Sheet3 /

Graph of Cost Tradeoffs for Known Demand



Inventory Policies Comparison: Known Demand (Instantaneous)

$$Q^* = \sqrt{\frac{2 \cdot D \cdot oc}{hc}} \qquad Q = \sqrt{\frac{2 \cdot 200 \cdot 50}{0.2 \cdot 10}} = 100$$

$$TC(Q) = IC + OC = \frac{hc \cdot Q}{2} + \frac{D}{Q} oc \qquad TC = \frac{100 \cdot 0.2 \cdot 10}{2} + \frac{200}{100} 50 = 200$$

$$R = d \cdot LT \qquad R = 200 \cdot 0 = 0$$

Inventory Policies Comparison Example: Results

Microsoft Excel - Inventory EOQ Variants Harvey Foods Example.xls

	A	B	C	D	E	F
1	Type	Q	R	SI	AI	TC
2	Deterministic Demand	100	0	0	50	200.0
3	+ Lead Time	100	100	0	50	200.0
4	Stochastic Demand	100	151	51	101	302.0
5	Shortage Cost (Seq)	100	151	51	101	311.2
6	Shortage Cost (Iter)	111	143	43	99	308.5
7	Type 2 Service (Seq)	100	126	26	76	252.0
8	Type 2 Service (Iter)	114	124	24	81	249.5
9	SL2 (Iter) + Stoc. Lead Time	119	139	39	99	281.5

Optimal EOQ Derivation

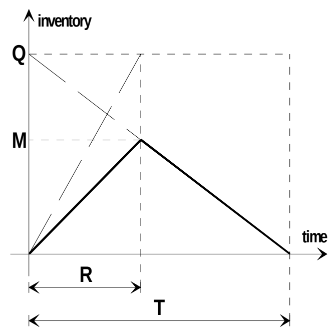
$$\frac{d(TC)}{dQ} = \frac{hc}{2} - \frac{D \cdot oc}{Q^2}$$

$$\frac{d^2(TC)}{dQ^2} = \frac{2 \cdot D \cdot oc}{Q^3} > 0$$

$$\left.\frac{d(TC)}{dQ}\right|_{Q \to 0} = -\left.\frac{D \cdot oc}{Q^2}\right|_{Q \to 0} = -\infty$$

$$\frac{d(TC(Q^*))}{dQ} = \frac{hc}{2} - \frac{D \cdot oc}{Q^{*2}} = 0 \qquad Q^{*2} = \frac{2 \cdot D \cdot oc}{hc}$$

Inventory Pattern for Batch Manufacturing



EOQ Formulas

$$M = Q - d \cdot R = Q - d\left(\frac{Q}{p}\right) = Q\left(1 - \frac{d}{p}\right)$$

$$TC = IC + MC$$

$$= \frac{HC \cdot M \cdot T}{2} + FC + VC \cdot Q$$

$$= \frac{HC \cdot Q \cdot (1 - d/p) \cdot Q}{2 \cdot d} + FC + VC \cdot Q$$

$$= FC + VC \cdot Q + \frac{HC(p-d)}{2 \cdot p \cdot d} Q^2$$

EOQ Formulas continued

$$tc = \frac{TC}{Q} = \frac{FC}{Q} + VC + \frac{HC(p-d)}{2pd} Q$$

$$\frac{d(tc)}{dQ} = -\frac{FC}{Q^2} + \frac{HC(p-d)}{2pd} Q = 0$$

$$Q^* = \sqrt{\frac{2 \cdot FC \cdot d}{HC(1 - \frac{d}{p})}}$$

$$\frac{d^2(tc)}{dQ^2} = \frac{2 \cdot FC}{Q^3} + \frac{HC \cdot (p-d)}{2pd} > 0$$

Batch Manufacturing Costs and Quantities

- Total Cost

$$TC = IC + MC = FC + VC \cdot Q + \frac{HC(p-d)}{2 \cdot p \cdot d} Q^2$$

- Optimal Batch Size

$$Q^* = \sqrt{\frac{2 \cdot FC \cdot d}{HC(1 - \frac{d}{p})}}$$

- Optimal Total Cost

$$TC^* = 2 \cdot FC + VC \sqrt{\frac{2pd \cdot FC}{HC(p-d)}}$$

Lead Time for Resupply Reorder Point

- Deterministic Demand (D) and Demand rate (d)
- Deterministic Lead Time (LT)
- Reorder Point = Quantity On Hand when Order is Placed

$$R = d \cdot LT$$

$$dlt = d \cdot LT$$

If Reorder Point Exceeds Maximum Inventory

- When lead time exceeds replenishment cycle length LT>Q/d, R > Q
- Two Equivalent Interpretations
  - Order when on-hand plus on-order inventory falls below reorder point
  - Order when on-hand inventory falls below reorder point modulo order quantity (or order quantity for exact multiple)

Inventory Policies Comparison: Known Demand

$$Q^* = \sqrt{\frac{2 \cdot D \cdot oc}{hc}} \qquad Q = \sqrt{\frac{2 \cdot 200 \cdot 50}{0.2 \cdot 10}} = 100$$

$$TC(Q) = IC + OC \qquad TC = \frac{100 \cdot 0.2 \cdot 10}{2} + \frac{200}{100} 50$$

$$= \frac{hc \cdot Q}{2} + \frac{D}{Q} oc \qquad = 200$$

$$R = d \cdot LT \qquad R = 200 \cdot 0.5 = 100$$

Inventory Policies Comparison Example: Results

Microsoft Excel - Inventory EOQ Variants Harvey Foods Example.xls

Type	Q	R	SI	AI	TC
Deterministic Demand	100	0	0	50	200.0
+ Lead Time	100	100	0	50	200.0
Stochastic Demand	100	151	51	101	302.0
Shortage Cost (Seq)	100	151	51	101	311.2
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Type 2 Service (Seq)	100	126	26	76	252.0
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SL2 (Iter) + Stoc. Lead Time	119	139	39	99	281.5

Inventory Systems Overview

❶ Introduction

❷ Independent Demand Systems

- •Continuous Review
- •Periodic Review

❸ Dependent Demand Systems

Independent Demand Systems

- Deterministic or Known Demand
- Stochastic or Uncertain Demand

Basic Pull Inventory Policies

- Single Order Quantity
  - Equal Marginal Profit and Loss
- Cyclical Order Quantities
  - EOQ (Infinite and finite production rate)

Single Order Quantity

- Single Ordering Opportunity
- Tradeoff between Shortage and Excess Costs
- “Newsboy” / Newsvendor Problem

Single Order Quantity Notation

$$Q = \text{purchased inventory}$$
$$D = \text{actual demand}$$
$$f(x) = \text{demand distribution}$$
$$F(x) = \text{cumulative demand distribution}$$
$$\bar{d} = \text{expected demand}$$
$$p = \text{sales price}$$
$$c = \text{purchase price}$$
$$s = \text{salvage value}$$

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Single Order Quantity Expected Cost

$$c_s = p - c \quad \text{if } Q \le D \quad (\text{marginal profit})$$
$$c_e = c - s \quad \text{if } Q > D \quad (\text{marginal loss})$$

$$G(Q) = c_e \int_0^Q (Q - x) f(x) dx + c_s \int_Q^\infty (x - Q) f(x) dx$$

$$\frac{dG(Q)}{dQ} = c_e \int_0^Q f(x) dx + c_s \int_Q^\infty -f(x) dx$$
$$= c_e F(Q) - c_s (1 - F(Q))$$

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Single Order Quantity Optimal Inventory

$$\frac{d^2 G(Q)}{dQ^2} = (c_s + c_e) f(Q) \ge 0 \qquad \forall Q$$

$$\left.\frac{dG(Q)}{dQ}\right|_{Q=0} = c_e F(0) - c_s (1 - F(0)) = -c_s < 0$$

$$\frac{dG(Q^*)}{dQ} = (c_e + c_s) F(Q^*) - c_s = 0$$

$$F(Q^*) = \frac{c_s}{c_e + c_s}$$

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Single Order Quantity by Equilibrium Theory

- At the optimal inventory level the expected profit of selling one more items equals the expected loss of one item of excess inventory

$$c_s (1 - F(Q^*)) = c_e F(Q^*)$$

$$F(Q^*) = \frac{c_s}{c_e + c_s}$$

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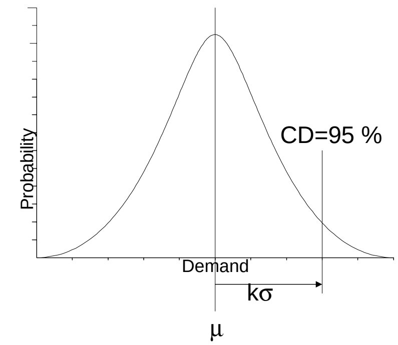
Optimal Inventory Level From Cumulative Frequency Lookup

- Compute demand frequencies and sort by increasing demand
- Compute cumulative demand frequencies

$$F\left(Q^*\right) = \frac{c_s}{c_e + c_s}$$

$$Q^* = \min\left\{ x \mid F(x) \geq \frac{c_s}{c_e + c_s} \right\}$$

Normally Distributed Demand



Single Order Quantity Optimal Inventory From Distribution

$$z^* = N^{-1}\left(\frac{c_s}{c_s + c_e}\right)$$

$$Q^* = \bar{d} + z^* \sigma_D$$

Testing If Demand Distribution Is Normal

- χ^2 Goodness of Fit Test
- Coefficient of Variation < 0.5
- Probability of Negative Demand Less Than 2 %
- Probability of Demand More Than Mean Plus Two Standard Deviations Less Than 2 %

Testing If Demand Distribution Is Normal (Formulas)

$$CV = \frac{\sigma}{\mu} \leq 0.5$$

$$P[x < 0] = F(0) \leq 0.02$$

$$P[x > \mu + 2\sigma] = 1 - F(\mu + 2\sigma) \leq 0.02$$

Single Order Quantity Example: Data

Microsoft Excel - Newsboy Example.xls

	A	B	C	D	E	F	G	H	I	J
1	15	19	9	12	9	22	4	7	8	11
2	14	11	6	11	9	18	10	0	14	12
3	8	9	5	4	4	17	18	14	15	8
4	6	7	12	15	15	19	9	10	9	16
5	8	11	11	18	15	17	19	14	14	17
6	13	12								

Data = Actual Sales Plus Unmet Demand

Single Order Quantity Example: Frequency Distributions

Microsoft Excel - Newsboy Example.xls

	B	C	D	E
1	*Bin*	*Frequency*	Rel Freq	*Cum Freq*
2	0	1	0.01923	0.0192
3	1	0	0.00000	0.0192
4	2	0	0.00000	0.0192
5	3	0	0.00000	0.0192
6	4	3	0.05769	0.0769
7	5	1	0.01923	0.0962
8	6	2	0.03846	0.1346
9	7	2	0.03846	0.1731
10	8	4	0.07692	0.2500
11	9	6	0.11538	0.3654
12	10	2	0.03846	0.4038
13	11	5	0.09615	0.5000
14	12	4	0.07692	0.5769
15	13	1	0.01923	0.5962
16	14	5	0.09615	0.6923
17	15	5	0.09615	0.7885
18	16	1	0.01923	0.8077
19	17	3	0.05769	0.8654
20	18	3	0.05769	0.9231
21	19	3	0.05769	0.9808
22	20	0	0.00000	0.9808
23	21	0	0.00000	0.9808
24	22	1	0.01923	1.0000
25	More	0	0.00000	
26		52	1.00000	

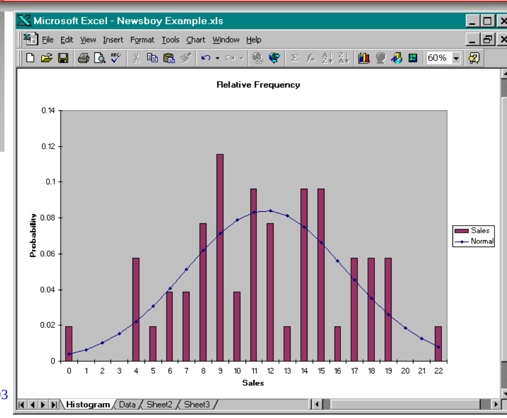
$$c_s = 75 - 25 = 50$$

$$c_e = 25 - 10 = 15$$

$$F\left(Q^*\right) = \frac{50}{50+15} = 0.77$$

$$Q^* = \min\left\{x \mid F(x) \geq 0.77\right\} = 15$$

Single Order Quantity Example: Histogram



Single Order Quantity Example: Optimal Inventory

Microsoft Excel - Newsboy Example.xls

	G	H	I	J	K
1	Mean	11.73			
2	Std Dev	4.741			
3					
4	Sales	75			
5	Purchase	25		Profit	50
6	Salvage	10		Loss	15
7					
8	Ratio	0.77			
9	Opt z	0.74			
10	Opt Inv.	15.2			

Histogram / Data / Inventory / S

$$11.73 + 0.74 \cdot 4.74 = 15.2 \qquad F = \frac{50}{65} = 0.77$$

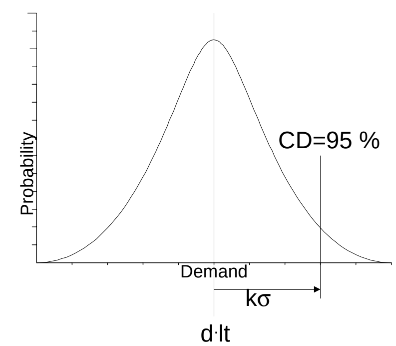
Basic Pull Inventory Policies

- Single Order Quantity
  - Equal Marginal Profit and Loss
- Cyclical Order Quantities
  - EOQ (Infinite and finite production rate)

Safety Stock Inventory Factor

- Customer Service Measured by:
  - Probability of Stockout (Type 1, α)
  - Fraction Delivered from Inventory (Type 2, β)
- Common Rule of Thumb:
  Safety Inventory Equals Time Length Multiplied by Demand (Linear Policy)
- Lead Time: Time From Placement to Arrival of Order

Demand During Lead Time and Stockout Probability (Type 1)



Reorder Point for Uncertain Demand (Sequential Decision)

- Sequentially Determine Q then R
- Approximation
- z Derived from Probability In Stock during Lead Time (Type 1)

$$Q^* = \sqrt{\frac{2 \cdot D \cdot oc}{hc}}$$

$$s_{dlt} = s_d \sqrt{lt} \qquad \sigma_{dlt}^2 = \sum_{1}^{lt} \sigma_d^2$$

$$R^* = d \cdot lt + z \cdot s_{dlt}$$

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Average Inventory and Total Cost (No Stockout Cost)

$$AI = CI + SI = \frac{Q}{2} + z \cdot s_{dlt}$$

$$TC = \frac{D}{Q} oc + hc \frac{Q}{2} + hc \cdot z \cdot s_{dlt}$$

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Inventory Policies Comparison: Type 1 Service Level, F(R)<α

$$Q^* = \sqrt{\frac{2 \cdot D \cdot oc}{hc}} \qquad Q = \sqrt{\frac{2 \cdot 200 \cdot 50}{0.2 \cdot 10}} = 100$$

$$s_{dlt} = s_d \sqrt{LT} \qquad s_{dlt} = \sqrt{0.5} \cdot 35.4 = 25$$

$$R^* = d \cdot LT + z \cdot s_{dlt} \qquad R = 200 \cdot 0.5 + 2.05 \cdot 25 = 151$$

$$TC = \frac{D}{Q} oc + hc \frac{Q}{2} + hc \cdot z \cdot s_{dlt}$$

$$TC = \left(\frac{100}{2} + 2.05 \cdot 25 \right) 0.2 \cdot 10 + \frac{200}{100} 50 = 303$$

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Inventory Policies Comparison Example: Results

Microsoft Excel - Inventory EOQ Variants Harvey Foods Example.xls

	A	B	C	D	E	F
1	Type	Q	R	SI	AI	TC
2	Deterministic Demand	100	0	0	50	200.0
3	+ Lead Time	100	100	0	50	200.0
4	Stochastic Demand	100	151	51	101	302.0
5	Shortage Cost (Seq)	100	151	51	101	311.2
6	Shortage Cost (Iter)	111	143	43	99	308.5
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Expected Units Out of Stock: Unit Normal Loss Function

$$n(R) = \int_R^\infty (x - R) f(x) dx$$

$$L(z) = \int_z^\infty (t - z) \phi(t) dt$$

$$n(R) = s_{dlt} L\left(\frac{R - d \cdot LT}{s_{dlt}}\right) = s_{dlt} \cdot L(z)$$

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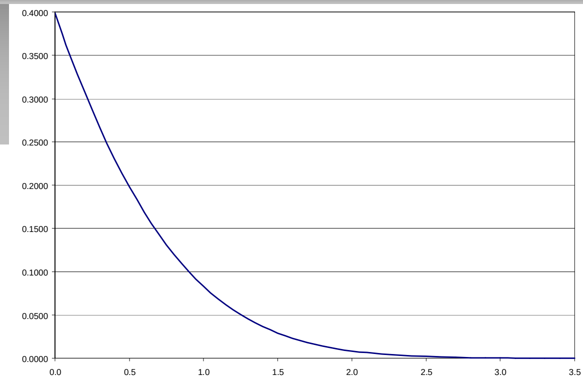
Unit Loss Function



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Total Cost for Uncertain Demand (Known Shortage Cost)

$$TC(Q) = \frac{D}{Q} oc + hc \frac{Q}{2} + hc \cdot z \cdot s_{dlt} + \frac{D}{Q} sc \cdot s_{dlt} \cdot L(z)$$

$$TC(Q,R) = \frac{D}{Q}\left(oc + sc \cdot s_{dlt} \cdot L(z)\right) + hc\left(\frac{Q}{2} + R - d \cdot LT\right)$$

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Inventory Policy Comparison: Known Stockout Costs

$$TC(Q,R) = \frac{D}{Q}\left(oc + sc \cdot s_{dlt} \cdot L(z)\right) + hc\left(\frac{Q}{2} + R - d \cdot LT\right)$$

$$TC = \frac{200}{100}\left(50 + 25 \cdot 25 \cdot 0.0073\right) + \\ = 10 \cdot 0.2\left(\frac{100}{2} + 151 - 200 \cdot 0.5\right) = 312$$

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Inventory Policies Comparison Example: Results

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4	Stochastic Demand	100	151	51	101	302.0
5	Shortage Cost (Seq)	100	151	51	101	311.2
6	Shortage Cost (Iter)	111	143	43	99	308.5
7	Type 2 Service (Seq)	100	126	26	76	252.0
8	Type 2 Service (Iter)	114	124	24	81	249.5
9	SL2 (Iter) + Stoc. Lead Time	119	139	39	99	281.5

Optimal Q and R (Iterative Method)

- Start with Q Determined by EOQ
- Determine Q and R Iteratively until No Further Change

Iterative Formulas for Determination of Q and R

$$Q = \sqrt{\frac{2D\left[oc + sc \cdot n(R)\right]}{hc}} = \sqrt{\frac{2D\left[oc + sc \cdot s_{dlt} \cdot L(z)\right]}{hc}}$$

$$P_{instock} = F(R) = \int_0^R f(x)dx = 1 - \frac{Q \cdot hc}{sc \cdot D}$$

Type 1 Service Level Iterations

Microsoft Excel - Inventory EOQ Variants Harvey F...

	J	K	L	M	N	O
54	Q	F(R)	z	R	L(z)	n(R)
55	100.0	0.9600	1.7507	143.8	0.0161	0.4037
56	109.6	0.9561	1.7076	142.7	0.0179	0.4487
57	110.7	0.9557	1.7033	142.6	0.0181	0.4536
58	110.8	0.9557	1.7028	142.6	0.0182	0.4541
59	110.8	0.9557	1.7027	142.6	0.0182	0.4542

Inventory Policies Comparison Example: Results

Microsoft Excel - Inventory EOQ Variants Harvey Foods Example.xls

	A	B	C	D	E	F
1	Type	Q	R	SI	AI	TC
2	Deterministic Demand	100	0	0	50	200.0
3	+ Lead Time	100	100	0	50	200.0
4	Stochastic Demand	100	151	51	101	302.0
5	Shortage Cost (Seq)	100	151	51	101	311.2
6	Shortage Cost (Iter)	111	143	43	99	308.5
7	Type 2 Service (Seq)	100	126	26	76	252.0
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Book / Overhead / Sheet3

Service Level Type 2 or Fill Rate

- Ratio of Long Range Expected Total Units Delivered out of Inventory Over Total Demand
- Complement of Expected Total Units Out of Stock Over Total Demand D
- Expected Units Out of Stock per Cycle n(R)

$$SL = 1 - \frac{(D/Q)n(R)}{D} = 1 - \frac{n(R)}{Q}$$

Inventory Comparison Type 2 Service (Sequential)

$$Q = \sqrt{\frac{2 \cdot oc \cdot D}{hc}} \qquad Q = \sqrt{\frac{2 \cdot 200 \cdot 50}{0.2 \cdot 10}} = 100$$

$$L(z) = \frac{Q(1-SL)}{s_{dlt}} \qquad L(z) = \frac{100(1-0.98)}{25} = 0.08$$

$$R = d \cdot LT + z \cdot s_{dlt} \qquad z = 1.02$$

$$R = 200 \cdot 0.5 + 1.02 \cdot 25$$

$$= 126$$

Inventory Comparison Cost: Type 2 Service (Sequential)

$$AI = CI + SI = \frac{Q}{2} + z \cdot s_{dlt}$$

$$TC = \frac{D}{Q} oc + hc \frac{Q}{2} + hc \cdot z \cdot s_{dlt}$$

$$TC = \left(\frac{100}{2} + 1.02 \cdot 25 \right) 0.2 \cdot 10 + \frac{200}{100} 50 = 251$$

Inventory Policies Comparison Example: Results

Microsoft Excel - Inventory EOQ Variants Harvey Foods Example.xls

	A	B	C	D	E	F
1	Type	Q	R	SI	AI	TC
2	Deterministic Demand	100	0	0	50	200.0
3	+ Lead Time	100	100	0	50	200.0
4	Stochastic Demand	100	151	51	101	302.0
5	Shortage Cost (Seq)	100	151	51	101	311.2
6	Shortage Cost (Iter)	111	143	43	99	308.5
7	Type 2 Service (Seq)	100	126	26	76	252.0
8	Type 2 Service (Iter)	114	124	24	81	249.5
9	SL2 (Iter) + Stoc. Lead Time	119	139	39	99	281.5

Optimal Q and R (Iterative Method) Type 2 Service

- Start with Q Determined by EOQ
- Determine Q and R Iteratively until No Further Change

Iterative Formulas for Q and R Type 2 Service

$$Q_i = \sqrt{\frac{2 \cdot oc \cdot D}{hc}}$$

$$Q = \frac{n(R)}{1-F(R)} + \sqrt{\frac{2 \cdot oc \cdot D}{hc} + \left(\frac{n(R)}{1-F(R)}\right)^2}$$

$$L(z) = \frac{Q(1-SL)}{s_{dlt}}$$

$$R = d \cdot LT + z \cdot s_{dlt}$$

Type 2 Service Level Iterations

Microsoft Excel - Inventory EOQ Variants Harvey Fo...

	J	K	L	M	N	O
70	Q	n(R)	L(z)	z	R	F(R)
71	100.0	2.00	0.0800	1.0210	125.5	0.8464
72	113.9	2.28	0.0911	0.9525	123.8	0.8296
73	114.3	2.29	0.0914	0.9507	123.8	0.8291
74	114.3	2.29	0.0914	0.9507	123.8	0.8291

Inventory Comparison Cost: Type 2 Service (Iterative)

$$AI = CI + SI = \frac{Q}{2} + z \cdot s_{dlt}$$

$$TC = \frac{D}{Q} oc + hc \frac{Q}{2} + hc \cdot z \cdot s_{dlt}$$

$$TC = \left(\frac{114}{2} + 0.95 \cdot 25 \right) 0.2 \cdot 10 + \frac{200}{114} 50 = 249$$

Inventory Policies Comparison Example: Results

Microsoft Excel - Inventory EOQ Variants Harvey Foods Example.xls

	A	B	C	D	E	F
1	Type	Q	R	SI	AI	TC
2	Deterministic Demand	100	0	0	50	200.0
3	+ Lead Time	100	100	0	50	200.0
4	Stochastic Demand	100	151	51	101	302.0
5	Shortage Cost (Seq)	100	151	51	101	311.2
6	Shortage Cost (Iter)	111	143	43	99	308.5
7	Type 2 Service (Seq)	100	126	26	76	252.0
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9	SL2 (Iter) + Stoc. Lead Time	119	139	39	99	281.5

Book / Overhead / Sheet3

Demand and Lead Time Uncertainty

- *Lead Time over the Total Supply Chain*

$$SI = z \cdot s_{dlt} = z \cdot \sqrt{lt \cdot Var_d + d^2 \cdot Var_{lt}}$$

$$CV_d = \frac{\sqrt{Var_d}}{d}$$

$$Var_d = \left(CV_d \cdot d \right)^2$$

$$SI = z \cdot \sqrt{lt \cdot CV_d^2 + Var_{lt}} \cdot d$$

Standard Deviation of the Demand During the Lead Time

$$s_{dlt} = \sqrt{lt \cdot Var_d + d^2 \cdot Var_{lt}}$$

$$s_{dlt} = \sqrt{0.5 \cdot 35.4^2 + 200^2 \cdot 0.125^2} = 35.4$$

Stochastic Lead Time and Service Level Type 2 Iterative

Microsoft Excel - Inventory EOQ Variants Harvey Foods Exa...

	J	K	L	M	N	O
84	Q	n(R)	L(z)	z	R	F(R)
85	100.0	2.00	0.0566	1.1958	142.3	0.8841
86	118.7	2.37	0.0672	1.1106	139.3	0.8666
87	119.4	2.39	0.0675	1.1079	139.2	0.8660
88	119.4	2.39	0.0675	1.1078	139.2	0.8660
89	119.4	2.39	0.0675	1.1078	139.2	0.8660

$$TC = \frac{D}{Q} oc + hc \frac{Q}{2} + hc \cdot z \cdot s_{dlt}$$

$$TC = \left(\frac{119}{2} + 1.11 \cdot 35.4 \right) 0.2 \cdot 10 + \frac{200}{119} 50 = 282$$

Inventory Policies Comparison Example: Results

Microsoft Excel - Inventory EOQ Variants Harvey Foods Example.xls

	A	B	C	D	E	F
1	Type	Q	R	SI	AI	TC
2	Deterministic Demand	100	0	0	50	200.0
3	+ Lead Time	100	100	0	50	200.0
4	Stochastic Demand	100	151	51	101	302.0
5	Shortage Cost (Seq)	100	151	51	101	311.2
6	Shortage Cost (Iter)	111	143	43	99	308.5
7	Type 2 Service (Seq)	100	126	26	76	252.0
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9	SL2 (Iter) + Stoc. Lead Time	119	139	39	99	281.5

Inventory Policies Comparison Exercise: Data

Microsoft Excel - Inventory EOQ Variants Tie Rack Ex...

	A	B	C	D
11	Purchase Price	p	0.11	$/unit
12	Lead Time	LT	1.5	months
13	Holding Cost Rate	hcr	0.01666667	/month
14	Shortage Cost	sc	0.01	$/unit
15	Ordering Cost	oc	10	$/order
16	Demand Rate	d	11107	units/month
17	SD Demand	sd	3099	
18	Probability No Stockout	F(R)	0.75	
19	Fill Rate	fr	0.75	
20	SD Lead Time	slt	0.5	months

Inventory Systems Overview

1. Introduction
2. Independent Demand Systems
   - Continuous Review
   - **Periodic Review**
3. Dependent Demand Systems

Periodic Review Inventory Systems

- Review at Fixed Intervals & Schedule
- Joint Ordering, Transportation, and Production Economies
- Simple Manual Implementation
- Reduced Administration Cost
- Slightly Higher Inventory Cost Compared to Continuous Review

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Periodic Review (s,S) Policies

- Definitions
  - i = on hand inventory
  - s = ordering trigger level
  - S = order up to level
- Policy

$$\begin{aligned} \text{if} \quad & i \le s \quad \text{order } S - i \\ & i > s \quad \text{do not order} \end{aligned}$$

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Periodic Review Inventory Policies

- Optimal Inventory Policies have Very Complex Derivations
- Simple Approximations have Acceptable Accuracy

$$s \approx R$$

$$S \approx R + Q$$

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Inventory Systems Overview

1. Introduction
2. Independent Demand Systems
3. Dependent Demand Systems

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Distribution Requirements Planning (DRP)

- Integrated Supply Scheduling Throughout Supply Chain
- MRP for Supply Chain
- Forecasted Demand for Products Delivered to Customers
- Bill of Materials (BOM) [a_{qp}]

Material Requirements Planning (MRP)

- Precise Timing of Material Flows to meet Production Planning
- Avoid Carrying Items as Work-In-Process (WIP) Inventory
- Scheduling of High-Value, Custom-Made Items
- When Demand is Reasonably Well Known

DRP Advantages

- Requires uniform and consistent data collection
- Advance planning for future demands
- New and irregular demands
- Uniform product shortage allocation
- Incorporate other planned demands
- Balance facility utilization
- Control and balance obsolescence

Clock MRP Example: K36 Demand

K36	Week								
	0	1	2	3	4	5	6	7	8
Gross Demand		100	150	120	150	100	90	110	120
Receipts									
Quantity on Hand	400								
Production Start									

Q=350
LT=2

Clock MRP Example: K36 Schedule

K36	Week								
	0	1	2	3	4	5	6	7	8
Gross Demand		100	150	120	150	100	90	110	120
Receipts					350			350	
Quantity on Hand	400	300	150	30	230	130	40	280	160
Production Start		0	350	0	0	350	0	0	0

Q=350
LT=2

Clock MRP Example: M21 Demand

M21	Week								
	0	1	2	3	4	5	6	7	8
Gross Demand		200	200	200	200	200	200	200	200
Receipts			600						
Quantity on Hand	500								
Production Start									

Q=600
LT=1

Clock MRP Example: M21 Schedule

M21	Week								
	0	1	2	3	4	5	6	7	8
Gross Demand		200	200	200	200	200	200	200	200
Receipts			600				600		
Quantity on Hand	500	300	700	500	300	100	500	300	100
Production Start		0	0	0	0	600	0	0	0

Q=600
LT=1

Clock MRP Example: R1063 Demand

R1063	Week								
	0	1	2	3	4	5	6	7	8
Service Parts		100	100	100	100	100	100	100	100
R36		0	350	0	0	350	0	0	0
M21		0	0	0	0	600	0	0	0
Gross Demand		100	450	100	100	1,050	100	100	100
Receipts									
Quantity on Hand	900								
Production Start									

Q=1000, LT=2, SS=250

Clock MRP Example: R1063 Schedule

	A	B	C	D	E	F	G	H	I	J
43	R1063	Week								
44		0	1	2	3	4	5	6	7	8
45	Gross Demand		100	450	100	100	1,050	100	100	100
46	Receipts					1,000	1,000			
47	Quantity on Hand	900	800	350	250	1,150	1,100	1,000	900	800
48	Production Start		0	1,000	1,000	0	0	0	0	0

Q=1000
LT=2
SS=250

Bill of Material Matrix

- # Immediate Components q Required to Produce 1 Unit of Product p = A_{qp}
  - Rows = Components q
  - Columns = Products p

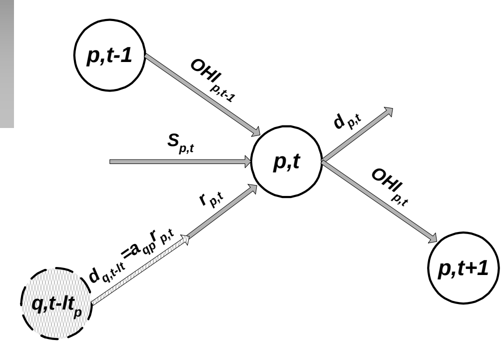
$$A_{qp} = \left[\quad \right]$$

DRP (MRP) Inventory Equation

- Notation
  - OHI = On-Hand Inventory
  - d = gross requirements (outflow)
  - s = scheduled receipts (inflow)
  - r = planned production receipts (inflow)

$$OHI_{p,t} = OHI_{p,t-1} - d_{p,t} + s_{p,t} + r_{p,t}$$
$$d_{q,t-lt_p} = a_{qp} r_{p,t}$$

DRP Inventory Transition Diagram



DRP (MRP) Characteristics

- Strongly Dependent on Quality and Stability of Final Product Forecast
  - Forecast changes/errors create WIP
- Uncapacitated (No Manufacturing or Transportation Resources)
  - Basic equations are deterministic
- Can Accommodate Irregular Demand

DRP (MRP) Implementation Details

- Production Quantity
  - First cut heuristic approximation by EOQ
  - More Sophisticated Algorithms
    - Silver-Meal heuristic
    - Wagner-Whitin (optimal dynamic programming)
- Safety Inventory
  - Newsboy Methodology

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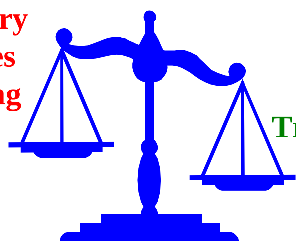
Supply Chain and Warehousing Trends

Inventory
Facilities
Handling



Transactions
Transportation
Information

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