Chapter 15. Material Handling Models

15.1. Automated Storage-Retrieval Systems (AS/RS)

Definitions

Unit Load

Automated storage and retrieval systems are primarily used for storing and handling unit loads. The *Effective Unit Load Dimensions* are used to size the system. They include the actual size of the unit load plus all the clearances between adjacent loads to accommodate access and fire prevention devices. The effective unit load dimensions are measured between the centroids of adjacent unit loads in the storage system.

Storage Policy

A storage policy is the set of rules that determine in which open location of the rack an arriving load will be stored. One of the most simple storage policies is random storage or RAN, which selects the storage location at random of the open locations. Usually this is modeled by assuming that the density function of the frequency of access is a uniform distribution over the face of the rack. Another policy often used is to store the arriving load in the "closest-open-location" or COL. It has been shown that this policy can be modeled by a uniform density function over the portion of the rack that is utilized, i.e., is filled with loads.

Chebyshev Travel

In many storage and retrieval systems the shuttle on the crane can move in the vertical direction while the crane itself is moving in the horizontal direction in the aisle. This is referred to as simultaneous travel and the actual travel time between two points in the rack can then be approximated by the Chebyshev travel time formula.

$$t_{CHEB} = \max\left\{\frac{\Delta_x}{v_x}, \frac{\Delta_y}{v_y}\right\} \tag{15.1}$$

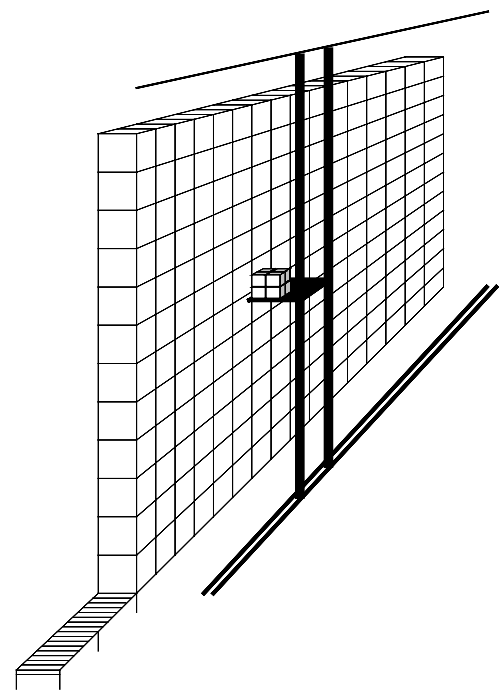

Figure 15.1. Unit Load AS/RS Illustration

Observe that in general there exist many different paths between two points which all take the same amount of time to traverse. The collection of these travel paths is a parallelogram based on the two points and where the angle of the parallelogram is determined by the speed ratio of the vertical shuttle speed and the horizontal crane speed. This parallelogram is illustrated in Figure 15.2

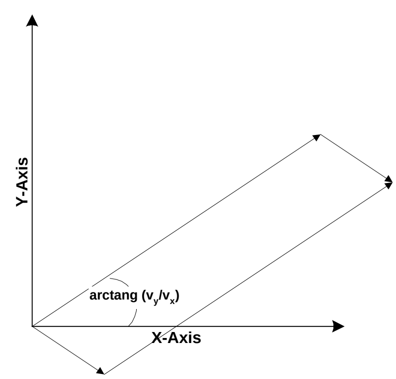

Figure 15.2. Parallelogram of Possible Chebyshev Travel Paths between Two Points

Command Cycle

The command cycle refers to the number of operations performed on a single trip of the crane in the automated storage and retrieval system. If the crane performs either a storage or retrieval, the number of operations is one and is called a single command. If the crane performs first a storage and then moves to another position in the rack and retrieves a load to the output point, then the number of operations performed per trip equals two and the crane is said to operate under dual command. By definition all dual command cycles consist of first a storage operation followed by a retrieval operation, unless explicitly stated otherwise. In person-aboard order picking systems, the crane might visit many locations where items are retrieved from the rack to the shuttle. This is called multiple command.

The expected or average time required to complete a single trip of the crane is called the cycle time. Depending on the crane cycle, we have single, dual, and multiple command cycle times. The cycle time includes the load pickup operation at the input point, the storage operation at the open rack location, the retrieval operation at the full rack location, and the deposit operation at the deposit point, if they are present in the cycle. The expected time to complete the travel of the crane during a cycle is called the single, dual, or multiple command travel time, respectively. It excludes the times required to execute the shuttle pick and place operations.

Dwell Point Policy

A dwell point policy specifies what the crane should do when it becomes idle, i.e., there are no further storage and retrieval request waiting to be executed. A dwell point policy might have two components corresponding to what to do after a storage and after a retrieval operation, respectively. Examples of dwell point policies are:

Always return to the input or pickup point.

Remain at the location of storage location or deposit point where the crane became idle.

Position the crane at the centroid location of the rack.

Cycle Time Computations for Automated Storage and Retrieval Systems

Introduction

Cycle times are very important performance measures both during design of the system and during operations. Cycle times are used to:

Determine the required number of aisles to satisfy throughput

Determine the number and location of the rack input and output station

Evaluate different crane operating policies such as the dwell point policy

Evaluate different storage policies such as random and class based storage

In addition to the rack sizes and the vehicle speeds and accelerations, the cycle times depend on the storage policy, dwell point strategy, command cycle mix, and pickup and deposit stations locations. Great care must be taken to apply the correct formula depending on all the above factors.

There exist basically three methods to determine the cycle times of a storage and retrieval system. If the system actually exists we can do time and motion sampling. Of course this is not possible for a planned system or to evaluate the effects of a new storage or dwell point policy. A second method is computer simulation, be either a simulation of the complete system or a simulation based upon random sampling. The third method estimates the cycle times based on analytical formulas for basis elements included in the cycle time. The amount of effort required determining the cycle time as well as the accuracy of the cycle time estimation increase sharply from low for the analytical formulas, to medium for simulation, to high for time and motion studies.

Random Storage Expected Travel Times

Random storage is the simplest storage policy to compute expected travel times for since the density function of the frequency of access of the rack locations is a uniform function.

The following notation will be used. For simplicity purposes all parameters and variables are expressed in compatible international standard units (ISO). In this section it is assumed that both input and output points are in the lower bottom corner of the rack unless otherwise indicated.

L = horizontal rack size in meters (m), also called rack width

H = vertical rack size in meters (m), also called rack height

v_x = horizontal speed (m/s)

v_y = vertical speed (m/s)

l = effective storage location horizontal size (m), also called location width

h = effective storage location vertical size (m), also called location height

Q = total number of locations that the rack system must provide

T_{PD} = execution time of a pickup or deposit operation

a = fraction of total number of crane cycles that are single command

T_x = horizontal rack size in the time domain (s)

T_y = vertical rack size in the time domain (s)

T = rack scale factor in the time domain (s)

b = rack shape factor

$f_x()$ = probability density function of the horizontal travel time

$f_y()$ = probability density function of the vertical travel time

$f_z()$ = probability density function of the Chebyshev travel time

$F_x()$ = cumulative distribution function of the horizontal travel time

$F_y()$ = cumulative distribution function of the vertical travel time

$F_z()$ = cumulative distribution function of the Chebyshev travel time

t_{IR} = one-way expected travel time from the input point to a random point in the unit rack

t_{RO} = one-way expected travel time from a random point to the output point in the unit rack

t_{RR} = expected travel time between two random points in the unit rack

t_{OI} = travel time between output and input points in the unit rack

t_{SC} = single command travel time in the unit rack (excluding pickup/deposit operations)

t_{DC} = dual command travel time in the unit rack (excluding pickup/deposit operations)

t_C = combined travel time in the unit rack (excluding pickup/deposit operations)

T_{SC} = single command cycle time

T_{DC} = dual command cycle time

T_C = combined (single and dual command) cycle time

u = expected number of operations per cycle

x = number of vertical columns in the rack, also horizontal rack size in locations

y = number of horizontal levels or rows in the rack, also vertical rack size in locations

z = number of aisles in the rack system

To provide better insight in the travel and cycle time computations we will apply two transformations. Since we are interested in travel times, we will transform first the rack from the (original) space domain to the time domain. We will then transform the rack from the time domain to the normalized time domain. This allows us to split the travel time computations into factors related to the size (or scale) of the rack and the shape of the rack. The transformations are illustrated in the next figure.

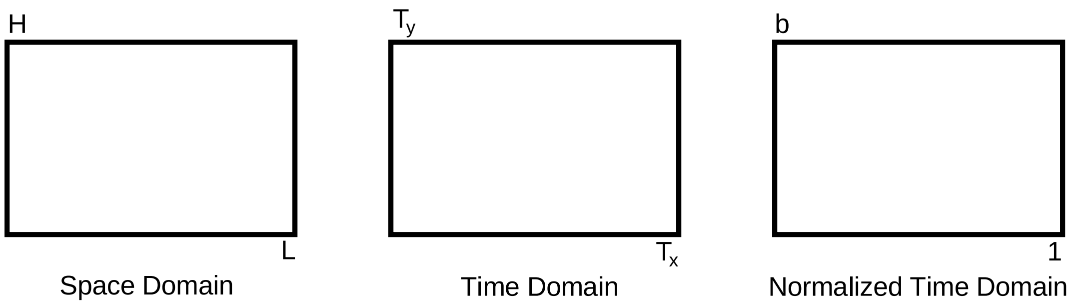

Figure 15.3. Rack Domain Transformations

The dimension of the rack in the time domain are given by:

$$T_x = \frac{l \cdot x}{v_x} = \frac{L}{v_x} \tag{15.2}$$

$$T_y = \frac{h \cdot y}{v_y} = \frac{H}{v_y} \tag{15.3}$$

The size of the rack in the time domain, also called its scale, is thus given by:

$$T = \max\left\{T_x, T_y\right\} \tag{15.4}$$

The ratio of the length to the width of the rack in the time domain is called its shape factor, which is computed as:

$$b = \frac{\min\left\{T_x, T_y\right\}}{T} \tag{15.5}$$

The dimensions of the rack in the normalized time domain are then 1 by b, where $0 \le b \le 1$. A rack with b equal to one is called "square in time". A square in time rack has the maximum amount of storage locations within a given distance to the pickup and deposit station. Obviously, it is advantageous to have racks be square in time. Even though in the space domain racks are long and not tall, in the time domain most shape factors fall between 0.8 and one, since the rack dimension and crane speeds have about the same ratio. Without loss of generality, it will be assumed from now on that the smaller (time) dimension is on the vertical axis.

Since the crane carriage and the crane shuttle move simultaneously, the time to travel between two points is given by the maximum of the horizontal and vertical travel time as computed by the Chebyshev travel time formula:

$$t_{CHEB} = \max\left\{\frac{\Delta_x}{v_x}, \frac{\Delta_y}{v_y}\right\} \tag{15.6}$$

The lines of equal travel or contour lines in the normalized time domain are illustrated in the next figure when the input/output point is in the lower left corner of the rack and the rack is longer than it is tall. In the left section of the rack the contour lines have an inverted L shape. In the right section of the rack the contour lines are vertical lines. These vertical lines can be thought of as inverted L shape lines where the horizontal part is clipped by the rack face.

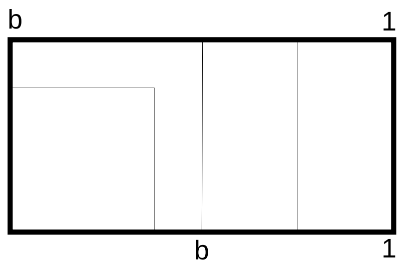

Figure 15.4. One Way Travel in an AS/RS Rack

The general formula for the computation of the expected travel time from the input/output point to a random point in the rack is given by

$$t_{IR} = \oint_{w \in \Omega} t(w) \cdot f(w) \cdot dw\,, \tag{15.7}$$

where *t(w)* is the travel time to a point *w* in the rack Ω and *f(w)* is the frequency of access density function to the point *w*. For random storage this frequency of access is equal to:

$$f(w) = \frac{1}{\Omega}\,. \tag{15.8}$$

The formula is complicated by the presence of the maximum operator in the Chebyshev travel time formula. Using the cumulative distribution functions of the travel times can eliminate this maximum operator. Since the horizontal and vertical travel times are independent, the probability that the maximum of the two is smaller than or equal to k is equal to the product of the probabilities that either is smaller than or equal to k, or

$$F_z(k) = \mathbf{P}\{z \le k\} = \mathbf{P}\{x \le k\} \cdot \mathbf{P}\{y \le k\} = F_x(k) \cdot F_y(k)$$

$$F_x(z) = \mathbf{P}\{t_x \le z\} = z$$

$$F_y(z) = \mathbf{P}\{t_y \le z\} = \begin{cases} z/b, & z \le b \\ 1, & z \ge b \end{cases}$$

$$F_z(z) = \mathbf{P}\{t_z \le z\} = \begin{cases} z^2/b, & z \le b \\ z, & z \ge b \end{cases}$$

The probability density function can then be found by taking the derivative, or

$$f_z(z) = \frac{dF_z(z)}{dz} = \begin{cases} 2z/b, & z \le b \\ 1, & z \ge b \end{cases}$$

The expected one-way travel time can then be found by integrating over the unit rack area with the above derived density function.

$$\begin{aligned} t_{IR} &= \oint_{\Omega} t_z(w) f_z(w) dw \\ &= \int_{z=0}^{z=b} \frac{2z^2}{b} dz + \int_{z=b}^{z=1} z dz \\ &= \frac{b^2}{6} + \frac{1}{2} \end{aligned} \tag{15.9}$$

The next step is to compute the expected travel time between two random points in the rack. In order to compute this, we need to compute the probability density function of the Chebyshev travel time. Again we will use the independence condition to derive the cumulative distribution function of the Chebyshev travel time in function of the cumulative distribution functions of the vertical and horizontal travel times between two points in the rack. We will call the travel time between two random points in the rack the range. The probability density function and the cumulative distribution function of the vertical range are:

$$\begin{aligned} f_{yrange}(r) &= 2 \int_{v=0}^{v=b-r} f_y(v) f_y(v+r) dv \\ &= 2 \int_{v=0}^{v=b-r} \frac{1}{b^2} dv \\ &= \frac{2}{b^2}(b-r) \end{aligned}$$

$$\begin{aligned} F_{yrange}(z) &= \int_{r=0}^{r=z} f_{yrange}(r) dr \\ &= \int_{r=0}^{r=z} \frac{2}{b^2}(b-r) dr \qquad 0 \le z \le b \\ &= \frac{2z}{b} - \frac{z^2}{b^2} \end{aligned}$$

$$F_{yrange}(z) = 1 \qquad b \le z \le 1$$

The probability density function and the cumulative distribution function of the horizontal range are:

$$\begin{aligned} f_{xrange}(r) &= 2 \int_{v=0}^{v=1-r} f_x(v) f_x(v+r) dv \\ &= 2 \int_{v=0}^{v=1-r} dv \\ &= 2(1-r) \end{aligned}$$

$$
\begin{aligned}
F_{xrange}(z) &= \int_{r=0}^{r=z} f_{xrange}(r)dr \\
&= \int_{r=0}^{r=z} 2(1-r)dr \\
&= 2z - z^2
\end{aligned}
$$

The probability density function and the cumulative distribution function of the Chebyshev range are:

$$
\begin{aligned}
F_{zrange}(z) &= F_{xrange}(z) \cdot F_{yrange}(z) \\
&= \begin{cases} (2z - z^2)\left(\dfrac{2z}{b} - \dfrac{z^2}{b^2}\right), & 0 \le z \le b \\ 2z - z^2, & b \le z \le 1 \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
f_{zrange} &= \frac{dF_{zrange}(z)}{dz} \\
&= \begin{cases} \dfrac{4z^3}{b^2} - \dfrac{6z^2}{b^2} - \dfrac{6z^2}{b} + \dfrac{8z}{b}, & 0 \le z \le b \\ 2 - 2z, & b \le z \le 1 \end{cases}
\end{aligned}
$$

The expected Chebyshev travel time between two uniformly distributed random points is then:

$$
\begin{aligned}
t_{RR} &= \oint_{\Omega} t_{zrange}(w) f_{zrange}(w) dw \\
&= \int_{z=0}^{z=b} \left(\frac{4z^4}{b^2} - \frac{6z^3}{b^2} - \frac{6z^3}{b} + \frac{8z^2}{b}\right)dz + \int_{z=b}^{z=1} (2-2z)dz \\
&= \frac{1}{3} + \frac{b^2}{6} - \frac{b^3}{30}
\end{aligned} \tag{15.10}
$$

Cycle Times for Random Storage (Single Corner P/D Point)

If the input and output station are both in the lower left corner of the rack and if the crane executes only one type of operation, i.e., only single commands or only dual commands, then we get the standard cycle time formulas. The single command cycle time is then composed of two one-way travels plus two operations, one to pick up the load and one to deposit the load.

$$
\begin{aligned}
T_{SC} &= T \cdot 2 \cdot t_{IR} + 2 \cdot T_{PD} \\
&= T \cdot \left(1 + \frac{b^2}{3}\right) + 2 \cdot T_{PD}
\end{aligned} \tag{15.11}
$$

The dual command cycle time is then composed of two one-way travels plus the travel between two random points, plus four operations, two pick up and two deposit operations.

$$
\begin{aligned}
T_{DC} &= T \cdot (2 \cdot t_{IR} + t_{RR}) + 4 \cdot T_{PD} \\
&= T \cdot \left(\frac{4}{3} + \frac{b^2}{2} - \frac{b^3}{30}\right) + 4 \cdot T_{PD}
\end{aligned} \tag{15.12}
$$

The average combined cycle time is then computed as the weighted average of the single and dual command cycle times:

$$T_C = a \cdot T_{SC} + (1-a) \cdot T_{DC} \tag{15.13}$$

Example of Simple Cycle Times for Random Storage

An AS/RS system is do be designed to store approximately 12,000 unit loads. Due to space limitations in the existing facility, it is know that the system will have eight tiers or levels of storage. The choice has been reduced to having either (a) 10 aisles of 75 columns (openings long), or (b) 9 aisles of 84 columns (openings long). The S/R machine will travel horizontally at an average speed of 450 feet per minute; simultaneously, it travels vertically an average speed of 80 feet per minute. The distance between the centroids of adjacent unit locations is 48 inches in the horizontal direction and 54 inches in the vertical direction. This centroid to centroid distance includes space for the structural elements of the rack and all fire and safety equipment. The P/D station is located at the end of the aisle at the floor level. The time required to pick up (P) or deposit (D) a load is 0.3 minutes. The throughput requirement on the system is to perform a total of 180 storage and 180 retrievals per hour. Randomized storage is used. It is anticipated that 40 % of the crane cycles are single command. We will determine for each of the alternatives listed above the overall expected cycle time and then determine if they will meet the throughput constraint and average crane utilization.

Since alternative (b) has fewer cranes and a longer aisle it will have the longest expected cycle times of the two. On the other hand, alternative (b) will be less expensive since it has the fewest cranes. So we will investigate this alternative first to see if it satisfies the throughput requirement.

$$H = y \cdot h = \frac{8 \cdot 54}{12} = 36 \textit{ feet}$$

$$L = x \cdot l = \frac{84 \cdot 48}{12} = 336 \textit{ feet}$$

$$T = \max\left\{\frac{L}{v_x}, \frac{H}{v_y}\right\} = \max\left\{\frac{336}{450}, \frac{36}{80}\right\} = \max\{0.7466, 0.45\} = 0.7466$$

$$b = \min\left\{\frac{L}{v_x}, \frac{H}{v_y}\right\} / T = \min\{0.7466, 0.45\} / 0.7466 = 0.603$$

$$a = 0.4$$

$$T_{PD} = 0.3$$

$$T_{SC} = T \cdot (1 + \frac{b^2}{3}) + 2 \cdot T_{PD} = 0.7466 \cdot (1 + \frac{0.603^2}{3}) + 2 \cdot 0.3 = 1.437$$

$$T_{DC} = T \cdot \left(\frac{4}{3} + \frac{b^2}{2} - \frac{b^3}{30}\right) + 4 \cdot T_{PD} = 0.7466 \cdot \left(\frac{4}{3} + \frac{0.603^2}{2} - \frac{0.603^3}{30}\right) + 4 \cdot 0.3 = 2.326$$

$$T_C = a \cdot T_{SC} + (1-a) \cdot T_{DC} = 0.4 \cdot 1.437 + 0.6 \cdot 2.326 = 1.970$$

$$u = a \cdot 1 + (1-a) \cdot 2 = 0.4 \cdot 1 + 0.6 \cdot 2 = 1.6$$

$$N = \left\lceil \frac{M \cdot S}{H} \right\rceil = \left\lceil \frac{360 \cdot \frac{1.970}{1.6}}{60} \right\rceil = \lceil 7.39 \rceil = 8$$

Since the required number of machines is 8 which is smaller than the designed number of machines, equal to 9, this alternative will meet the throughput requirements and the average machine utilization is 7.39 / 9 = 82.1 %. In practice, alternative (a) no longer has to be evaluated but we include it here for completeness sake.

$$H = y \cdot h = \frac{8 \cdot 54}{12} = 36 \textit{ feet}$$

$$L = x \cdot l = \frac{75 \cdot 48}{12} = 300 \textit{ feet}$$

$$T = \max\left\{\frac{L}{v_x}, \frac{H}{v_y}\right\} = \max\left\{\frac{300}{450}, \frac{36}{80}\right\} = \max\{0.667, 0.45\} = 0.667$$

$$b = \min\left\{\frac{L}{v_x}, \frac{H}{v_y}\right\} / T = \min\{0.667, 0.45\} / 0.667 = 0.675$$

$$a = 0.4$$

$$T_{PD} = 0.3$$

$$T_{SC} = T \cdot (1 + \frac{b^2}{3}) + 2 \cdot T_{PD} = 0.667 \cdot (1 + \frac{0.675^2}{3}) + 2 \cdot 0.3 = 1.368$$

$$T_{DC} = T \cdot \left(\frac{4}{3} + \frac{b^2}{2} - \frac{b^3}{30}\right) + 4 \cdot T_{PD} = 0.667 \cdot \left(\frac{4}{3} + \frac{0.675^2}{2} - \frac{0.675^3}{30}\right) + 4 \cdot 0.3 = 2.234$$

$$T_C = a \cdot T_{SC} + (1-a) \cdot T_{DC} = 0.4 \cdot 1.368 + 0.6 \cdot 2.234 = 1.888$$

$$u = a \cdot 1 + (1-a) \cdot 2 = 0.4 \cdot 1 + 0.6 \cdot 2 = 1.6$$

$$N = \left\lceil \frac{M \cdot S}{H} \right\rceil = \left\lceil \frac{360 \cdot \frac{1.888}{1.6}}{60} \right\rceil = \lceil 7.08 \rceil = 8$$

Since the required number of machines is 8 which is smaller than the designed number of machines, equal to 10, this alternative will meet the throughput requirements and the average machine utilization is 7.08 / 10 = 70.8 %.

Cycle Times for Random Storage (General P/D Points)

Both pickup and deposit points are not always located in the lower left corner of the rack. The overall expected cycle time is then most easily computed based on a two-dimensional matrix which shows the previous and next operation, their associated probabilities and the expected cycle time for each combination. The fraction of single command storage operations is denoted by *a* in the following Table.

Table 15.1. Cycle Time Computation Matrix Template

	Next Operation	SC Storage	SC Retrieval	Dual
Prev. Operation	Probability	a/2	a/2	(1-a)
SC Storage	a/2			
SC Retrieval	a/2			
Dual	(1-a)			

We will compute the expected operation time in the following Automated Storage/Retrieval System using a continuous approximation. The horizontal dimension (L) equals 200 feet; the vertical dimension (H) equals 80 feet. The horizontal speed equals 250 feet per minute and the vertical speed equals 50 feet per minute. The input point is a the lower left corner with coordinates (0,0) and the output point is at the lower right corner with coordinates (200,0). All coordinates are expressed in feet. The products are stored in the rack following random storage and the rack is considered filled to capacity. Forty percent of the crane cycles are dual command cycles. The dwell point strategy of the crane is to remain at the point where it became idle, i.e., at the storage location after the completion of a storage operation and at the output point after the completion of a retrieval or dual command operation. We will assume that pickup and deposit times are equal to 30 seconds. We will execute all our computations to four significant digits and round intermediate results to four significant digits.

First, we will compute the shape and scale factor of the rack.

$$T = \max\left\{\frac{200}{250}, \frac{80}{50}\right\} = \max\{0.8, 1.6\} = 1.6$$

$$b = \min\left\{\frac{200}{250}, \frac{80}{50}\right\} \Big/ 1.6 = \min\{0.8, 1.6\} / 1.6 = 0.5$$

$$a = 0.6$$

Observe that the vertical travel time is the longest, i.e. the rack is vertically dominated or taller than it is long (in time units).

Second, we will compute the expected travel times from the input/output point to a random point in the rack and between two random points in the rack for the normalized rack in the time domain. Since the output point is located symmetrically to the input point, the expected travel times from either point to a random point are the same.

$$t_{IR} = \left(\frac{1}{2} + \frac{0.5^2}{6}\right) = 0.5417$$

$$t_{RR} = \left(\frac{1}{3} + \frac{0.5^2}{6} - \frac{0.5^3}{30}\right) = 0.3708$$

$$t_{OI} = b = 0.5$$

Third, we compute the overall expected cycle time for the specified dwell point strategy based on the cycle time probability matrix, which is shown below. Observe that in the normalized time domain rack the length of the horizontal trip from output point to input point, denoted by t_{OI}, is equal to 0.5 for this example.

Table 15.2. Cycle Time Computation Matrix

	Next Operation	SC Storage	SC Retrieval	Dual
Prev. Operation	Probability	a/2	a/2	1-a
SC Storage	a/2	2 t_{IR}	2 t_{IR}	2 t_{IR} + t_{RR}
SC Retrieval	a/2	2 t_{IR} + t_{OI}	2 t_{IR}	2 t_{IR} + t_{RR} + t_{OI}
Dual	1-a	2 t_{IR} + t_{OI}	2 t_{IR}	2 t_{IR} + t_{RR} + t_{OI}

Since the last two rows of the matrix are identical for this example, we can simplify the computation:

$$t_C = \frac{a}{2}\left[\frac{a}{2}2t_{IR} + \frac{a}{2}2t_{IR} + (1-a)(2t_{IR} + t_{RR})\right] +$$

$$\left(1-\frac{a}{2}\right)\left[\frac{a}{2}(2t_{IR} + 0.5) + \frac{a}{2}2t_{IR} + (1-a)(2t_{IR} + t_{RR} + t_{OI})\right]$$

$$= 2 \cdot t_{IR} + (1-a) \cdot t_{RR} + \left(1-\frac{a}{2}\right)^2 \cdot t_{OI}$$

$$= 2 \cdot 0.5417 + (1-0.6) \cdot 0.3708 + (1-0.3)^2 \cdot 0.5 = 1.4767$$

Fourth, we transfer back the results from the normalized time rack to the original rack by multiplying with the rack scale factor and by adding the expected pickup and deposit time per cycle.

$$u = a \cdot 1 + (1-a) \cdot 2 = 0.6 \cdot 1 + (1-0.6) \cdot 2 = 1.4$$

$$T_C = t_C \cdot T + 2 \cdot u \cdot T_{PD} = 1.6 \cdot 1.4767 + 2 \cdot 1.4 \cdot 0.5 = 2.363 + 1.4 = 3.763$$

If the output point is moved to the coordinates (200, 20) while all other parameters remain the same, we can compute the expected one-way travel time to the output point and the expected time between two random points in the corresponding unit rack in the following way.

The difficult part is the computation of the expected time from a random point to the output point since we need to based this computation on two sub-racks created by the horizontal dividing line with vertical coordinate equal to 200. The expected time is then computed based on the relative size of these two sub racks. Observe that this computation has to be done in the time domain (not the normalized time domain) since the scale factors for each of the sub racks are different. Let A denote the area of a sub rack.

$$T_{TOP} = \max\left\{\frac{200}{250}, \frac{60}{50}\right\} = \max\{0.8, 1.2\} = 1.2$$

$$b_{TOP} = \frac{0.8}{1.2} = 0.667$$

$$T_{BOT} = \max\left\{\frac{200}{250}, \frac{20}{50}\right\} = \max\{0.8, 0.4\} = 0.8$$

$$b_{BOT} = \frac{0.4}{0.8} = 0.5$$

$$t_{OR,TOP} = \left(\frac{1}{2} + \frac{0.6667^2}{6}\right) = 0.5741$$

$$T_{OR,TOP} = 1.2 \cdot 0.5741 = 0.6889$$

$$t_{OR,BOT} = \left(\frac{1}{2} + \frac{0.5^2}{6}\right) = 0.5417$$

$$T_{OR,BOT} = 0.8 \cdot 0.5417 = 0.4333$$

$$T_{OR} = \frac{A_{TOP} \cdot T_{OR,TOP} + A_{BOT} \cdot T_{OR,BOT}}{A_{TOP} + A_{BOT}} = \frac{200 \cdot 0.4333 + 600 \cdot 0.6889}{800} = 0.6250$$

$$t_{OR} = \frac{T_{OR}}{T} = \frac{0.6250}{1.6} = 0.3906$$

Now we can compute the overall cycle time based on the probability matrix, which is shown below. The difference with before is the fact that t_{IR} is no longer equal to t_{RO}.

Table 15.3. Cycle Time Computation Matrix

	Next Operation	SC Storage	SC Retrieval	Dual
Prev. Operation	Probability	a/2	a/2	1-a
SC Storage	a/2	2 t_{IR}	$t_{IR + tRO}$	t_{IR} + t_{RO} + t_{RR}
SC Retrieval	a/2	2 t_{IR} + t_{OI}	2 t_{RO}	t_{IR} + t_{RO} + t_{RR} + t_{OI}
Dual	1-a	2 t_{IR} + t_{OI}	2 t_{RO}	t_{IR} + t_{RO} + t_{RR} + t_{OI}

Since the last two rows of the matrix are identical for this example, we can simplify the computation:

$$\begin{aligned}
t_C &= \frac{a}{2}\left[\frac{a}{2}2t_{IR} + \frac{a}{2}\left(t_{IR}+t_{RO}\right) + (1-a)(t_{IR}+t_{RO}+t_{RR})\right] + \\
&\qquad \left(1-\frac{a}{2}\right)\left[\frac{a}{2}(2t_{IR}+t_{OI}) + \frac{a}{2}2t_{RO} + (1-a)(t_{IR}+t_{RO}+t_{RR}+t_{OI})\right] \\
&= 0.3\cdot\left[0.3\cdot 2\cdot 0.5417 + 0.3\cdot(0.5417+0.3906) + 0.4(0.5417+0.3906+0.3708)\right] + \\
&\qquad 0.7\cdot\left[0.3(2\cdot 0.5417+0.5) + 0.3(2\cdot 0.3906) + 0.4(0.5417+0.3906+0.3708+0.5)\right] \\
&= 1.3392
\end{aligned}$$

Fourth, we transfer back the results from the normalized time rack to the original rack by multiplying with the rack scale factor and by adding the expected pickup and deposit time per cycle.

$T_C = 1.6\cdot 1.3392 + 2\cdot 1.4\cdot 0.5 = 2.1428 + 1.4 = 3.5428$ minutes.

Cycle Times for Random Storage for Order Picking Trucks

The order picking truck is another material handling equipment type operating in the aisles and is closely related to Automated Storage and Retrieval systems. Usually, the truck does not have a top rail to steady its vertical mast. In addition, safety rules usually prohibit the truck to move while a person is on the shuttle on the mast. As a consequence the movements of the order picking truck in the horizontal and vertical direction occur in a sequential fashion. The travel time can then be approximated with the rectilinear norm. Using the notations derived for the AS/RS, the cycle times can be derived as follows.

Since the horizontal and vertical travel occur sequentially and hence independent, they can be estimated separately. Assume without loss of generality that the unit rack is longer than it is tall, i.e., its horizontal dimension is equal to one and its vertical dimension is equal to b. In the unit or normalized rack the average horizontal travel is then 0.5 and the average vertical travel is then b/2. The expected one way travel time is then given by:

$$t_{IR} = \frac{1+b}{2} \tag{15.14}$$

The lines of equal travel or contour lines in the normalized time domain are illustrated in the next figure when the input/output point is in the lower left corner of the rack and the rack is longer than it is tall.

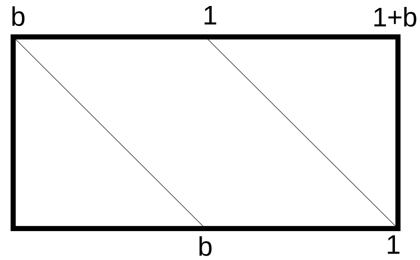

Figure 15.5. One Way Travel in an Order Picking Truck Rack

The expected one way travel time can also be derived by the following integration:

$$t_{IR} = \frac{1}{b}\int_{x=0}^{1}\int_{y=0}^{b}(x+y)dydx$$
$$= \frac{1}{b}\int_{x=0}^{1}(bx+\frac{b^2}{2})dx$$
$$= \frac{1+b}{2}$$

The single command cycle time is then given by:

$$T_{SC} = 2\cdot T\cdot t_{IR} + 2\cdot T_{PD} = T(1+b)+2\cdot T_{PD} \tag{15.15}$$

Exercises

Material Handling Equipment Selection and Sizing

A company manufactures two part types using three machines. Six distinct operations are performed in the manufacturing plant. Two material handling devices are considered to transport the parts between the machines. Table 15.4 shows the cost and required time of a processing operation of a single part on a machine, where the time is given in parentheses. Table 15.5 shows the cost and required time of transporting a unit of a part type on a material handling device, where again the time is given in parentheses. Finally, Table 15.6 shows the required production volume and operations sequence of each part type. A missing value indicates that this particular combination of part type and material handling device or machine is not feasible, i.e. the operation cannot be performed on that machine or the part cannot be transported by that material handling device. Each machine and material handling device is available for 4000 minutes. The total budget for equipment fixed costs for the planning horizon is $1,400,000. The costs per machine and material handling device are given in Tables 15.1 and 15.2.

Table 15.4. Manufacturing Operation versus Machine Data

	MC1	**MC2**	**MC3**
Cost	$140,000	$188,000	$95,000
Operation 1	20 (18)	12 (17)	
Operation 2		17 (23)	23 (12)
Operation 3	15 (14)		12 (13)
Operation 4	8 (5)	9 (6)	10 (4)
Operation 5		12 (17)	
Operation 6	15 (21)		

Table 15.5. Material Handling Operation versus Device Data

	MH1	MH2
Cost	$255,000	$195,000
Part 1	15 (3)	13 (4)
Part 2	17 (8)	

Table 15.6. Part Type Production Data

	Volume	Operation Sequence
Part 1	240	O1-O2-O3-O6
Part 2	150	O2-O5-O4-O3

Draw a diagram of the production and material handling system, representing the material flows in the system. Draw this diagram separately for each part type. The nodes represent the different machines. The arcs represent the different material handling systems (use solid lines for material handling device one and dashed lines for material handling device two). Clearly mark each node. On each arc clearly indicate the required time per operation of executing that particular production or handling operation. Clearly define the necessary variables and display all material flow variables in their correct place in the diagrams. Write out the objective function and all the necessary constraints for this particular system using the defined variables and with numerical constants. Do not use any summation signs but write out each individual term. Solve this formulation with the mixed integer programming solver of your choice.

AS/RS Throughput Requirements

A unit load automated storage and retrieval system is being designed for the reserve storage area of a mail order retailer. The planned system has 8 aisles, each with an aisle-captive crane. The horizontal and vertical speed of the cranes is 380 and 60 feet per minute, respectively. The rack is 80 columns long and 12 rows high. The centroid to centroid distance between two adjacent columns is 56 inches and is 58 inches between two adjacent rows. The pickup and deposit point for each aisle is at the front of the rack and at ground level. It is estimated that 40 % of all crane cycles are single command cycles. The required throughput of the rack is 120 storage operations and 120 retrievals per hour. The pickup and deposit time of the shuttle is equal to 30 seconds. Products are stored randomly in each aisle and randomly between aisles. Storage unit loads are waiting on eight conveyor lines, one in front of each individual crane. To avoid the construction of long conveyors holding the unit loads to be stored the retailer has specified that the expected waiting for a storage request should be less than 8 minutes. It has been agreed upon that the crane cycle times can be approximated with sufficient accuracy by a distribution with a coefficient of variation equal to 2.

Compute the expected cycle time for the cranes, the expected utilization of the cranes, and the expected waiting time of the storage operations. Does this systems satisfy the throughput and waiting time requirements? Given your computations are all assumptions for this system valid?

Simulation

Introduction

While material handling models are used for the preliminary design and sizing of material handling systems, detailed design, verification, and validation are most often based on digital simulation.

The more recent introduction of relatively inexpensive personal computers that have large internal memory and fast processor speed has allowed for the development of more interactive and graphical simulation systems. Several simulation systems even create three dimensional scale representations of the material handling systems complete with camera mounting and fly-throughs. Other systems allow the kinematic representation of material handling systems and their interaction with humans to test for ergonomic sound design. The graphical feedback allows the engineer a better overview of the system and thus enables faster design, analysis, and reporting and sales presentations to non-technical personnel.

A second recent development has been the inclusion of templates or smart objects for material handling systems. This relieves the user of the detailed coding of these complex objects and requires only that the user specifies certain parameters.

There still exists the widespread misconception that simulation is an optimization tool. While simulation lets you evaluate a number of alternative system configurations, it is intrinsically a descriptive evaluation tool. It cannot determine by itself the best value for a particular decision parameter, as a normative optimizer could. This misconception is often further encouraged by the language used in the advertising and sales literature of the simulation systems.

Some of the simulation systems currently on the market are Automod from AutoSimulations, Quest from Deneb, Micro Saint from Micro Analysis Design, and VisFactory from EAI, and Promodel by Promodel Corp. The Institute of Industrial Engineers (www.iienet.org) publishes yearly a simulation software buyers guide in their Solutions magazine.