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Allocating space in a forward pick area of a distribution center for small parts

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The forward pick area of a distribution center is a cache of conveniently located products from which order pickers can quickly draw, but which must be replenished from bulk or reserve storage. The quantities stored forward determine the amount of work required to sustain the forward pick area. Two stocking strategies that are commonly used in industry are analyzed and compared with the optimal stocking strategy for small parts.

Keywords: Warehousing, distribution, deterministic inventory theory, forward pick area, order picking

1. The forward pick area

The *forward pick area* of a warehouse functions as a "warehouse within the warehouse": many of the most popular StockKeeping Units (SKUs) are stored there in small amounts, so that order picking can be concentrated within a relatively small area. This reduces unproductive travel by order pickers and enables closer supervision. The trade-off is that the forward pick area must be replenished from a bulk storage or reserve area elsewhere in the warehouse, as suggested in Fig. 1. A typical forward pick area for small parts is an aisle (or more) of carton flow rack(s) through which runs a conveyor. Such an arrangement is common in high-volume distribution centers in North America, especially those supporting retail sales.

Because it is relatively inexpensive to pick from a forward pick area, that space is particularly valuable. During the planning horizon (which, for convenience, we take to be a year), forward space is reserved, with each SKU therein allocated a carefully determined volume. The challenge is to extract maximum value from that space.

The question of how much space should be allocated to each SKU is, in practice, answered primarily by rules of thumb. Hackman and Rosenblatt (1990) were the first to describe a mathematical model to allocate space in a forward pick area. They used a fluid model that treats the volume of each SKU as continuously divisible and incompressible. We extend their results in several ways, and compare their solution to the two standard stocking strategies commonly used in industry.

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To our knowledge only one other paper has studied the stocking of a forward pick area. Van der Berg *et al.* (1998) adapted the Hackman–Rosenblatt model to the storage of unit-loads (typically pallets), which lends the economics of restocking a simple combinatorial nature. They observe that, while "there are no savings possible by assigning more than one unit-load of a product to the forward area," one can control the timing of restocks by storing extra pallets forward. This allows restocks to be deferred until after the period of order picking, so that there is no interference.

The timing of restocks is not an issue in our model. Typical forward storage for small parts is flow rack, which can picked and restocked concurrently, with no interference.

Others have explored additional issues regarding forward pick areas, such as size (Frazelle *et al.*, 1994; Gu, 2005), limits on restocking (Frazelle *et al.*, 1994; Van den Berg *et al.*, 1998), dynamic reallocation of space (Hun, 2003), or avoiding restocks by replenishing directly from receiving (Hollingsworth, 2003).

2. Estimating restocks

We assume that SKUs have already been chosen for storage in the forward pick area as in Hackman and Rosenblatt (1990), and so the only variable cost is the labor to restock as necessary. Assuming the units of storage are less-than-pallet quantities, then, as in Hackman and Rosenblatt (1990), the number of restocks may be estimated as follows.

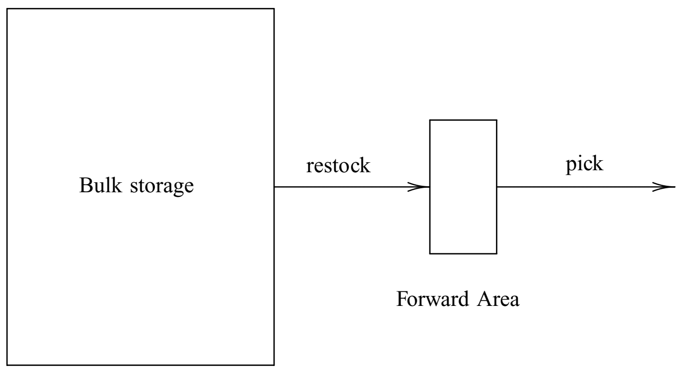

Fig. 1. The forward area concentrates picking but must be restocked from a bulk or reserve area.

Assumption 1. (Fluid model for small parts.) If SKU i flows through the warehouse at rate f_i cubic feet per year then, if v_i cubic feet of SKU i is stored in the forward pick area, it will require about

$$\frac{f_i}{v_i} \tag{1}$$

restocks per year.

These restocks are internal to the distribution center and are determined by rates of flow of the SKUs and the quantities stored, not by the company purchasing department.

There are some assumptions implicit in this model of restocks. We assume that a pick quantity never exceeds the full allocation of a SKU in the forward pick area. (In practice unusually large order quantities are typically filled from bulk storage.) Also, if a pick quantity exceeds the amount available in the forward pick area at that time, a restock is triggered.

In addition we assume that the entire restock quantity for a SKU can be carried in one trip. In this case the work to restock a SKU consists of the following components.

1. Travel between the forward pick and bulk areas: The magnitude of this cost is typically determined by the warehouse layout and not by the locations of individual SKUs.
2. Travel within the bulk area to locate stock: This is variable but unpredictable because of "random storage" in the bulk area. It is reasonable to assume an average value here. (We will relax this assumption later.)
3. Travel within the forward pick area to the location(s) to be restocked: This is a small component of the cost because a forward pick area is a relatively small part of the warehouse.
4. Handling storage units: This cost is is determined by the total volume of product sold and is fixed with respect to the decision of how much to store forward. For example, if a SKU sells 100 cartons annually from the forward pick area, then all 100 cartons must be handled, independently of the quantity stored forward.

Because these cost components are either small or fixed with respect to the decision of quantity to store, we take the number of restocks as a measure of the cost of maintaining the forward pick area.

3. Optimally allocating space

Let the physical volume of available storage be normalized to unity and let f_i be scaled accordingly. Let v_i represent the fraction of space allocated to SKU i. Then, following Hackman and Rosenblatt (1990), the problem of allocating space among n SKUs to minimize annual restocks may be expressed as

$$\min \sum_{i=1}^{n} f_i/v_i, \tag{2}$$

$$\sum_{i=1}^{n} v_i \leq 1, \tag{3}$$

$$v_i > 0, \quad i = 1, \ldots, n. \tag{4}$$

An economic order quantity-like solution to problem (2)–(4) follows by induction on the number of SKUs.

Theorem 1. (Hackman and Rosenblatt (1990), Equation (5)). *To minimize the total restocks over all SKUs $j = 1, \ldots, n$, each SKU i should be stored in the amount*:

$$v_i^* = \frac{\sqrt{f_i}}{\sum_{j=1}^{n} \sqrt{f_j}}. \tag{5}$$

We refer to the volumes of Equation (5) as the Optimal allocation.

First we record some previously unremarked properties of the Optimal allocations.

Theorem 2. *Under Optimal allocations each unit of storage space is restocked at the same frequency.*

Proof. Under Optimal allocations, the restocks per unit of space are

$$\frac{f_i/v_i^*}{v_i^*} = \left(\sum^{n} \sqrt{f_j}\right)^2,$$

which is constant and identical for all SKUs i. ■

This means that restocks should be distributed *uniformly* throughout the volume of the forward pick area. Roughly speaking, the restockers should visit each section of shelving at about the same frequency. This provides a useful way to benchmark a forward pick area without any measurements whatsoever. Simply ask restockers whether they tend to visit some parts of the forward pick area more often than others; if so, then the storage policy is out of balance and there is excessive restocking.

We have just seen that Optimal allocations call for SKU i to receive a fraction of the available space equal to $\sqrt{f_i}/(\sum_j \sqrt{f_j})$. A similar result holds for allocation of restocking effort.

Lemma 1. *Under Optimal allocations, SKU i incurs a fraction of the total restocks equal to* $\sqrt{f_i}/(\sum_j \sqrt{f_j})$.

4. Two common storage strategies

The Optimal storage policy of Equation (5) is generally unknown to industry. To learn industry practice, we have asked hundreds of people in the warehousing industry about how they stock a forward pick area. All answers have been one of the two following (which were also observed by Van den Berg *et al*. (1998) to be typical).

1. *Allocate the same amount of space to each SKU.* We call this the Equal Space strategy (EQS) and model it by $v_i = 1/n$, from which it follows that SKU i is restocked nf_i times a year.
2. *Store an equal time supply of each SKU.* We call this the Equal Time strategy (EQT). It requires that $v_i/f_i = v_j/f_j$ for any two SKUs. Substituting this in the expression $\sum_i v_i = 1$ yields $v_i = f_i/\sum_j f_j$, from which it follows that each SKU i is restocked at the common rate of $\sum_j f_j$ times a year.

Our idealizations of these two stocking strategies are also fluid models. This simplicity enables us to compare the three strategies in some detail. Of course the fluid models ignore the geometries of the SKUs and of storage and so the recommended allocations of space might not be precisely realizable in practice. However, when SKUs are relatively small, such as for pharmaceuticals, office supplies and cosmetics, then the values can be rounded off to match the sizes of the available storage containers (cartons, bins, etc.).

It seems obvious to most people that Equal Space is not the best storage strategy because it ignores all differences in SKU popularity and size. In our surveys, people in the warehousing industry unanimously expressed the belief that Equal Time allocations reduce restocks because a more popular SKU will be allocated more space. (Indeed, two respondents said they had, at some cost, revised their warehouse management systems to store Equal Time rather than Equal Space allocations, believing this to be an improvement.) This observation is folk wisdom in the industry; but it is wrong.

Theorem 3. *For a given set of SKUs, Equal Time allocations require the same total restocks,* $n\sum_j f_j$, *as Equal Space allocations.*

Proof. Simple algebra gives the counts of Table 1. ■

This also reveals an interesting duality between Equal Space and Equal Time allocations.

Corollary 1. *The space and the labor consumed by SKU i in the forward pick area under each of the two storage strategies are as listed in Table 2.*

Table 1. Comparison of the Equal Space and Equal Time Allocations

	Equal space	*Equal time*
Allocation v_i for SKU i	$1/n$	$f_i/\sum_j f_j$
Restocks for SKU $i = f_i/v_i$	nf_i	$\sum_j f_j$
Total restocks over all SKUs	$n\sum_i f_i$	$n\sum_j f_j$

For comparison, under the Optimal strategy SKU i gets the fraction $\sqrt{f_i}/(\sum_j \sqrt{f_j})$ of the space and incurs the same fraction of restocks.

5. Comparison with the Optimal strategy

5.1. *Total restocks*

Frazelle suggests "an arbitrary allocation of space… or space for a quantity sufficient to satisfy the expected weekly or monthly demand" (Tompkins and Harmelink, 1994). About three-quarters of the distribution centers we surveyed store Equal Time amounts in the forward pick area. Needless to say, an "arbitrary allocation of space" is suboptimal; and, as we have shown, an Equal Time allocation is no better than an Equal Space allocation, both of which are suboptimal. Suboptimal allocations give too much space to some SKUs and too little to others. Any SKU that is allocated too little space must be restocked too often and so generates extra work. Any SKU that is allocated too much space leaves too little space for the other SKUs. When the excessive work is accumulated over tens of thousands of SKUs, the total can be significant when compared to the optimal storage amounts.

The Equal Space and Equal Time allocations incur more restocks than necessary; but how severe is the waste? We answer this by studying the ratio of the number of restocks under the Equal Space (EQS) or Equal Time allocations (EQT) to the number incurred under the Optimal allocations (OPT).

We have computed the value of EQT/OPT for several warehouses and found that EQT/OPT ≈ 1.45 for 6000 fast-moving SKUs of a major drug store chain. This suggests that this particular warehouse, which had 20 restockers, may have needed only 14. Similarly, for 4000 SKUs of a telecommunications company we computed a ratio of 2.44, which means that storing SKUs in Equal Time allocations incurred more than twice as many restocks as necessary.

It is difficult to make a general statement about the magnitude of EQT/OPT except in some special cases, such as the following.

Table 2. Comparison of the Equal Space and Equal Time allocations

	Equal space	*Equal time*
Fraction of space to SKU i	$1/n$	$f_i/\sum_j f_j$
Fraction of restocks to SKU i	$f_i/\sum_j f_j$	$1/n$

Theorem 4. *If the values of the $\sqrt{f_i}$ are drawn independently from a distribution with well-defined mean and variance (and coefficient of variation CV), then*:

$$\frac{EQT}{OPT} \approx 1 + CV^2.$$

This suggests that, the more diverse the rates of flow of the SKUs, the more important it is to allocate space optimally, rather than by Equal Space or Equal Time strategies.

Proof. From Theorems 1 and 3:

$$\frac{\text{EQT}}{\text{OPT}} = \frac{n \sum f_i}{\left(\sum \sqrt{f_i}\right)^2} = \frac{\sum f_i / n}{\left(\sum \sqrt{f_i}/n\right)^2}.$$

Estimate the sample mean μ, sample second moment and sample variance σ^2 of the values $\sqrt{f_i}$ as $(\sum \sqrt{f_i})/n$, $(\sum f_i)/n$ and $\sum(\sqrt{f_i} - \mu)^2/(n-1)$, respectively; then because n is large (in the thousands for a typical large North American distribution center) the following is a good approximation:

$$\frac{\text{EQT}}{\text{OPT}} \approx \frac{\mu^2 + \sigma^2}{\mu^2}. \qquad \blacksquare$$

We examined actual values of the $\sqrt{f_i}$ from several warehouses and all the distributions are similar in displaying broad dynamic range and in being highly skewed, with a very few SKUs having large values of flow and many others having much smaller values, as illustrated in Fig. 2. There are many distributions sharing these general properties, but a power law seems a likely possibility. If the $\sqrt{f_i}$ are described by a power law $p(x) = Cx^{-\alpha}$ with power $\alpha > 3$ then following the analysis of Newman (2005) we have that:

$$\text{EQT/OPT} \approx \frac{(\alpha - 2)^2}{(\alpha - 1)(\alpha - 3)}. \tag{6}$$

When $\alpha \leq 3$ the variance is undefined and we cannot estimate EQT/OPT. For the values in Fig. 2, a maximum likelihood estimate of α is 3.19 (Newman, 2005), for which EQT/OPT = 3.39. Alternatively, if the $\sqrt{f_i}$ follow an exponential distribution, the coefficient of variation is unity and so EQT/OPT $\approx$ 2. These several pieces of evidence all suggest that it is reasonable to expect a significant reduction in restocking when moving to the Optimal stocking strategy.

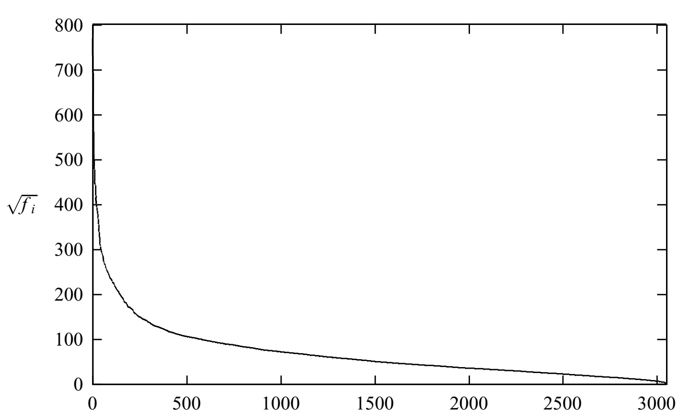

Fig. 2. The distribution of the square roots of flows of 3051 SKUs, ranked from largest to smallest.

5.2. *Manageability*

Equal Space and Equal Time allocations each offer a kind of uniformity that can simplify warehouse management. Under Equal Space allocations all storage slots are the same size and so a newly arrived SKU will fit in any available space in the forward pick area. Under Equal Time allocations each SKU is restocked at the same frequency and so it may be easier to manage the process.

How manageable are the Optimal allocations? We will show that Optimal allocations share space among SKUs more evenly than Equal Time allocations and share restocks more evenly than Equal Space allocations, and so is, by these measures, an intermediate solution.

Figure 3 shows histograms of allocation sizes and of restocking frequencies based on data from a major chain retailer in the US. In each case the distributions of Optimal allocations (foreground) have significantly less variability.

Figure 4 provides another way of looking at this point. It was constructed by ranking the same set of SKUs from largest to smallest value of flow f_i (which means they are also ranked by size of allocation for each of the allocation strategies) and then plotting the cumulative space consumed by the top fraction of SKUs. The curve corresponding to Equal Time allocations is everywhere above that of the Optimal allocations, which in turn is everywhere above the Equal Space allocations. For example, the 20% of SKUs with the most space fill almost 80% of the forward pick area under Equal Time allocations, 50% under Optimal allocations and 20% under Equal Space allocations.

As we shall show, this behavior holds in general. Assume that the f_i have been labeled so that $f_1 \geq f_2 \geq \cdots \geq f_n$.

Lemma 2. *The k largest of the Optimal allocations never consume more space than they would under Equal Time allocations*:

$$\sum_{i=1}^{k} \frac{\sqrt{f_i}}{\sum_{j=1}^{n} \sqrt{f_j}} \leq \sum_{i=1}^{k} \frac{f_i}{\sum_{j=1}^{n} f_j}, \quad k = 1, 2, \ldots, n. \tag{7}$$

Proof. Because the flows have been labeled in decreasing order, $(\sqrt{f_{k+1}})(\sqrt{f_i}) \leq f_i$ for all $i \leq k$. Summing both sides over i:

$$\sqrt{f_{k+1}}\left(\sum^{k} \sqrt{f_i}\right) \leq \sum^{k} f_i \quad \text{for all } k,$$

$$f_{k+1}\left(\sum^{k} \sqrt{f_i}\right) \leq \sqrt{f_{k+1}}\left(\sum^{k} f_i\right).$$

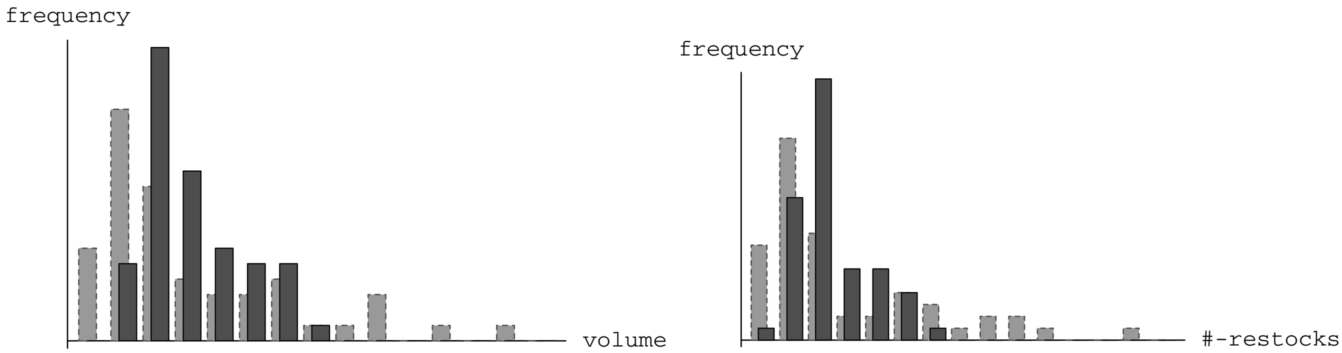

Fig. 3. Optimal allocations (foreground) have less variability in the allocated volume than Equal Time allocations (left) and less variability in number of restocks than Equal Space allocations (right).

Adding $(\sum^k f_i)(\sum^k \sqrt{f_i})$ to each side and then factoring gives:

$$\left(\sum^k \sqrt{f_i}\right)\left(\sum^k f_i + f_{k+1}\right) \leq \left(\sum^k f_i\right)\left(\sum^k \sqrt{f_i} + \sqrt{f_{k+1}}\right),$$

$$\left(\sum^k \sqrt{f_i}\right)\left(\sum^{k+1} f_i\right) \leq \left(\sum^k f_i\right)\left(\sum^{k+1} \sqrt{f_i}\right),$$

$$\frac{\sum^k \sqrt{f_i}}{\sum^k f_i} \leq \frac{\sum^{k+1} \sqrt{f_i}}{\sum^{k+1} f_i}.$$

Since the last statement holds for all k, it follows that:

$$\frac{\sum^k \sqrt{f_i}}{\sum^k f_i} \leq \frac{\sum^n \sqrt{f_i}}{\sum^n f_i},$$

$$\frac{\sum^k \sqrt{f_i}}{\sum^n \sqrt{f_i}} \leq \frac{\sum^k f_i}{\sum^n f_i},$$

and the result follows. ■

Interestingly, if we were to plot a companion to Fig. 4 but showing the distribution of relative restock rates, the plot would look exactly the same, except for interchanging the labels "Equal Time" and "Equal Space". This follows from the sort of duality observed in Corollary 1, Theorem 3 and Lemma 1.

In a sense, Equal Time allocations are more extreme than Optimal allocations. This is useful to know when, for example, planning bin sizes to hold the SKUs of a forward pick area.

Corollary 2. *The largest of the Optimal allocations is never larger than that of the Equal Time allocations. Similarly, the smallest of the the Optimal allocations is never smaller than that of the Equal Time allocations.*

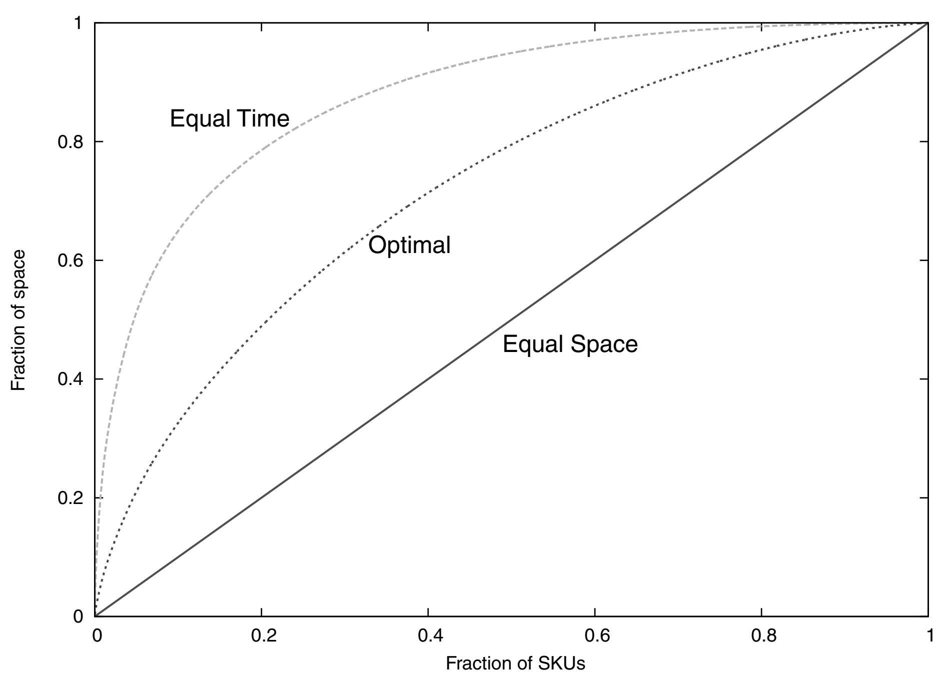

Fig. 4. Along the horizontal axis 3051 SKUs are ranked from largest to smallest flow ($f_1 \geq \cdots f_n$) The curves display the cumulative fraction of space consumed by each allocation strategy.

However, we can say more: Optimal allocations share space more evenly than Equal Time allocations and share labor (restocking) more evenly than Equal Space allocations.

Theorem 5. *For any given set of SKUs*:

1. *The sizes of the Optimal allocations have a sample variance that does not exceed that of the Equal Time allocations.*
2. *The rates of restocking the Optimal allocations have a sample variance that does not exceed that of the Equal Space allocations.*

Proof. Lemma 2 is equivalent to saying that the Optimal allocations are *majorized* by the Equal Time allocations: See, for example, Marshall and Olkin (1979) , wherein Dalton is credited with establishing that when one vector is majorized by another, the sample variance of the one cannot exceed that of the other.

Corollary 1 to Theorem 3 shows that the frequencies of restocking Optimal allocations are majorized by the frequencies incurred by Equal Space allocations and so the second claim holds. ■

As described by Marshall and Olkin (1979), Dalton showed that, in addition to sample variance, several other measures ϕ of dispersal respect the sense of majorization in that if vector $\mathbf{v}$ is majorized by vector $\mathbf{w}$ then $\phi(v) \leq \phi(\mathbf{w})$. This is illustrated in the data of Fig. 4, where Equal Time allocations display about ten times the variance displayed by Optimal allocations. Furthermore, the largest Equal Time allocation is about six times that of the Optimal allocations; and the smallest is about one-tenth that of the Optimal allocations.

All these measures of dispersion tell the same story: Optimal allocations share the space more evenly amongst the SKUs than do Equal Time allocations; and Optimal allocations share the work more evenly amongst SKUs than do Equal Space allocations. Equal Space/Time allocations sacrifice one type of manageability for another; Optimal allocations balance the two—and reduce labor.

6. A case study: Exchanging time for space, saving both

In storing Optimal allocations one may claim a saving in the form of space instead of labor. For example, Theorem 4 suggests that one might squeeze SKUs into half the space occupied by Equal Time or Space allocations without increasing the labor to sustain the forward pick area. In effect, the capacity of the forward pick area will have been increased by half.

We made such a trade-off of labor for space at the national distribution center of Revco Drugstores (now part of CVS Drugstores) in Knoxville, Tennessee, USA. They stored 3 weeks supply of each of about 6000 SKUs in 325 bays of carton flow rack that served as a forward pick area for the most popular products. A steady expansion in the number of SKUs had filled all available space and Revco was considering expansion into a nearby warehouse.

Table 3. The total cost of restocking

EQS	*EQT*	*OPT*
$n \sum_i c_i f_i$	$(\sum_i c_i)(\sum_i f_i)$	$(\sum_i \sqrt{c_i f_i})^2$

We projected that if the 325 bays were stocked optimally then the required restocks would drop by 40%. In addition by squeezing the SKUs into 285 bays of rack, 40 bays were left empty for growth and the restocks were still reduced by 20%. In this way we both reduced labor *and* created space for Revco simply by storing SKUs in optimal amounts.

7. Extensions

7.1. *Differing costs per restock*

Sometimes the cost of restocking a SKU depends significantly on the identity of the SKU, as would be the case if some overstock were held in an outlying warehouse or if bulk storage was zoned. This can be modeled by charging a cost c_i per restock of SKU i. Following Hackman and Rosenblatt (1990), simply replace any appearance of f_i with the weighted flow $\hat{f}_i = c_i f_i$ and our results describing the Optimal allocations still follow. In contrast, Equal Space/Time allocations ignore differences in the costs of restocking.

Simple algebra yields the total cost of restocking under each of the three strategies listed in Table 3.

The total cost of Optimal allocations is, of course, still the smallest, as may be confirmed by the Cauchy–Schwarz inequality:

$$\left(\sum_i a_i b_i\right)^2 \leq \left(\sum_i a_i^2\right)\left(\sum_i b_i^2\right).$$

Letting $a_i = 1$ and $b_i = \sqrt{c_i f_i}$ shows that OPT $\leq$ EQS; and letting $a_i = \sqrt{c_i}$, $b_i = \sqrt{f_i}$ shows that OPT $\leq$ EQT.

If the cost c_i to restock SKU i depends on where in bulk storage or where in the forward area SKU i is stored, then both Optimal and Equal Space allocations can take advantage of that by storing SKUs with large flows f_i in convenient locations (small c_i). No such savings are possible under Equal Time allocations because every SKU is restocked at the same frequency and so there are no savings possible by careful placement of SKUs. A restocker must visit the least convenient locations in bulk storage about as often as the most convenient.

7.2. *Reorder points and safety stock*

Optimal allocations can easily be adapted to account for reorder points and safety stock levels. To guard against stockout in the forward pick area each SKU must be stored in sufficient quantity to cover the mean lead time demand l_i. We can enforce this by revising the statement of Problem 2 to include constraints:

$$v_i \geq l_i. \tag{8}$$

This extended model can be solved efficiently by an algorithm of Luss and Gupta (1975) that repeatedly allocates space according to Equation (5), identifies SKUs that received less than their minimum required space l_i and increases their allocation to l_i, then reallocates the remaining space among the remaining SKUs.

In addition, if the allocation of each SKU i is to include safety stock s_i (which we assume has been exogenously determined), then a total volume of $S = \sum_i s_i$ within the forward pick area must be devoted to safety stock, leaving the remaining $1 - S$ to hold cycle stock and this is the space that is allocated to minimize total restock costs.

In contrast, it can be problematical to adapt Equal Space or Equal Time allocations to account for lead time demand and safety stock. For example, Equal Space allocations must include the safety stock as part of the allocation (otherwise allocations are unlikely to be equal). If the mean lead time demand l_i plus safety stock s_i of SKU i were to exceed its allocation $v_i = 1/n$ under Equal Space, then presumably SKU i would have to be excluded from the forward pick area. Under Optimal allocations, space is reallocated from other SKUs and so the decision of which SKUs to include remains more clearly separated from the allocation decision.

8. Critique of model

We have analyzed a continuous model that ignores geometries of SKUs and storage medium. That the continuous model is relevant is guaranteed by the fact that it is the ideal toward which all warehouse managers strive, by sizing shelves and packaging SKUs to reduce the space wasted by imperfect fit. In any event all three stocking strategies must eventually be converted to discrete allocations—specifications of exactly how many cartons of each SKU and how they are arranged. Currently, warehouses approximate Equal Time and Equal Space allocations, and the same can be easily done for Optimal allocations. These approximations will be accurate to the extent that the storage containers are small with respect to the shelves, as might be expected in warehouses distributing service parts, cosmetics, pharmaceuticals or office supplies.

The predictions of the continuous model were confirmed when we slotted the 3051 SKUs of Fig. 4 using a discrete model that reflected the geometry of shelving and cartons (a topic for a future paper). The discrete allocations specified exactly how many cartons were to be placed on each shelf, in what orientation, stack height and how many lanes. The discrete approximations to the Optimal allocations required less than half the restocking labor of the discrete approximations to the Equal Space allocations.

A further word is necessary regarding Equal Time allocations. Our model has omitted the possibility that one might schedule Equal Time allocations to even out restocking, and so level the labor requirement, or else concentrate restocking to create opportunities for batching restocks (that is, retrieving in one trip multiple products to be restocked). Might one of these strategies enable Equal Time allocations to reduce the cost per restock so that total cost of restocking is comparable with Optimal allocations? Perhaps, but it is difficult to say.

To be preferred, the total savings from batching restocks under Equal Time allocations must compensate for the additional restocks incurred, which, from Theorem 4 and the discussion following, might easily be two to three times as many as under Optimal allocations. Thus, for Equal Time allocations to be competitive, two or three SKUs must be carried together from bulk storage during each trip to restock. However, this reduces only one component of the work to restock: travel between the forward pick area and bulk storage. Because multiple locations must be visited within bulk storage and again within the forward pick area, the travel in these areas per trip must increase, and batching k restocks delivers less than k times the efficiency. This means that Equal Time allocations must enable batching of *more* than two or three SKUs per restock to be competitive with Optimal allocations. If batching of restocks is allowed for Optimal allocations, Equal Time allocations then would have to batch even more SKUs, perhaps five or six or more, to be competitive. This begins to seem impractical if these must all fit on a pallet that is not shrink-wrapped, and in quantities each sufficient to fill a lane of carton flow rack that might be 8 to 12 feet deep. Can such a load be conveyed without toppling?

There is also the question of whether, under Equal Time allocations, there are sufficient SKUs that require restocking at the same time to allow significant batching. The timing of restocks is driven by customer orders, over which warehouse management has little control. The flow of a SKU can be predicted with much greater accuracy over a year than over a day, and so our model predicts the total number of restocks much more accurately than their timing. Consequently one should not expect all SKUs with Equal Time allocations to require restocking simultaneously. It seems reasonable to expect more opportunities to batch restocks under Equal Time allocations, but unless there is little variability in daily order quantities, the actual opportunities for batching restocks might not be much greater than under Optimal allocations.

Of course there are occasions in which Equal Time allocations make sense, such as for products with a very short, common life cycle, in which case the forward pick area might be stocked with just enough of each SKU to carry through the selling season. However, we believe that Equal Time is used most often in the simple, mistaken belief that it reduces restocks compared to Equal Space allocations.

9. Conclusions

Almost everyone in industry stocks their forward pick areas with what we believe to be insufficient regard to the labor to maintain them. This may be due to an understandable focus on reducing the work at the front-end (order picking), which typically consumes more labor than any other warehouse process. However, simply by storing product in the right quantities, one can reduce the work to maintain the forward pick area without affecting any other operation. Furthermore, this can be done incrementally by adjusting quantities whenever a SKU is restocked.

Each of the strategies for stocking a forward pick area has advantages and disadvantages. Equal Space and Equal Time allocations require the same work to maintain a forward pick area, while Optimal allocations may be expected to require significantly less work. Equal Space allocations have the advantage of requiring no knowledge of the size or popularity of the SKUs; and the uniformity of storage may make it easier to manage SKUs with short life cycles, such as apparel or cosmetics. To use either Optimal or Equal Time allocations one must forecast the flow of each SKU over the planning period. This requires knowing the physical dimensions of the product, which is common in industrialized countries but far from universal. In addition, one must forecast demand and these forecasts are more likely to be most reliable for mature commodity products.

Finally, there may be second order benefits from changing from Equal Space/Time allocations to Optimal allocations. If safety stock is a non-decreasing function of the total number of restocks during the year, as would be the case if, for example, the workforce devoted to restocking is of fixed size, then there is a virtuous cycle in which reducing restocks reduces the required lead time, which reduces both mean lead time demand and required safety stock. This, in turn, frees up more space to be used for cycle stock, which further reduces required restocks, completing the virtuous cycle. The virtuous cycle suggests that it might be worth re-computing mean lead time demands and safety stocks and then re-allocating space, perhaps even a few times. Computational experiments suggest that, taken together, these echoing improvements can generate an additional 3 to 6% reduction in restocks.

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Biographies

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