Joint Monitoring of PID-Controlled Processes

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Statistical process control (SPC) monitoring of the special causes of a process, along with engineering feedback control such as proportional-integral-derivative (PID) control, is a major tool for on-line quality improvement. In this paper, a strategy to jointly monitor the PID-controlled outputs and the manipulated inputs using bivariate SPC is proposed, and a specific design procedure that takes into account both the controller and the disturbance model is provided. The run length properties of the joint monitoring schemes, using Hotelling's approach and Bonferroni's approach, are systematically investigated. The robustness of the proposed monitoring schemes with respect to disturbance model uncertainty is studied, and a decision rule to select an appropriate monitoring scheme is proposed.

Introduction

An effective strategy for on-line quality improvement is to use statistical process control (SPC), along with feedback control, to monitor the special causes of a process (see, e.g., Box and Kramer (1992); Vander Wiel et al. (1992); and Montgomery et al. (1994)). Several fundamental issues and technical challenges need to be addressed in order to develop an effective SPC algorithm for a controlled process.

Joint Consideration of Controlled Process Output and Manipulated Input

In most controlled processes, the only monitored variable, if any, is the controlled output (see, e.g., Montgomery et al. (1994)). However, monitoring the controlled outputs alone may not be sufficient because the process changes can be compensated by control actions and are therefore hard to detect using the controlled outputs. MacGregor (1991), in his discussion of Montgomery and Mastrangelo (1991), suggested the potential value of monitoring the manipulated inputs. Faltin and Tucker (1991) and Faltin et al. (1993) addressed the issue of what quantities in a controlled process should be monitored.

Messina et al. (1996) considered the monitoring of the manipulated variable of the minimum mean squared error control under an autoregressive moving average (ARMA) disturbance process and proposed joint monitoring strategies based on a comparison of monitoring the outputs alone and monitoring the inputs alone. Although they thoroughly studied the effectiveness of monitoring the manipulated inputs, they did not provide either specific design procedures or quantitative performance analysis of the joint monitoring strategies. Our research indicates that the performance of joint monitoring cannot be predicted simply from monitoring the outputs or monitoring the inputs alone. A comprehensive study is required for different process models and corresponding control parameters.

It is worth noting that monitoring the controlled outputs focuses on the "transient period," while monitoring the manipulated inputs focuses on the

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"static period." Once a step-change or mean-shift in the input is added to the system, the output will experience a time period with larger dynamic responses. This time period is called a "transient period" in control literature (Franklin, Powell, and Emami-Naeini (1986)) and is considered a "window of opportunity" for detection from the output (Vander Wiel (1996)). After the transient period, the system output will remain in a small static range, referred to as a "static period," in which some larger than usual manipulated input can still be observed because of the mean-shift (MacGregor (1991)). In this paper, a novel strategy is proposed to jointly monitor the proportional-integral-derivative (PID)-controlled output and the manipulated input using bivariate SPC. A step-by-step procedure is proposed, and its performance analysis is presented.

Handling of Autocorrelated and Cross-Correlated Output and Input

Feedback controlled processes are often autocorrelated, and various studies have been done on autocorrelated SPC (see, e.g., Vasilopoulos and Stamboulis (1978); Alwan and Roberts (1988); Montgomery and Mastrangelo (1991); and Wardell, Moskowitz, and Plante (1994)). However, monitoring PID-controlled processes, which are different from autocorrelated SPC processes, has some unique features: (1) more complicated autocorrelated structures due to feedback control, (2) the PID parameters impact the SPC performance, and (3) correlation between input and output. These features have not been considered in the existing literature.

In this paper, a novel strategy is proposed to design joint SPC algorithms based on the PID control structures and parameters. The relationships among the controlled output and input, the model parameters, and the PID control parameters are established. They provide a basis for deriving an SPC algorithm that incorporates the autocorrelations and crosscorrelations inherent in the process.

Robustness to Process Model Parameter Uncertainty

In a controlled process, a model is generally required in the PID controller design. The model is normally obtained from physical or engineering knowledge or estimated from process data. The accuracy of the model varies with the complexity of the process. Various research studies have been done in the engineering control community on PID controller robustness and sensitivity to model parameter uncertainties. The literature on SPC robustness with respect to model uncertainty is scanty because the issue is not important for conventional SPC, in which an independent and identically distributed disturbance process is assumed. However, when monitoring autocorrelated processes, their disturbance models and autocorrelation structures need to be taken into consideration. Thus, robustness to model uncertainty may have an impact on SPC performance. Recently, Adams, Woodall, and Superville (1994), in their discussion of Wardell, Moskowitz, and Plante (1994), noted the importance of SPC robustness with respect to disturbance model uncertainty. Vander Wiel (1996) gave a simple example to show the sensitivity of the average run length (ARL) of the residual-based cumulative sum for an integrated moving average (IMA) disturbance process when the model parameter is incorrectly estimated.

In this paper, new criteria for SPC robustness are defined, and based on these criteria, the robustness of the proposed monitoring schemes is systematically investigated throughout the ARMA(1,1) disturbance parameter space. Finally, an effective on-line monitoring scheme selection strategy is proposed to consider the model uncertainty and provide a robust SPC scheme for the PID-controlled process.

Process Models and the PID Control Schemes

Consider a process under feedback control, and assume, without loss of generality, that the target value is zero. The process output, e_t, which can be viewed as the deviation from target, is the sum of two components: the output from the process dynamics, Y_t, and the process disturbance, D_t. Let X_t denote the control action, with the initial input assumed to be zero. Throughout this paper we will consider a dynamic model for process output with $Y_t = X_{t-1}$. This implies that the control action immediately has its full effect on the process output in one run, which is reasonable in most run-to-run (RTR) manufacturing processes (Del Castillo and Hurwitz (1997)).

The output e_t can be written as

$$e_t = Y_t + D_t = X_{t-1} + D_t, \qquad (1)$$

where D_t is the stationary ARMA(1,1) disturbance process with

$$D_t = \phi D_{t-1} + a_t - \theta a_{t-1}, \qquad (2)$$

where $|\phi| < 1$, $|\theta| < 1$, and a_t represents white noise. Some of the results in this paper can be extended to other models. For example, when ϕ is close to 1, D_t is approximately an IMA(0,1,1) model. The monitoring of an IMA(0,1,1) process has been thoroughly investigated by Vander Wiel (1996). He pointed out that a mean-shift in the process results in a patterned shift in the mean of forecast errors or controlled outputs. Initially, the mean shifts by the same amount as the process level; later it decays geometrically to zero as the forecast or control "recovers" from the upset. For simplicity, we will restrict our attention to the ARMA(1,1) disturbance processes.

In industrial practice, the PID controllers are by far the most commonly used (Astrom (1988)) and typically are the only feedback control instruments available in a process. A PID control scheme can be expressed as

$$X_t = -k_P e_t - k_I \sum_{j=0}^{\infty} e_{t-j} - k_D(e_t - e_{t-1}), \qquad (3)$$

where k_P, k_I, and k_D are constants. In some situations, only one or two of these three modes of action are used. For example, we can set k_P and k_D to zero, which is called the integral (I) control or EWMA control and has been widely studied in the literature. Or we can set k_D to zero, which is referred to as proportional-integral (PI) control (see Tsung, Wu, and Nair (1998)). In this paper, a PID control is used to minimize the process variation due to process disturbance by manipulating the process input. Based on this criterion, the PID control parameters—k_P, k_I, and k_D—can be chosen from Figure 1, which is taken from Tsung and Shi (1998).

The PID parameter zones in Figure 2(a) are divided according to the sign of k_P and the zero or nonzero value of k_I: Zone A is the region with positive k_P and nonzero k_I; Zone B is the region with positive k_P and zero k_I; Zone C is the region with negative k_P and zero k_I. These zones will be used to describe and explain the performance of the proposed monitoring schemes in later sections. Figure 2(a) shows that the sign of k_P can affect the feedback control performance as well as the SPC monitoring performance. One explanation of this effect is that the proportional control with negative k_P may produce a larger oscillation than that with positive k_P, thereby increasing the power of detection. The SPC monitoring performance also varies with zero or nonzero k_I values; it is known in control theory that if there is an I control involved in a process with a sudden mean-shift, the mean of the controlled output will return to zero after the transient period (Franklin, Powell, and Emami-Naeini (1986)). Hence, SPC monitoring may not be effective if it misses the transient period. There is, however, no significant difference in the SPC performance for different k_D values from our observation.

Figure 2(b) shows different correlation patterns between e_t and X_t in different zones. In Zone A, the magnitude of correlation gets smaller, approaching zero as ϕ gets close to 1. In Zone B, the correlation is between −0.8 and −1.0. In Zone C, the correlation is between 0.8 and 1.0. Both the PID parameter values and the corresponding correlation patterns will be shown to have significant effects on the improved performance of joint monitoring over individual monitoring.

Procedures for Joint Monitoring Schemes

Two joint monitoring procedures based on Bonferroni's and Hotelling's approaches are studied in

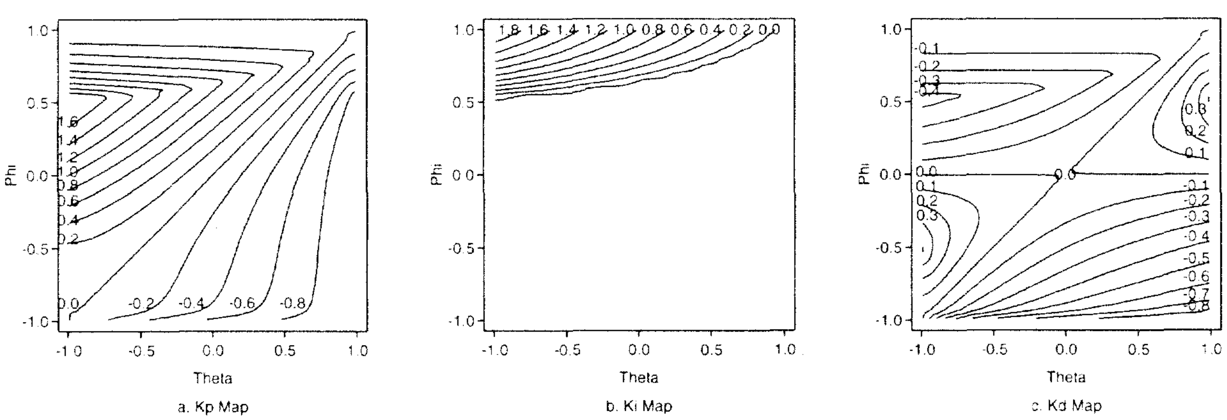

FIGURE 1. PID Design Maps for ARMA(1,1) Disturbance Processes.

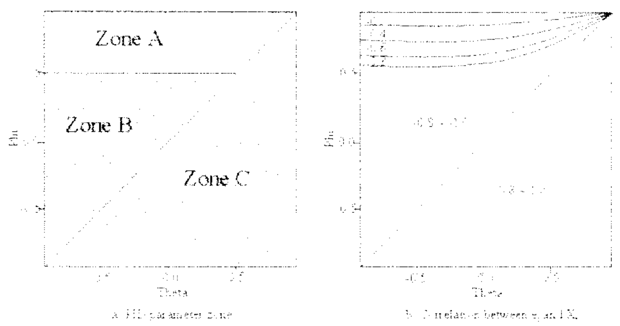

FIGURE 2. PID Parameter Zones for ARMA(1,1) Disturbance Processes and the Corresponding Correlation Between e_t and X_t.

this section. Neither approach can be used for the pure P-controlled processes as the manipulated inputs are proportional to the controlled outputs. Using Hotelling's approach for the P-controlled processes, the determinant of the variance-covariance matrix is singular (i.e., $|\boldsymbol{\Sigma}_{e,X}| = 0$ as e_t is proportional to X_t); using Bonferroni's approach for the P-controlled processes, the Type I error is incorrectly designed. It is known that a pure P-controlled process implies that the disturbance process is assumed to be MA(1) (i.e., $D_t = a_t - \theta a_{t-1}$ with $\theta = -k_P$). Thus, when ϕ is close to zero—that is, when the disturbance process is close to an MA(1) model where the best choices of k_I and k_D are close to zero (see Figure 1)—neither approach is recommended. For this process, it is equally effective to monitor either the controlled outputs or the manipulated inputs alone.

Because of its simplicity, Bonferroni's approach was recommended by Alt (1985) and Doganaksoy, Faltin, and Tucker (1991). It is based on the Bonferroni inequality,

$$P(e \cap X) = 1 - P(\bar{e} \cup \bar{X}) \geq 1 - (P(\bar{e}) + P(\bar{X})). \qquad (4)$$

where $\bar{e}$ and $\bar{X}$ are the complements of e and X. From Equation (4), choosing the control limits to ensure $P(\bar{e}) = P(\bar{X}) = \alpha/2$ leads to the overall probability $P \geq 1 - \alpha$. Because the inequality in Equation (4) is not sharp, it is a conservative (but simple) procedure. To obtain the control limits of e_t and X_t, their standard deviations, σ_e and σ_X, must be evaluated. The standard deviation σ_e is a function of the disturbance model parameters (ϕ, θ) and the PID parameters (k_P, k_I, k_D) and can be expressed as

$$\sigma_e = \sqrt{I + 2\rho_1(II + III)}\sigma_D, \qquad (5)$$

where ρ_1 is the first-order autocorrelation of D_t, σ_D is the standard deviation of D_t, and I, II, and III can be obtained from Equation (A1) in Appendix A. Note that ρ_1 and σ_D can be easily obtained during disturbance-process modeling (Box, Jenkins, and Reinsel (1994)). Similarly, σ_X is also a function of k_P, k_I, and k_D and ϕ and θ and can be expressed as

$$\sigma_X = \sqrt{I' + 2\rho_1(II' + III')}\sigma_D, \qquad (6)$$

where I', II', and III' can be calculated from Equation (B1) in Appendix B.

The center lines of the joint charts are the means of e_t and X_t, which are assumed to be zero. The control limits are set at some constants, L_e and L_X, multiplied by the standard deviations (σ_e and σ_X) above and below the center lines. Here the constants are chosen to be

$$L_e = L_X = z_{(1-\alpha/4)},$$

so that the desired overall false alarm rate is bounded by α. The control limits of the joint charts, CL_e and CL_X, can then be written as

$$CL_e = \pm L_e\sigma_e, \quad CL_X = \pm L_X\sigma_X. \qquad (7)$$

The joint decision rule suggests that the controlled process is out of control when either the controlled output or the manipulated input is outside the control limits.

Another procedure is based on Hotelling's T^2 statistic and has been discussed by Hotelling (1947), Montgomery and Klatt (1972), Alt (1985), and Jackson (1985). This approach can measure the overall distance of the observations from the reference values.

Hotelling's (1947) multivariate control chart approach gives an out-of-control signal when

$$\chi_t^2 = \mathbf{x}_t'\boldsymbol{\Sigma}^{-1}\mathbf{x}_t > CL_T,$$

where the monitored characteristics are $\mathbf{x}_t = (e_t, X_t)'$; the reference value, $(\mathrm{E}(e_t), \mathrm{E}(X_t))$, is assumed to be a zero vector; and $\boldsymbol{\Sigma}$ is the covariance matrix of $\mathbf{x}_t$ and is defined as

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_e^2 & \sigma_{e,X}^2 \\ \sigma_{e,X}^2 & \sigma_X^2 \end{bmatrix},$$

where σ_e and σ_X are obtained from Equations (5) and (6). In addition, $\sigma_{e,X}$ is expressed as

$$\sigma_{e,X} = \sqrt{V + \rho_1(VI + VII + VIII + IX)}\sigma_D.$$

where V, VI, VII, $VIII$, and IX are obtained from Equation (C1) in Appendix C.

Because X_t is a linear combination of the ϵ_t's, $\mathbf{x}_t$ has a bivariate normal distribution, implying that χ_t^2 has a χ^2 distribution with two degrees of freedom. Thus, the control limit for Hotelling's approach is determined to be

$$CL_T = \chi^2_{\alpha,2}, \quad (8)$$

so that the desired false alarm rate is α. The joint decision rule suggests that the controlled process is out of control when χ_t^2 exceeds the control limit. In some situations the covariance matrix can be estimated from historical data, and the distribution of χ_t^2 changes from a χ^2 to an F distribution.

Although Hotelling's approach requires the extra knowledge of the covariance $\sigma_{\epsilon,X}$, it is superior to Bonferroni's approach in that it gives exact Type I errors, while Bonferroni's approach gives conservative Type I errors. On the other hand, if the calculation of $\sigma_{\epsilon,X}$ from the estimated model causes a significant error, then the advantage of Hotelling's approach may be eroded, and it may even perform worse than Bonferroni's approach. A comparison of the two approaches is given in the next section.

Performance Analysis

To measure the effectiveness of the joint monitoring schemes, we need to investigate their ARL properties. In order to make a fair comparison, the control limits are adjusted so that the ARL is the same for all charts when there is no shift in the mean. In this study the in-control ARL value of 370 is used. The chart with the lowest out-of-control ARL when a mean-shift, μ_t, has occurred is considered superior. The mean-shift μ_t is defined by

$$\mu_t = \begin{cases} 0 & \text{for } t < 0 \\ \text{constant} & \text{for } t \geq 0. \end{cases}$$

Because the formulas for the ARL's are extremely difficult to compute, they are determined via simulation. The simulation procedure is similar to that reported by Wardell, Moskowitz, and Plante (1994). Disturbance processes are generated according to Equation (2), with a step-change in the mean introduced at time zero. The observed control inputs and outputs are calculated using Equation (3) and Equation (1). Then the number of time periods is measured until the first out-of-control condition is signaled for each chart. The process is repeated 500 times in order to obtain an ARL with a standard error of 4.5%.

To compare the performances of the joint monitoring schemes, contour plots of the ARL's for individual monitoring of the controlled outputs using Shewhart charts are given in Figure 3. For smaller mean-shifts ($\leq 1\sigma_D$), individual monitoring is not effective in Zone B, where most of the ARL's are higher than 50. Its performance is even worse in Zone A, where most of the ARL's are higher than 100. However, its performance is better in Zone C, where the ARL's are lower than 30. For larger mean-shifts ($\geq 2\sigma_D$), individual monitoring still does not perform well in Zone B, where most of the ARL's are higher than 10. The ARL's in Zone A are mostly higher than 5, and in Zone C the ARL's are mostly lower than 3.

Figures 4 and 5 contain the contour plots of the ARL's for joint monitoring using Bonferroni's and Hotelling's approaches, respectively. Overall, the joint monitoring schemes using Hotelling's approach and Bonferroni's approach can significantly improve the individual monitoring performance in different zones. The major findings are:

1. Figure 4(b) shows that, for larger mean-shifts ($\geq 2\sigma_D$), Bonferroni's approach performs quite well throughout the parameter space, where most of the ARL's are lower than 10. In Zone A, especially, the Bonferroni approach shows a significant improvement over individual monitoring.
2. Figure 4(a) shows that, for smaller mean-shifts ($\leq 1\sigma_D$), Bonferroni's approach also results in significant improvement over individual monitoring in Zone A. It also performs well (ARL's < 30) in Zone C. However, in Zone B it does not perform well (ARL's > 100) near the region $\phi = 0$ and $\theta < 0$.
3. Figure 5(b) shows that, for larger mean-shifts ($\geq 2\sigma_D$), Hotelling's approach performs well (ARL's < 10) for most of the parameter space. It displays significant improvement over individual monitoring in Zone A, as well as in some regions ($\phi < -0.3$) of Zone B.
4. Figure 5(a) shows that, for smaller mean-shifts ($\leq 1\sigma_D$), Hotelling's approach performs better than individual monitoring in Zone A, but near the region $\phi = 1$ it is not very efficient. Hotelling's approach performs comparably to individual monitoring in Zones B and C.

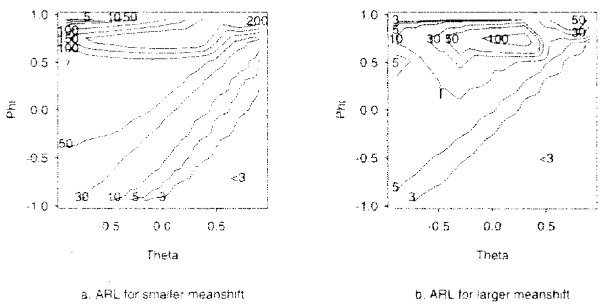

FIGURE 3. ARL Plots for Individual Monitoring of the Controlled Outputs.

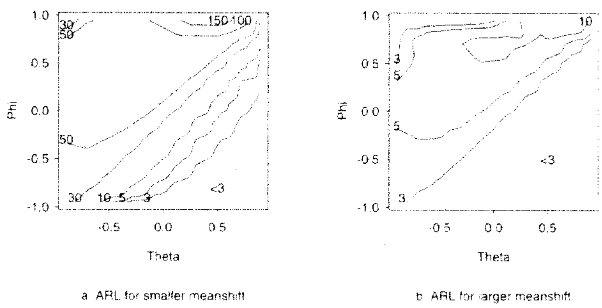

FIGURE 5. ARL Plots for Joint Monitoring by Hotelling's Approach.

In summary, Bonferroni's approach has better overall performance in Zone A. Hotelling's approach has better overall performance in Zone B. In Zone C, both approaches perform well. This can be explained in part by the correlation between e_t and X_t (described in Figure 2(b)). For small correlations, Bonferroni's approach performs better, while for large correlations, Hotelling's approach performs better. There is, however, no clear-cut pattern for processes with negative k_P.

Robustness Analysis

The performance analysis in the previous section is based on the assumption that the models for the disturbance process and its parameters are precisely known. However, uncertain knowledge and changes in the manufacturing environment often lead to misidentification of the disturbance models and parameters, resulting in inadequate monitoring (i.e., frequent false alarms, low detectability, etc.). Here we investigate the robustness of SPC over the entire parameter space of the ARMA(1,1) disturbance processes.

Suppose that the estimated parameters of the ARMA(1,1) disturbance process in Equation (2) are $\widehat{\phi}$ and $\widehat{\theta}$; then the optimal PID control parameters for the estimated disturbance model are $\widehat{k}_P = k_P(\widehat{\phi},\widehat{\theta})$, $\widehat{k}_I = k_I(\widehat{\phi},\widehat{\theta})$, and $\widehat{k}_D = k_D(\widehat{\phi},\widehat{\theta})$, respectively, and the control limits based on the estimated disturbance model are $\widehat{CL} = CL(\widehat{k}_P,\widehat{k}_I,\widehat{k}_D,\widehat{\phi},\widehat{\theta})$. The corresponding ARL, when there is a mean-shift μ_t, is written as $\text{ARL}(\widehat{k}_P,\widehat{k}_I,\widehat{k}_D,\widehat{CL}|\phi,\theta,\mu_t)$.

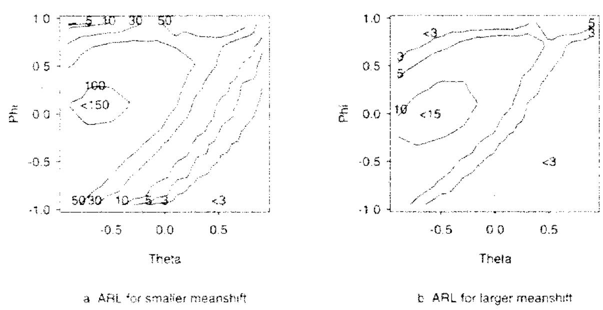

FIGURE 4. ARL Plots for Joint Monitoring by Bonferroni's Approach.

The robustness in terms of the worst-case ARL is measured by

$$R_{out-of-control} \equiv \text{Max}\left\{ \text{ARL}(\widehat{k}_P,\widehat{k}_I,\widehat{k}_D,\widehat{CL}|\phi,\theta,\mu_t) : \max(|\phi-\widehat{\phi}|,|\theta-\widehat{\theta}|) \le \delta \right\} \quad (9)$$

and

$$R_{in-control} \equiv \text{Min}\left\{ \text{ARL}(\widehat{k}_P,\widehat{k}_I,\widehat{k}_D,\widehat{CL}|\phi,\theta) : \max(|\phi-\widehat{\phi}|,|\theta-\widehat{\theta}|) \le \delta \right\} \quad (10)$$

over regions with estimation error, δ, as much as 0.1. Lower $R_{out-of-control}$ and higher $R_{in-control}$ values are desired. When the model contains error, the SPC performance, as measured by the ARL, will vary from the designed value. By specifying a bound on the estimation error, Equations (9) and (10) define the worst ARL, that is, the maximum ARL for the *out–of–control* condition and the minimum ARL for the *in–control* condition. Robustness to estimation error is achieved if the maximum ARL for the *out–of–control* condition is small and, similarly, if the minimum ARL for the *in–control* condition is large.

Figure 6 shows the contour plots of the

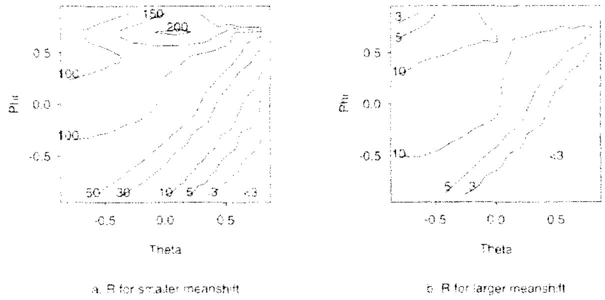

FIGURE 6. $R_{out-of-control}$ Plots for Bonferroni's Approach.

$R_{out-of-control}$ for Bonferroni's approach. For both larger and smaller mean-shifts, there is not much loss in detectability, compared with the plots in Figure 4. Only in Zone A is it not as robust. Figure 7 shows the contour plots of the $R_{out-of-control}$ for Hotelling's approach. There is also not much loss in detectability, compared with the plots in Figure 5. In Zone A and the region $\phi > 0$ in Zone B, it is not robust.

In Figure 8, the contour plots indicate that the $R_{in-control}$ of Bonferroni's approach and of Hotelling's approach are worse when ϕ gets close to 1 or -1. This is because the autocorrelation of the disturbance processes gets larger as $|\phi|$ gets larger, which leads to a higher false alarm frequency.

Monitoring Scheme Selection

The results in the previous section demonstrate that Bonferroni's approach and Hotelling's approach perform differently in different zones, and therefore it is hard to judge which scheme is superior. Thus, an effective decision rule to select appropriate monitoring schemes under different situations is needed. Here the designed in-control ARL and out-of-control ARL based on the estimated disturbance

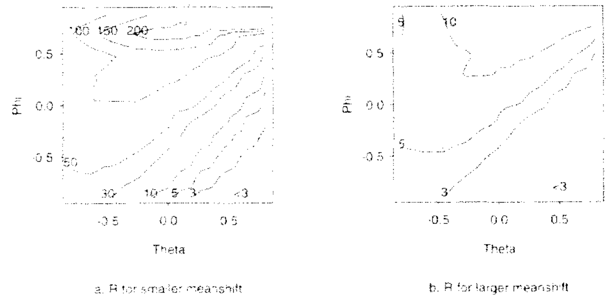

FIGURE 7. $R_{out-of-control}$ Plots for Hotelling's Approach.

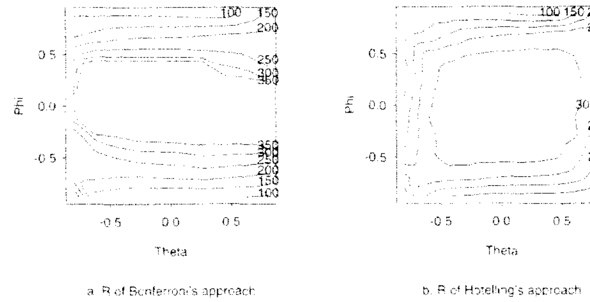

FIGURE 8. $R_{in-control}$ Plots.

model and the corresponding control limits are denoted by ARL_0^* and ARL_1^*, respectively; the observed in-control ARL and out-of-control ARL of the true disturbance process are denoted by ARL_0 and ARL_1, respectively.

As long as the open-loop data are collected and the disturbance processes are modeled as previously described, the parameter estimates $\widehat{\phi}$ and $\widehat{\theta}$ can be obtained. These estimates are used to obtain the corresponding PID controller design from Figure 1 as if they were the true parameters. Based on $\widehat{\phi}$ and $\widehat{\theta}$ and k_P, k_I and k_D, the control limits are calculated for the designed ARL_0^* and ARL_1^*. Because the true parameters ϕ and θ are unknown, the estimation errors, $\Delta\phi$ and $\Delta\theta$ (where $\Delta\phi = \phi - \widehat{\phi}$ and $\Delta\theta = \theta - \widehat{\theta}$), are assumed to be normally distributed. Based on this, 1,000 simulated processes with $\phi = \widehat{\phi} + \Delta\phi$ and $\theta = \widehat{\theta} + \Delta\theta$ and where $(\Delta\phi, \Delta\theta)$ are generated from a normal distribution with a mean of 0 and a variance of 0.01 are used to calculate the ARL_1 and ARL_0 of the true disturbance process. The appropriate monitoring scheme can then be selected based on the smaller Type I error and/or Type II error. In other words, the monitoring scheme with higher probability $\Pr\{\mathrm{ARL}_0 > \mathrm{ARL}_0^*\}$ and/or higher probability $\Pr\{\mathrm{ARL}_1 < \mathrm{ARL}_1^*\}$ in the presence of estimation error will be selected.

Hence, the selection index, SI, is defined as

$$SI = (1 - w)\Pr\{\mathrm{ARL}_0 > \mathrm{ARL}_0^*\} + w\Pr\{\mathrm{ARL}_1 < \mathrm{ARL}_1^*\}.$$

where w, $0 \le w \le 1$, is a customized weighting factor. For w's larger than 0.5, more weight is given to Type II errors; otherwise, more weight is given to Type I errors. Thus, the decision rule suggests that the monitoring scheme with the higher SI is selected. This selection strategy is adaptive as the disturbance process modeling is updated.

A TPS Assembly Process Monitoring Example

A simulated case with real context and data from a throttle position sensor (TPS) assembly process is used to demonstrate the applicability and efficiency of joint monitoring strategies. The TPS in Figure 9 is a potentiometer, mounted on the throttle body of a vehicle, which detects the opening of the throttle plate and sends this information to the power train control module.

The process outputs in Figure 10(a) are the rotor-end play readings (i.e., the distances between the potentiometer arms and the substrates) collected from a TPS assembly process of an automotive supplier. The variability of the rotor-end play may cause malfunction of the TPS. For example, large rotor-end play will cause open circuit conditions, while small rotor-end play will cause "sticking" of the TPS when the potentiometer arm inside comes in contact with the underlying substrate. Thus, to reduce process variation, it is necessary to control the rotor-end play in a RTR base by adjusting the screw heights of the press. It is also important to monitor the RTR-controlled process using SPC for long-term process improvement.

A step-by-step procedure to implement the joint monitoring scheme is as follows:

1. After open-loop data collection and time series model-fitting, an ARMA(1,1) model with $\phi = 0.66$ and $\theta = 0.35$ was determined to be adequate for this process. Based on the PID design maps (Figure 1), the PID scheme with $k_P = 0.47$, $k_I = 0$, and $k_D = -0.17$ was chosen for the RTR control of the TPS process.
2. Based on the model and control parameters, the proposed joint monitoring design, using the two approaches with known covariance, was determined by Equations (7) and (8). The control limits of Bonferroni's approach were ± 3.24 and ± 1.13, and the control limit of Hotelling's approach was 11.8.
3. Figure 4 shows the SPC performance plot of Bonferroni's approach, and Figure 5 shows the SPC performance plot of Hotelling's approach. From the plots we can see that, for larger mean-shifts, the ARL's of both approaches are 10 or less. For smaller mean-shifts, the ARL's of both approaches are about 50.

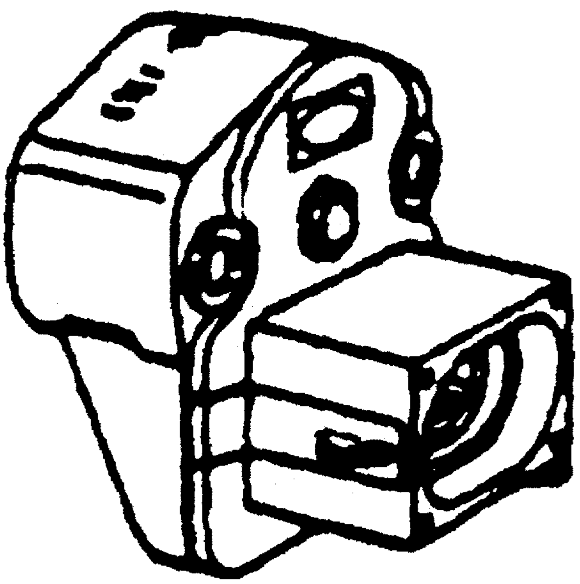

FIGURE 9. Throttle Position Sensor (TPS).

To select a robust monitoring scheme among those with similar ARL performance, the selection strategy

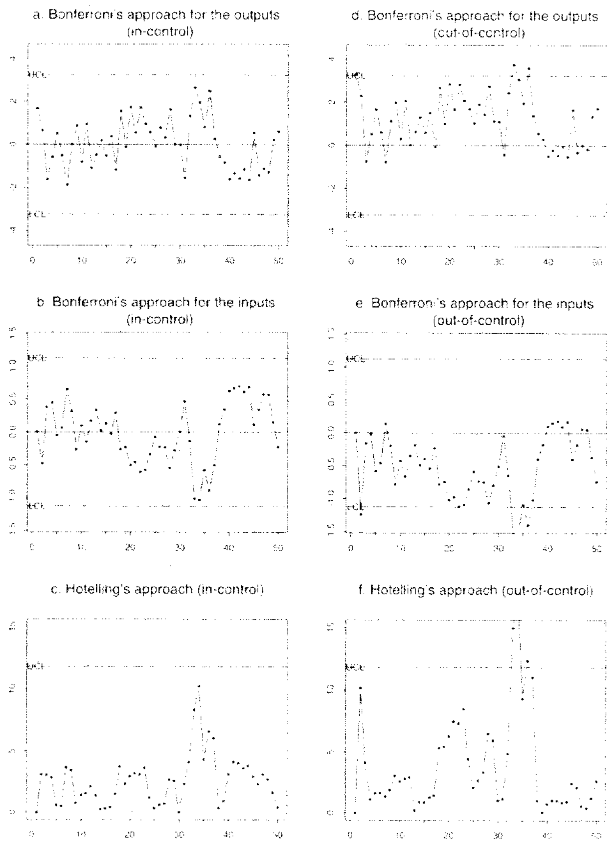

FIGURE 10. Joint Monitoring for a PID-Controlled TPS Assembly Process.

TABLE 1. The Selection of TPS Assembly Process Monitoring Schemes ($w = 0.5$)

Schemes	ARL_0^*	ARL_1^*	$\Pr\{ARL_0 > ARL_0^*\}$	$\Pr\{ARL_1 < ARL_1^*\}$	SI
Hotelling	372.8	8.1	0.14	0.49	0.32
Bonferroni	377.9	7.3	0.48	0.57	0.53*

* Indicates the suggested monitoring scheme based on SI.

in the previous section is applied, assuming normal parameter estimation errors. For this process, larger mean-shifts ($\geq 2\sigma_D$), as well as frequent false alarms, are the major concerns. The SI results are calculated using the proposed strategy and are shown in Table 1.

The designed ARL_0^*'s and ARL_1^*'s of Bonferroni's approach and of Hotelling's approach are very close. Therefore it is hard to make a decision based just on their values. However, based on the probabilities $\Pr\{ARL_0 > ARL_0^*\}$ and $\Pr\{ARL_1 < ARL_1^*\}$ and the selection index SI with $w = 0.5$, Bonferroni's approach is chosen.

Figures 10(a) and 10(b) show the performance of Bonferroni's approach for the *in−control* situation based on the data, and Figure 10(c) shows the corresponding performance of Hotelling's approach. The figures indicate that there was no false alarm triggered during joint monitoring by either approach.

The study of the *out−of−control* situation is based on the original data with a simulated sustained shift of $2\sigma_D$ introduced at time $t = 0$. Figures 10(d) and 10(e) show the plots of the *out−of−control* performance of Bonferroni's approach. The plots indicate that Bonferroni's approach can efficiently detect the *out−of−control* condition at the first observation. Figure 10(f) shows the corresponding plots of Hotelling's approach. These plots show that, for this case, Hotelling's approach is much worse than Bonferroni's approach: it does not detect the mean-shift until the 33rd observation. This result is consistent with the suggestion made by the selection strategy, thus demonstrating the effectiveness of the proposed strategy. Also, the parameters estimated from the example fall in Zone A, where the Bonferroni method was shown to be superior. This also agrees with the simulation results on robustness.

Conclusion

A joint monitoring strategy is presented for PID-controlled processes to monitor the controlled process input and output simultaneously. Detailed design procedures are developed, and a performance analysis is conducted. The results indicate that joint monitoring schemes using either Hotelling's approach or Bonferroni's approach can overcome the shortcomings of conventional SPC for controlled processes, and that both are quite efficient over a wide region of the parameter space.

In addition, a new robustness measure for the SPC algorithm is proposed to study the sensitivity to process model uncertainty. The robustness analysis shows that the performance of the joint monitoring algorithm varies with the process model. As a result, an effective decision-making strategy is proposed to adaptively select an appropriate monitoring scheme. By using this strategy, the most robust SPC algorithm is selected according to the process model and robustness performance measure. Application of the joint monitoring schemes to a TPS assembly process also demonstrates their effectiveness for both in-control and out-of-control situations.

Appendix A: The Variance of the Controlled Outputs

The variance of the PID-controlled outputs is summarized as follows (see Tsung and Shi (1998) for a detailed derivation):

$$\sigma_e^2 \equiv (I + 2\rho_1(II + III))\sigma_D^2,$$

where ρ_1 is the first-order autocorrelation of $\{D_t\}$, σ_D^2 is the variance of $\{D_t\}$, and the three terms are

$$\begin{aligned} I =& \frac{\Gamma^2}{1-\epsilon^2} + \frac{\Theta^2}{1-\zeta^2} + \frac{\Lambda^2}{1-\eta^2} + \frac{2\Gamma\Theta}{1-\epsilon\zeta} \\ &+ \frac{2\Gamma\Lambda}{1-\epsilon\eta} + \frac{2\Theta\Lambda}{1-\zeta\eta} + 1 \\ II =& \frac{\Gamma}{1-\phi\epsilon} + \frac{\Theta}{1-\phi\zeta} + \frac{\Lambda}{1-\phi\eta} \qquad \text{(A1)} \\ III =& \frac{\Gamma\epsilon}{1-\epsilon\phi}\left(\frac{\Gamma}{1-\epsilon^2} + \frac{\Theta}{1-\epsilon\zeta} + \frac{\Lambda}{1-\epsilon\eta}\right) \end{aligned}$$

$$+\frac{\Theta\zeta}{1-\zeta\phi}\left(\frac{\Gamma}{1-\epsilon\zeta}+\frac{\Theta}{1-\zeta^2}+\frac{\Lambda}{1-\zeta\eta}\right)$$
$$+\frac{\Lambda\eta}{1-\eta\phi}\left(\frac{\Gamma}{1-\epsilon\eta}+\frac{\Theta}{1-\zeta\eta}+\frac{\Lambda}{1-\eta^2}\right),$$

where ϵ, ζ, and η are three roots of $\varphi(B) = 1 - \alpha B - \beta B^2 - \gamma B^3 = 0$ (with $\alpha = 1 - k_P - k_I - k_D$, $\beta = k_P + 2k_D$, and $\gamma = -k_D$) and where

$$\Gamma = \frac{\epsilon^2(-1+\epsilon)}{(\epsilon-\zeta)(\epsilon-\eta)}$$
$$\Theta = \frac{\zeta^2(-1+\zeta)}{(\zeta-\epsilon)(\zeta-\eta)}$$
$$\Lambda = \frac{\eta^2(-1+\eta)}{(\eta-\epsilon)(\eta-\zeta)}.$$

Appendix B: The Variance of the Manipulated Inputs

Similar to the derivation in Appendix A, the variance of the manipulated input is

$$\sigma_X^2 = (I' + 2\rho_1(II' + III'))\sigma_D^2,$$

where the three terms are

$$I' = \frac{\Gamma'^2}{1-\epsilon^2}+\frac{\Theta'^2}{1-\zeta^2}+\frac{\Lambda'^2}{1-\eta^2}+\frac{2\Gamma'\Theta'}{1-\epsilon\zeta}+\frac{2\Gamma'\Lambda'}{1-\epsilon\eta}+\frac{2\Theta'\Lambda'}{1-\zeta\eta}+(k_P+k_I+k_D)^2$$

$$II' = \left(\frac{\Gamma'}{1-\phi\epsilon}+\frac{\Theta'}{1-\phi\zeta}+\frac{\Lambda'}{1-\phi\eta}\right)(k_P+k_I+k_D)$$

$$III' = \frac{\Gamma'\epsilon}{1-\epsilon\phi}\left(\frac{\Gamma'}{1-\epsilon^2}+\frac{\Theta'}{1-\epsilon\zeta}+\frac{\Lambda'}{1-\epsilon\eta}\right)+\frac{\Theta'\zeta}{1-\zeta\phi}\left(\frac{\Gamma'}{1-\epsilon\zeta}+\frac{\Theta'}{1-\zeta^2}+\frac{\Lambda'}{1-\zeta\eta}\right)+\frac{\Lambda'\eta}{1-\eta\phi}\left(\frac{\Gamma'}{1-\epsilon\eta}+\frac{\Theta'}{1-\zeta\eta}+\frac{\Lambda'}{1-\eta^2}\right) \quad \text{(B1)}$$

with the same ϵ, ζ, and η as defined in Appendix A and with

$$\Gamma' = \frac{-\epsilon^3(k_P+k_I+k_D)+\epsilon^2(k_P+2k_D)-\epsilon k_D}{(\epsilon-\zeta)(\epsilon-\eta)}$$
$$\Theta' = \frac{-\zeta^3(k_P+k_I+k_D)+\zeta^2(k_P+2k_D)-\zeta k_D}{(\zeta-\epsilon)(\zeta-\eta)}$$
$$\Lambda' = \frac{-\eta^3(k_P+k_I+k_D)+\eta^2(k_P+2k_D)-\eta k_D}{(\eta-\epsilon)(\eta-\zeta)}.$$

Appendix C: The Covariance Between e_t and X_t

The covariance between e_t and X_t is

$$\sigma_{e,X} = \text{cov}(e_t, X_t) \equiv (V + \rho_1(VI + VII + VIII + IX))\sigma_D^2,$$

where the five terms are

$$V = \frac{\Gamma\Gamma'}{1-\epsilon^2}+\frac{\Theta\Theta'}{1-\zeta^2}+\frac{\Lambda\Lambda'}{1-\eta^2}+\frac{\Gamma\Theta'}{1-\epsilon\zeta}+\frac{\Gamma'\Theta}{1-\epsilon\zeta}+\frac{\Gamma\Lambda'}{1-\epsilon\eta}+\frac{\Gamma'\Lambda}{1-\epsilon\eta}+\frac{\Theta\Lambda'}{1-\zeta\eta}+\frac{\Theta'\Lambda}{1-\zeta\eta}+k_P+k_I+k_D$$

$$VI = \left(\frac{\Gamma}{1-\phi\epsilon}+\frac{\Theta}{1-\phi\zeta}+\frac{\Lambda}{1-\phi\eta}\right)(k_P+k_I+k_D)$$

$$VII = \frac{\Gamma'}{1-\phi\epsilon}+\frac{\Theta'}{1-\phi\zeta}+\frac{\Lambda'}{1-\phi\eta} \quad \text{(C1)}$$

$$VIII = \frac{\Gamma'\epsilon}{1-\epsilon\phi}\left(\frac{\Gamma}{1-\epsilon^2}+\frac{\Theta}{1-\epsilon\zeta}+\frac{\Lambda}{1-\epsilon\eta}\right)+\frac{\Theta'\zeta}{1-\zeta\phi}\left(\frac{\Gamma}{1-\epsilon\zeta}+\frac{\Theta}{1-\zeta^2}+\frac{\Lambda}{1-\zeta\eta}\right)+\frac{\Lambda'\eta}{1-\eta\phi}\left(\frac{\Gamma}{1-\epsilon\eta}+\frac{\Theta}{1-\zeta\eta}+\frac{\Lambda}{1-\eta^2}\right)$$

$$IX = \frac{\Gamma\epsilon}{1-\epsilon\phi}\left(\frac{\Gamma'}{1-\epsilon^2}+\frac{\Theta'}{1-\epsilon\zeta}+\frac{\Lambda'}{1-\epsilon\eta}\right)+\frac{\Theta\zeta}{1-\zeta\phi}\left(\frac{\Gamma'}{1-\epsilon\zeta}+\frac{\Theta'}{1-\zeta^2}+\frac{\Lambda'}{1-\zeta\eta}\right)+\frac{\Lambda\eta}{1-\eta\phi}\left(\frac{\Gamma'}{1-\epsilon\eta}+\frac{\Theta'}{1-\zeta\eta}+\frac{\Lambda'}{1-\eta^2}\right)$$

with ϵ, ζ, η, Γ, Θ, Λ, Γ', Θ', and Λ' as defined in Appendices A and B.

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