Unit Load Storage Systems Design

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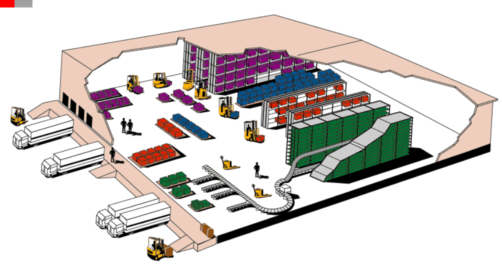
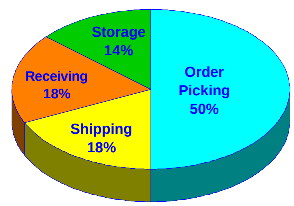
Overview

1. Introduction to Unit Load Storage Systems
2. Rack Storage Systems
3. Block Stacking Storage Systems
4. Storage Assignment Formulation & Algorithms
5. Conclusions

Warehousing Illustration



Warehousing Costs



Warehousing Objectives

- Minimize the Expected Travel Time
  - Minimize MH Equipment and Personnel

$$Min \sum_j f_j \cdot t_j$$

- Minimize the Required Storage Space
  - Minimize Capital Investment

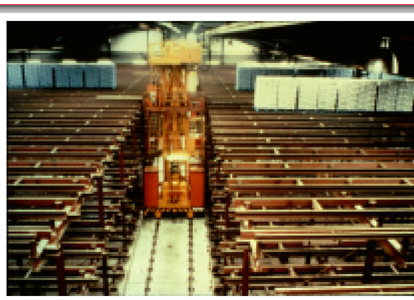
$$Min \;\; N$$

Pallet Rack Example

Multiple Aisles Automated Storage System Example

Block Stacking Storage Example

Deep Lane Storage Example



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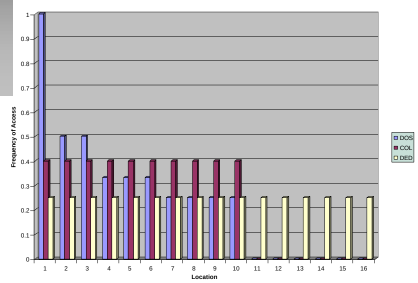
Unit Load Storage Policies

- No Information Policies
  - Random, Closest-Open-Location
- Product Characteristics Policies
  - Product Turnover Based
  - Class Product Turnover Based
- Item Characteristics Policies
  - Duration Of Stay
  - Zone Duration of Stay

Main Storage Principle to Minimize Travel

Place unit loads that generate the highest frequency of access in locations with the lowest expected distance

Shared Storage Example: Frequency of Access Distribution



Product Turnover Pure Dedicated

- Sort and Locate Products by Decreasing Turnover to Locations with Lowest Expected Distance

$$f_p = \frac{F_p}{q_p}$$

Product Turnover Class Based Storage

- Pure Dedicated is Very Space Inefficient
- 3 to 5 Classes based on Frequency of Access
- Dedicated Space for Each Class
- Class Space Determined by Simulation
- Inside Class Use Random or Closest Open Location
- Shared Storage Policy

Duration of Stay Shared Storage Observations

- Exploits that First and Last Unit Load in Batch are Different
- Cross Docking (DOS = 0)
- Minimizes Both Storage Space and Travel Time for a Perfectly Balanced Warehouse
- Very Constrained Perfectly Balanced Replenishment Pattern $n_p(t)$

Duration of Stay Shared Storage: Conclusions

- More Uncorrelated Products Favors DOS Policy
- More Balanced Warehouse Favors DOS Policy and Any Shared Policy

Storage Policy Conclusions

- Real Systems are Not Perfectly Balanced
- Duration of Stay Saves Travel and Storage Space
- 2 & 3 Class Product Performs Well
- Pure Turnover Dedicated Worst of All
- Savings Magnitude Depends on Replenishment Pattern
- DOS Data Requirements Indicate Warehouse Management System

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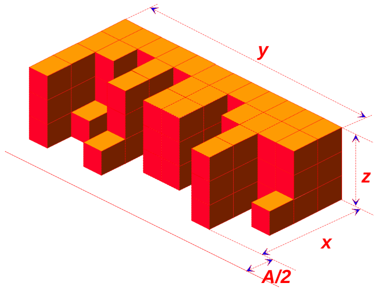
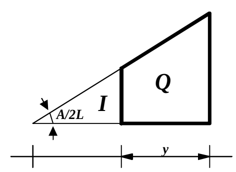
Block Stacking Storage System

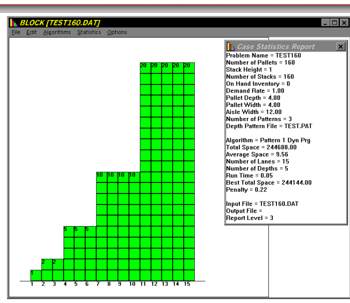
Block Stacking Illustration



Optimal Triangular Pattern



Block Stacking Design Program



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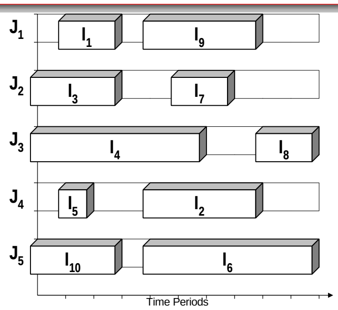
Occupancy Gantt Chart



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Unit Load Storage Formulation

$$\min \sum_{i=1}^{M}\sum_{j=1}^{N} c_{ij} x_{ij}$$

$$s.t. \quad \sum_{j=1}^{N} x_{ij} = 1 \qquad \forall i$$

$$\sum_{i=1}^{M} b_i x_{ij} \le 1 \qquad \forall j$$

$$x_{ij} \in \{0,1\}$$

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Occupancy and Constraint Matrices

$$B = [b_i] = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} I & I & I & .. & I \\ B & 0 & 0 & .. & 0 \\ 0 & B & 0 & .. & 0 \\ .. & .. & .. & .. & .. \\ 0 & 0 & 0 & .. & B \end{bmatrix}$$

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Vector Assignment Formulation

$$\max \sum_{i=1}^{M}\sum_{j=1}^{N} r_{ij} x_{ij}$$

$$r_{ij} = C - c_{ij} > 0 \qquad s.t. \quad \sum_{j=1}^{N} x_{ij} \le 1 \qquad \forall i$$

$$\sum_{i=1}^{M} b_i x_{ij} \le 1 \qquad \forall j$$

$$x_{ij} \in \{0,1\}$$

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Vector Assignment Formulation

- Neither AP nor 3DAP
- Consecutive Ones in Occupancy Matrix B
- Block Diagonal Structure of Constraint Matrix
- Integrality Property Not Satisfied

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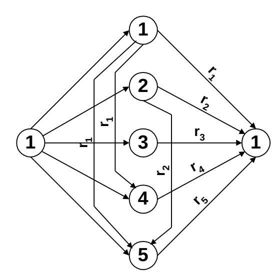
Longest Path Subproblem

$$B = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$



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Lagrangean Relaxation

$$\max \sum_{i=1}^{M}\sum_{j=1}^{N} r_{ij}x_{ij} + \sum_{i=1}^{M} m_i\left(1-\sum_{j=1}^{N} x_{ij}\right)$$

$$\max \sum_{i=1}^{M}\sum_{j=1}^{N} \left(r_{ij}-m_i\right)x_{ij} + \sum_{i=1}^{M} m_i$$

$$s.t. \quad \sum_{i=1}^{M} b_i x_{ij} \le 1 \qquad \forall j$$

$$x_{ij} \in \{0,1\}, m \ge 0$$

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Computational Experiment

Activities	Resources	Heuristic	Upper Bound	% Gap	Optimal	Penalty
10	10	861	861	0.0	861	0.0
15	15	1357	1357	0.0	1357	0.0
30	9	2491	2491	0.0	2491	0.0
30	10	2631	2631	0.0	2631	0.0
30	15	2627	2627.1	0.0	2627	0.0
100	20	9024	9276.8	2.8	9270	-2.7
200	25	18869	18959.8	0.5		

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VAP Conclusions

- Very Large Integer Problem
- Very Tight LP Relaxation
- Efficient Subproblem and Problem Size Indicate Decomposition
- Very Small Gap for Lagrangean Relaxation Upper Bound
- Acceptable Penalty for Primal Heuristic

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Unit Load Storage Systems: Conclusions

- Products are Not Uniform
- Duration-of-Stay of Individual Loads
- Fairly Simple Statistical Analysis Yields Large Savings
- Optimal, Mathematical Programming based Procedures for Small Systems

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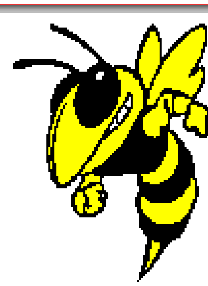
Thank You



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