Operating Window Experiments: A Novel Approach to Quality and Reliability Improvement

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Abstract

The operating window method is a novel tool in quality engineering. It has been practiced in some industrial sectors but has received scant attention in academic research. If a critical factor for competing failure modes can be identified, the probability of failures can be reduced by widening the operating window of this factor, thus improving the system's quality and reliability. A rigorous foundation is given for some existing practice, particularly the operating window SN ratio. A new strategy is given for design, analysis and system optimization, which improves over existing practice. Illustration is given with the analysis of a wave soldering experiment.

[Technical Report #371 , August 2001, Department of Statistics, University of Michigan, Ann Arbor, MI, USA]

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Introduction

The performance of a product/process is often evaluated based on failure rate or defect rate. This metric has the advantage that it can be easily converted into monetary units and therefore is useful for managerial decision making. The failures can be thought of as extremities of certain continuous functional characteristics of the system. Measuring a functional characteristic is thus more appropriate for assessing the system performance. Statistically, the information contained in a continuous variable is much more than observing some of its categories. Looking at the extremities only can be misleading too. The importance of using a functional characteristic for system optimization was emphasized by Taguchi and was supported by many researchers. See the discussions in Nair (1992). Though this is the case in many real applications, it is difficult to identify the right functional characteristic. Even after identifying it, sometimes it is difficult to measure due to an expensive or inadequate measurement system. Operating window response is introduced as an alternative in such circumstances.

The *operating window* is defined as the boundaries of a critical parameter at which certain failure modes are excited. See Figure 1 for a paper feeder example. The concept of operating window was first developed by Don Clausing in the late 70's when he was working at the Xerox corporation. He used an operating window response for the design of a friction-retard paper feeder in a copier machine. Later the concept found applications in diverse areas such as wave soldering, printed circuit board manufacturing, imaging, resistance welding, and drug dose studies. Details about the operating window can be found in Clausing (1994), Taguchi (1993), La Vallee (1992), Parks (1992), Peace (1993), Mori (1995), Fowlkess and Creveling (1995), and Wu and Wu (2000) among others.

The concept of operating window is best explained using the paper feeder example. The function of the feeder is to transport a single sheet of paper from the paper stack to a desired position. Two failure modes are usually encountered. The feeder may fail to feed the sheet, which is known as misfeed. On the other hand, the feeder may pickup more than one sheet of paper, which is known as multifeed. The objective is to design the feeder to minimize

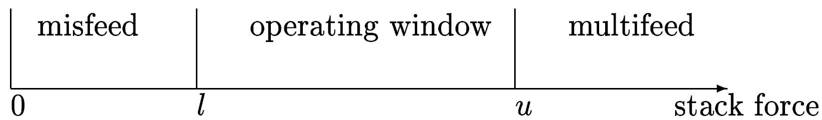

Figure 1: Operating window for a paper feeder

both failure modes. A natural way to do the optimization is to observe the occurence of the two failure modes by feeding, say 1000 sheets at different design settings and choosing a design that minimizes the total failure rate. This is not a good metric for experimentation because a very large sample size would be needed to distinguish between two design settings particularly when the probability of failure is small. The total failure rate can also introduce spurious interactions among control factors. See Taguchi (1987, Section 2.1) and Phadke (1989, Section 6.2) for some examples. This problem can be alleviated to some extent by doing separate optimizations for each of the failure modes. It is difficult to obtain a unique optimum with this approach because of the conflicting nature of the failures.

It is possible to think about some functional characteristics for optimization. The time of arrival of the sheet at the output is a possibility (Clausing, 1994, page 204). If the sheet arrives late, the feeder is tending towards a misfeed; if the sheet arrives early, it is tending towards a multifeed. Another possibility is the friction force. If the feed roller-to-paper friction is less, it leads to misfeed. If the paper-to-paper friction is large, it leads to multi feed. Unfortunately these continuous variables are rather difficult to measure.

An alternative is to construct an operating window response. Stack force is a critical parameter of the paper feeder and is easy to measure. A small force leads to misfeed and a large force leads to multifeed. We can find two threshold values of the force at which the misfeed stops (l) and at which the multifeed starts (u). Then (l, u) forms the operating window. See Figure 1. We call the stack force an *operating window factor*. In the literature it is sometimes referred to as an *operating window signal factor*. Since signal factor refers

Table 1: Examples of operating window factor

Process/Product	Failure or defect type 1	Failure or defect type 2	Operating window factor
Paper feeder	misfeed	multifeed	stack force
Wave soldering	voids	bridges	temperature
Resistance welding	under weld	expulsion	time
Image transfer	opens	shorts	exposure energy
Threading	loose	tight	depth of cut
Picture printing	black	blur	water quantity
Medication	no effect	side effect	dosage

to a factor that is used to express an intent to a system in order to achieve it, it is not an appropriate terminology in the present context. Some examples of operating window factor are given in Table 1.

In reality there are no clear boundaries separating failure modes. So the boundaries are defined with respect to a threshold failure rate. In customer's view point the threshold rate should be very small, such as 0.01% or 0.001%. In experiments values such as 50% are used in order to reduce the experimentation time. Then l is the value of the force at which 50% misfeed occurs and u is the force at which 50% multifeed occurs.

Figure 2 shows the effect of noise factors on operating window. Because the noise factors are uncontrollable during the production/usage conditions, the available operating window for reliable operation reduces to $(\max_i l_i, \min_i u_i)$. Intuitively, the two failure modes can be minimized by keeping the operating window factor at the center of the operating window. A large operating window then corresponds to a robust system. Thus *the objective of achieving a robust design is translated as maximizing the operating window.*

There are several control factors in a paper feeder such as wrap angle, belt tension, radius of retard roll, width of feed belt, take away roll velocity, etc. The noise factors include paper

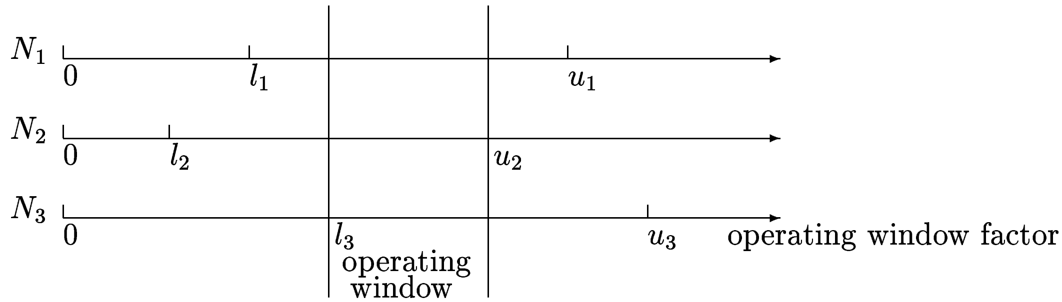

Figure 2: Noise factors reduce the operating window

weight and type, environment, alignment errors, etc. The l and u are then determined for several control and noise factor settings. The optimization is done in two steps:

1. Find a control factor setting to maximize the signal-to-noise ratio

$$SN = -\log\left(\frac{1}{n}\sum_{i=1}^{n} l_i^2 \frac{1}{n}\sum_{i=1}^{n}\frac{1}{u_i^2}\right) \tag{1}$$

2. Adjust the operating window factor to the middle of the operating window.

The SN ratio in (1) was developed at the Fuji Xerox corporation. It is derived as follows (see Taguchi, 1993). The two failure modes can be eliminated if l is reduced to 0 and u is increased to ∞. Therefore l is a smaller-the-better characteristic and u is a larger-the-better characteristic. Using Taguchi's SN ratios for smaller-the-better and larger-the-better problems

$$SN_l = -\log\left(\frac{1}{n}\sum_{i=1}^{n} l_i^2\right) \text{ and } SN_u = -\log\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{u_i^2}\right),$$

where the summation is taken over different noise levels ($i = 1, 2, \cdots, n$), the SN ratio for the operating window is then defined as the sum of the SN ratios of l and u, i.e.,

$$SN = -\log\left(\frac{1}{n}\sum_{i=1}^{n} l_i^2\right) - \log\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{u_i^2}\right).$$

At this point of discussion it may not be clear that why the SN ratio is defined as in (1) instead of a more straight forward definition like $\log\left(\frac{1}{n}\sum(u_i - l_i)^2\right)$. It may not even be clear how the maximization of operating window is related to the robustness of the system. In this article we attempt to answer these questions and provide a statistical foundation and theory for operating window experiments.

The article is organized as follows. In the following section we derive the SN ratio as a performance measure independent of adjustment. We discuss some modeling strategies and develop a two-step optimization procedure. A generalized performance measure using a more flexible model is derived. Then the estimation of the operating window factor thresholds is considered. We suggest an efficient sequential design to find the operating window factor thresholds. The analysis of operating window experiments is illustrated with an example on wave soldering. Some concluding remarks are given in the end.

Performance Measure and SN Ratio

In this section we derive the SN ratio in (1) as a performance measure independent of adjustment (PerMIA). See Leon, Shoemaker, and Kacker (1987) and Leon and Wu (1992) for details on PerMIA.

Let $\mathbf{X}$ be the set of control factors, $\mathbf{N}$ the set of noise factors, and M the operating window factor. Then for a given $\mathbf{N}$, the probability of failure due to defect type 1 (say, misfeed) p_1 and the probability of failure due to defect type 2 (say, multifeed) p_2 are functions of $\mathbf{X}$, $\mathbf{N}$, and M. Let $p_1 = f_1(\mathbf{X}, \mathbf{N}, M)$ and $p_2 = f_2(\mathbf{X}, \mathbf{N}, M)$. A typical behavior of these functions with respect to M are shown in Figure 3. Let γ be the failure threshold, such as 50%. The lower and upper operating window factor thresholds can be solved from $f_1(\mathbf{X}, \mathbf{N}, l) = \gamma$ and $f_2(\mathbf{X}, \mathbf{N}, u) = \gamma$. Thus l and u are functions of $\mathbf{X}$ and $\mathbf{N}$.

Consider the following one-parameter family of functions to approximate f_1 and f_2,

$$p_1 = \frac{1}{1 + \theta_1 M^2} \text{ and } p_2 = \frac{1}{1 + \theta_2 / M^2}. \tag{2}$$

At $M = l$ we have $p_1 = \gamma$ and at $M = u$ we have $p_2 = \gamma$. Solving for θ_1 and θ_2, we get

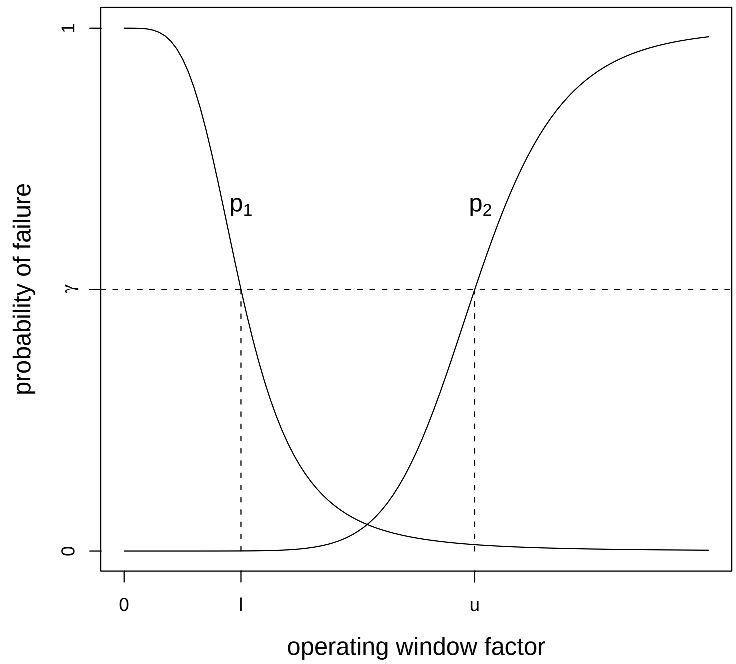

Figure 3: Probability of failure curves against operating window factor

$\theta_1 = (1-\gamma)/\gamma l^2$ and $\theta_2 = (1-\gamma)u^2/\gamma$. Substituting in (2), we have

$$p_1 = \frac{1}{1 + \frac{M^2}{l^2}\frac{(1-\gamma)}{\gamma}} \text{ and } p_2 = \frac{1}{1 + \frac{u^2}{M^2}\frac{(1-\gamma)}{\gamma}}. \tag{3}$$

To evaluate the performance of a control factor setting, we need to define a loss function. In the case of fraction defective data, Taguchi (1987) suggests to use $L = cp/(1-p)$ as the loss function, where p is the fraction defective and c is a cost-related constant. This is because the extra number of units to be produced to get a non-defective unit has a geometric distribution with mean $p/(1-p)$. The loss can be considered to be proportional to the expected number of extra units. In the case of two types of failures, we can similarlly define the loss function as

$$L = c_1 \frac{p_1}{1-p_1} + c_2 \frac{p_2}{1-p_2}. \tag{4}$$

Substituting (3) in (4) we get

$$L = c_1 \frac{\gamma}{1-\gamma}\frac{l^2}{M^2} + c_2 \frac{\gamma}{1-\gamma}\frac{M^2}{u^2}.$$

The expected loss taken over the distribution of $\mathbf{N}$ is

$$EL = c_1 \frac{\gamma}{1-\gamma} \frac{1}{M^2} E(l^2) + c_2 \frac{\gamma}{1-\gamma} M^2 E(\frac{1}{u^2}).$$

We can set the value of M to minimize the expected loss. Solving for M from

$$\frac{\partial}{\partial M} EL = c_1 \frac{\gamma}{1-\gamma} \frac{-2}{M^3} E(l^2) + c_2 \frac{\gamma}{1-\gamma} 2ME(\frac{1}{u^2}) = 0,$$

we get

$$M^* = \left[\frac{c_1}{c_2} \frac{E(l^2)}{E(1/u^2)} \right]^{1/4}. \tag{5}$$

Then EL at the M^*,

$$EL^* = \frac{2\gamma}{1-\gamma} \sqrt{c_1 c_2 E(l^2) E(1/u^2)}$$

can be used to evaluate a control factor setting $\mathbf{X}$. The EL^* is a PerMIA by definition. Thus the objective is to find an $\mathbf{X} \in D$ minimizing EL^*, where D is the feasible region of $\mathbf{X}$. In experiments using fractional factorial designs or orthogonal arrays (Wu and Hamada, 2000) D will usually be a cuboid representing the experimental region. An equivalent performance measure to maximize is

$$PM = -\log \left[E(l^2) E(1/u^2) \right]. \tag{6}$$

When the noise factors have random levels in an experiment, the PM can be estimated using its sample analog, *which is the same as the SN ratio in (1).*

It is interesting to note that PM does not depend on γ or c_1/c_2. Thus PM can be used irrespective of the failure threshold value chosen for determining l and u. The cost ratio c_1/c_2 plays a role only in the adjustment step in (5). If $c_1 > c_2$, then M is adjusted towards the upper boundary and vice versa, which make intuitive sense. This is similar in the spirit of optimization to the asymmetric loss function discussed in Moorhead and Wu (1998). For the case of single noise level with $c_1 = c_2$, the adjustment step in (5) reduces to adjusting the operating window factor to $\sqrt{lu}$. Thus in our formulation the adjustment in (5) is *closer to a geometric mean of the two thresholds* than to the arithmetic mean discussed in Clausing (1994).

Note that $E(l^2) = E^2(l) + Var(l)$ and $E(1/u^2) \approx 1/E^2(u) + 3Var(u)/E^4(u)$. Therefore PM is a composite measure for doing the following jobs: (i) minimize $E(l)$, (ii) minimize $Var(l)$, (iii) maximize $E(u)$, and (iv) minimize $Var(u)$. Thus operating window optimization is much more than just widening the operating window. This suggests that a performance measure like $\sum(u_i - l_i)^2$ cannot be used for evaluating the robustness of a system.

For the two-step optimization discussed in the introduction, two models, one on SN and another on the midpoint of the operating window have to be fitted. Based on the developments in this section, we suggest a different modeling and optimization strategy. For each control factor setting, compute

$$\hat{PM}_l = -\log\left(\frac{1}{n}\sum_i l_i^2\right) \text{ and } \hat{PM}_u = -\log\left(\frac{1}{n}\sum_i 1/u_i^2\right).$$

We prefer this notation to SN_l and SN_u, because we consider that signal-to-noise ratio is a misnomer for smaller-the-better and larger-the-better cases as no ratios are involved.

Fit two linear models for $\hat{PM}_l$ and $\hat{PM}_u$ in terms of the $\mathbf{X}$. Now the two-step optimization can be stated as follows:

1. Find $\mathbf{X}^* \in D$ to maximize $\hat{PM}(\mathbf{X}) = \hat{PM}_l(\mathbf{X}) + \hat{PM}_u(\mathbf{X})$.
2. Adjust M to $\left(\frac{c_1}{c_2}\right)^{1/4} \exp\left\{\left(\hat{PM}_u(\mathbf{X}^*) - \hat{PM}_l(\mathbf{X}^*)\right)/4\right\}$.

(7)

The approach will be explained with an example in a later section.

Generalized Performance Measure

The discussion in the previous section immediately motivates us to consider more flexible models for p_1 and p_2 such as

$$p_1 = \frac{1}{1 + \left(\frac{M}{l}\right)^{\alpha_1} \frac{(1-\gamma)}{\gamma}} \text{ and } p_2 = \frac{1}{1 + \left(\frac{u}{M}\right)^{\alpha_2} \frac{(1-\gamma)}{\gamma}}, \tag{8}$$

where α_1 and α_2 are some positive constants. The α_1 and α_2 control the rate at which p_1 and p_2 change with M. See Figure 4. The models in (3) are special cases of (8) with $\alpha_1 = \alpha_2 = 2$.

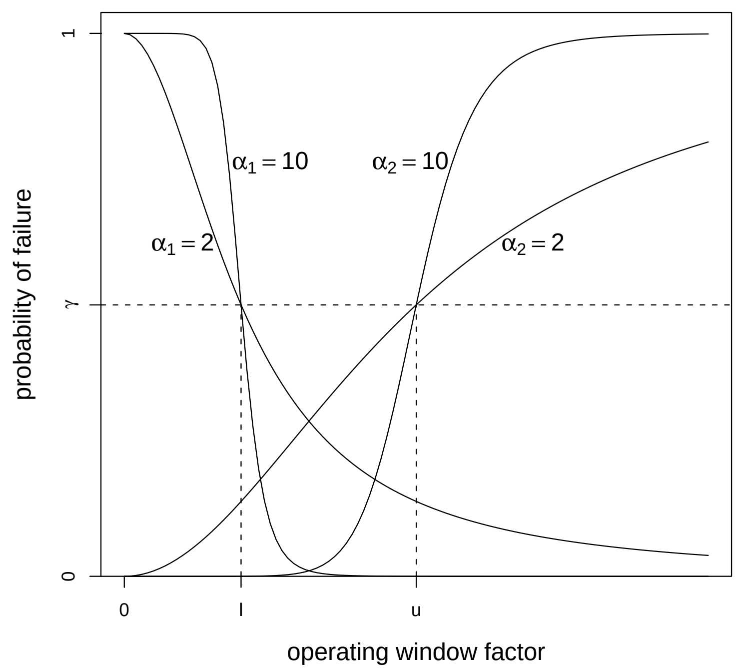

Figure 4: Probability of failure curves for different α values

There is no reason to believe that this choice is the best for every process. Using the loss function in (4), the PerMIA can be similarly derived as

$$GPM = -\log \left[E^{1/\alpha_1}(l^{\alpha_1}) E^{1/\alpha_2}(1/u^{\alpha_2}) \right]. \tag{9}$$

where GPM denotes generalized performance measure. The GPM is very similar to the PM in (6) as they differ only in the norms for l and u. In the case of random noise levels, the GPM can be estimated by

$$G\hat{P}M = -\log \left[\left(\frac{1}{n} \sum_{i=1}^{n} l_i^{\alpha_1} \right)^{1/\alpha_1} \left(\frac{1}{n} \sum_{i=1}^{n} \frac{1}{u_i^{\alpha_2}} \right)^{1/\alpha_2} \right]. \tag{10}$$

By considering the limiting case of $G\hat{P}M$ when $\alpha_1 \to \infty$ and $\alpha_2 \to \infty$, we have

$$G\hat{P}M_\infty = \log \left(\min_i u_i \right) - \log \left(\max_i l_i \right).$$

The interval

$$\left(\max_i l_i, \min_i u_i \right) \tag{11}$$

is the resultant *operating window* obtained by the superposition of the operating windows at several noise levels. See Figure 2. Thus in the limiting case the robustness objective becomes maximizing the logarithmic width of the resultant operating window. An important consequence of this result is that we may not need to estimate the operating window factor thresholds at many noise levels. We may be able to find a compound noise factor level favoring failure type 1 and estimate l, and another compound noise factor level favoring failure type 2 and estimate u, which would suffice to compute $G\hat{P}M_{\infty}$. This approach can be used efficiently if we can choose an appropriate compound noise factor and also assume α_1 and α_2 to be large.

Estimation

The l and u are estimated by observing the number of failures at different levels of the operating window factor. We use the example in Clausing (1994, page 216) to explain the approach. Suppose the failure threshold is 50%. Then, for a particular control and noise factor setting, the following experiments are conducted:

> ... set the stack force to 0.5 pound and make 10 attempts to feed, with seven misfeeds. At 0.6 pound there are five misfeeds, and at 0.7 pound there are two misfeeds. We conclude that 0.6 pound is the threshold force for misfeed ...

A similar experiment is performed to obtain the threshold force for multifeed.

The number of failures observed in a given sample is random. Therefore observing five misfeeds out of 10 attempts does not ensure that we will observe again 50% misfeeds at the same level of force. A better estimate of the threshold can be obtained by using all the data collected. Assume that the model (2) is true. Then using logistic regression (McCullagh and Nelder, 1989), we get

$$\log \frac{p_1}{1-p_1} = -1.1763 - 2\log M.$$

From which $\hat{l} = \exp(-1.1763/2) = 0.555$. Using the standard error of estimate, an approximate 90% confidence interval for l can be obtained as $(0.41, 0.75)$. If we use only the data at

$M = 0.6$, then the confidence interval would be $(0.33, 0.93)$, which is almost twice as wide as the previous one. Therefore the *complete* data should be used for estimating the operating window factor thresholds.

If we use the model in (8), then we get $\hat{l} = 0.585$ and $\hat{\alpha}_1 = 11.05$. We see that the value of α_1 is very different from 2. We should use the complete data set in the experiment to estimate α_1 and α_2. Unfortunately most of the experiments reported in the literature that we have come across do not give the complete data set. The data are either not recorded or possibly thrown away after getting l and u in some manner. The complete data contains valuable information that will help us to fit more elaborate models. Analysis of operating window experiments with the complete set of data is a research topic under investigation. For the rest of the discussions in this article we will work with model (3).

Sequential Design

As in the example given in the previous section, usually a trial and error method is used to find the operating window factor thresholds. We can make use of sequential designs for binary data available in the statistical literature to obtain the thresholds in a more systematic and efficient way. Here we adapt the logit-MLE method of Wu (1985) to this problem.

Suppose we want to find the misfeed threshold. Let y_i be the number of misfeeds observed in the i^{th} experiment out of m samples. Suppose we have the following data $(M_1, y_1), (M_2, y_2), \cdots, (M_k, y_k)$ after k experiments. Let $\hat{l}_k$ be the MLE of l from this data based on model (3). The $\hat{l}_k$ can be solved from the equation

$$\sum_{i=1}^{k} \frac{1}{1 + \frac{M_i^2}{\hat{l}_k^2} \frac{(1-\gamma)}{\gamma}} = \sum_{i=1}^{k} \frac{y_i}{m}.$$

Then the next experiment should be carried out at $M_{k+1} = \hat{l}_k$. The search is stopped when $|M_{k+1} - M_k|$ is less than a small pre-chosen value.The convergence of the sequence $M_1, M_2, \cdots$ to l is discussed and proved in Wu (1985) and Ying and Wu (1997). A similar set of sequential experiments can be used to find the multifeed threshold.

We use the same example in the previous section to explain this approach. Suppose we observe seven misfeeds in 10 attempts at $M_1 = 0.5$. Then corresponding to 50% failure threshold, $\hat{l}_1 = 0.76$. Therefore the next experiment should be carried out at $M_2 = 0.76$. If we did the experiment at $M_2 = 0.6$ (as in the original example) and observed five misfeeds, then the next experiment should be at $M_3 = \hat{l}_2 = 0.67$, and so on.

An Example

We will illustrate the analysis of operating window experiments using an example from Peace (1993). Electronic components are assembled on a printed circuit board using wave soldering. Insufficient heat energy results in solder voids, whereas too much heat energy causes solder bridges. The objective of the study was to minimize both voids and bridges through parameter design optimization. The top side board temperature is taken as the operating window factor. The operating window factor thresholds are defined as follows:

l: top side board temperature at which solder voids stop.

u: top side board temperature at which solder bridges begin.

Peace did not mention the failure threshold value, but this will not be required in the analysis. The control factors and their levels are given in Table 2. The printed circuit board carrier is taken as the noise factor and has five levels. The data on l and u from a 2^{15-11}_{III} design are given in Table 3.

The $P\hat{M}_l = -\log\left(\frac{1}{5}\sum l_i^2\right)$ and $P\hat{M}_u = -\log\left(\frac{1}{5}\sum 1/u_i^2\right)$ are calculated for each run. The half-normal plots of the effects are shown in Figure 5. Because it is a saturated resolution III design, the two-factor and higher order interactions cannot be estimated and are assumed to be negligible. We see that the effects A, D, G, L, and N are significant for $P\hat{M}_l$ and the effects H, J, and M are significant for $P\hat{M}_u$. This is confirmed using Lenth's method at 5% level IER (see Wu and Hamada, 2000, Section 3.13). It is a coincidence that the two groups of factors do not overlap. The fitted models are (the factor levels are coded as -1 and +1),

$$P\hat{M}_l = -10.8776 + 0.0314A + 0.0377D + 0.0187G + 0.0272L + 0.0265N$$

and

$$P\hat{M}_u = 11.0204 - 0.0701H - 0.0920J + 0.0457M.$$

Table 2: Control factors and levels

Factors	Notation	Levels	
		$-$	$+$
PCB finish	A	hot air level	bare copper
Solder mask	B	liquid photo-imagable	dry film
PCB thickness	C	.062 inches	.090 inches
Plated through hole size	D	.033 inches	.039 inches
Flux air knife pressure	E	40 psi	60 psi
Flux air knife angle	F	0 degrees	45 degrees
Flux wave height	G	low	high
Solder wave height	H	low	high
Flux composition	I	low solids	medium solids
Pad size	J	small	large
Solder pump size	K	55%	80%
Flux density	L	.86 gms/ml	.87 gms/ml
Solder temperature	M	460 ^{0}F	500 ^{0}F
Solder waves	N	single	dual
PCB orientation	O	0 degrees	45 degrees

Table 3: Data of wave soldering experiment

Run	$A\ B\ C\ D\ E\ F\ G\ H\ I\ J\ K\ L\ M\ N\ O$	l	u
1	$-\ -\ -\ -\ -\ -\ -\ -\ -\ -\ -\ -\ -\ -\ -$	247 245 242 245 240	253 260 265 265 250
2	$-\ -\ -\ -\ -\ -\ -\ +\ +\ +\ +\ +\ +\ +\ +$	235 232 230 232 230	231 235 238 238 240
3	$-\ -\ -\ +\ +\ +\ +\ -\ -\ -\ -\ +\ +\ +\ +$	229 223 220 225 220	273 280 290 280 275
4	$-\ -\ -\ +\ +\ +\ +\ +\ +\ +\ +\ -\ -\ -\ -$	234 230 235 233 228	222 230 235 230 228
5	$-\ +\ +\ -\ -\ +\ +\ -\ -\ +\ +\ -\ -\ +\ +$	242 235 234 235 230	228 235 234 235 230
6	$-\ +\ +\ -\ -\ +\ +\ +\ +\ -\ -\ +\ +\ -\ -$	242 230 238 234 237	252 260 258 264 257
7	$-\ +\ +\ +\ +\ -\ -\ -\ -\ +\ +\ +\ +\ -\ -$	237 234 235 230 232	248 255 245 260 252
8	$-\ +\ +\ +\ +\ -\ -\ +\ +\ -\ -\ -\ -\ +\ +$	238 235 236 235 230	234 240 246 235 240
9	$+\ -\ +\ -\ +\ -\ +\ -\ +\ -\ +\ -\ +\ -\ +$	241 240 235 240 235	270 275 285 270 275
10	$+\ -\ +\ -\ +\ -\ +\ +\ -\ +\ -\ +\ -\ +\ -$	230 225 222 215 215	215 225 222 215 225
11	$+\ -\ +\ +\ -\ +\ -\ -\ +\ -\ +\ +\ -\ +\ -$	224 220 215 212 212	261 268 265 262 272
12	$+\ -\ +\ +\ -\ +\ -\ +\ -\ +\ -\ -\ +\ -\ +$	231 230 228 228 226	225 230 238 228 236
13	$+\ +\ -\ -\ +\ +\ -\ -\ +\ +\ -\ -\ +\ +\ -$	239 235 235 230 235	235 235 235 240 245
14	$+\ +\ -\ -\ +\ +\ -\ +\ -\ -\ +\ +\ -\ -\ +$	239 235 238 235 230	235 235 238 245 250
15	$+\ +\ -\ +\ -\ -\ +\ -\ +\ +\ -\ +\ -\ -\ +$	223 220 215 218 218	245 255 265 238 258
16	$+\ +\ -\ +\ -\ -\ +\ +\ -\ -\ +\ -\ +\ +\ -$	222 220 215 224 218	255 260 265 244 268

Following the two-step optimization procedure in (7), the factor levels that maximize $\hat{PM} = \hat{PM}_l + \hat{PM}_u$ are $A_+D_+G_+H_-J_-L_+M_+N_+$. At this optimum level the $\hat{PM}_l$ and $\hat{PM}_u$ are -10.7361 and 11.2282 respectively. Assuming $c_1 = c_2$, the operating window factor should be adjusted to

$$\exp\left(\frac{11.2282 + 10.7361}{4}\right) = 242.5.$$

We have also plotted the effects of $\hat{PM} = \hat{PM}_l + \hat{PM}_u$ in the first panel of Figure 6. Note that $\hat{PM}$ is the same as the SN ratio in (1). None of the effects seems significant. The Lenth's method at 5% level IER identifies only the effect J to be significant. A possible explanation for this discrepancy is the following. A factor that increases l tends to increase u also. Therefore such a factor will have opposite effects on $\hat{PM}_l$ and $\hat{PM}_u$. This may lead to a non-monotonic relationship between the factor and $\hat{PM} = \hat{PM}_l + \hat{PM}_u$. The non-monotonicity in the response can be manifested as nonlinearity and interactions among control factors which may not be estimable from the experiment. Modeling $\hat{PM}_l$ and $\hat{PM}_u$ separately avoids this problem.

The half-normal plot of $G\hat{P}M_\infty$ is shown in the second panel of Figure (6). The conclusions from this plot are essentially the same as the previous analysis. It is not surprising that even though the underlying models are very different, the analysis of $\hat{PM}$ and $G\hat{P}M_\infty$ yields the same results. This is because the sensitivity of the performance measure to α_1 and α_2 is small.

Conclusions

In this article we have given a comprehensive review of the operating window experiments and attempted to provide a rigorous statistical foundation. We consider operating window experiments to be a powerful tool for quality and reliability improvement, though a lot of changes and improvements from the existing practice should be made on the design and analysis of the experiments.

The use of the operating window SN ratio in (1) is justified by using the theory of PerMIA under some restrictive modeling assumptions. Based on the derivation, we have

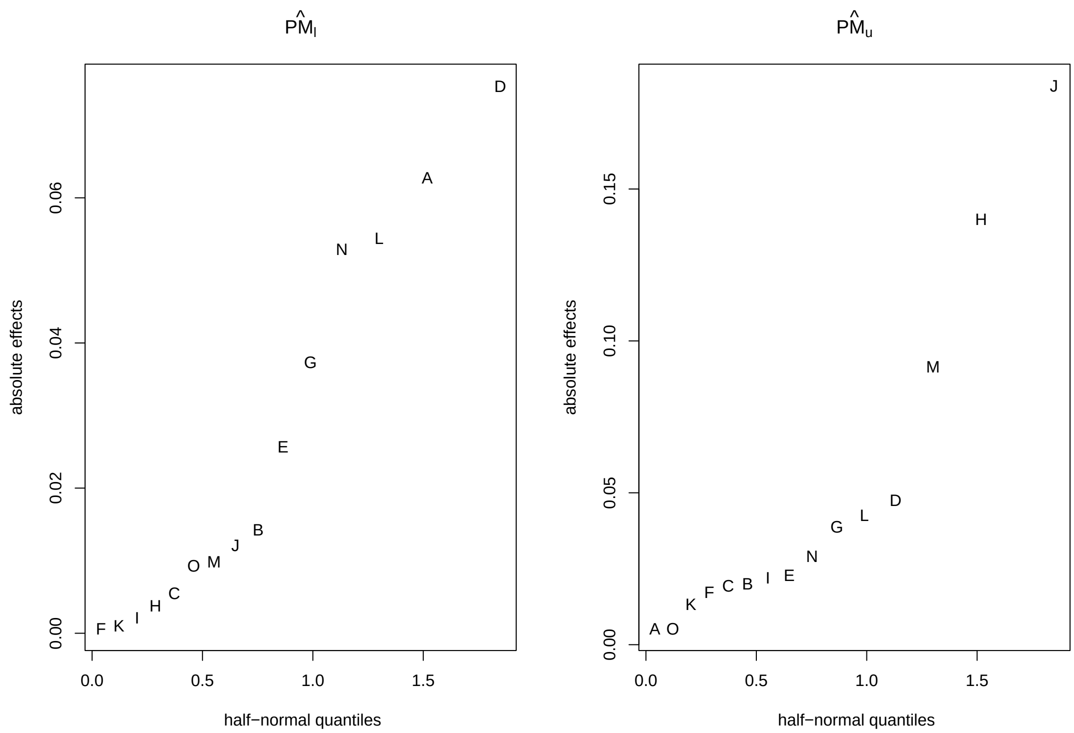

Figure 5: Half-normal plots of $\hat{PM}_l$ and $\hat{PM}_u$

suggested a new modeling and optimization strategy. The performance measure was further generalized using more flexible models for the probability of failures. We have suggested using generalized linear models to estimate the operating window factor thresholds from the data. The need of using complete set of data in the experiment is emphasized for a more informative analysis. We have recommended the logit-MLE method for sequentially searching the operating window factor thresholds instead of using a trial and error approach. The superiority of the proposed analysis strategy is illustrated with the analysis of a wave soldering experiment.

Acknowledgment

The research was supported by NSF DMS-0072489.

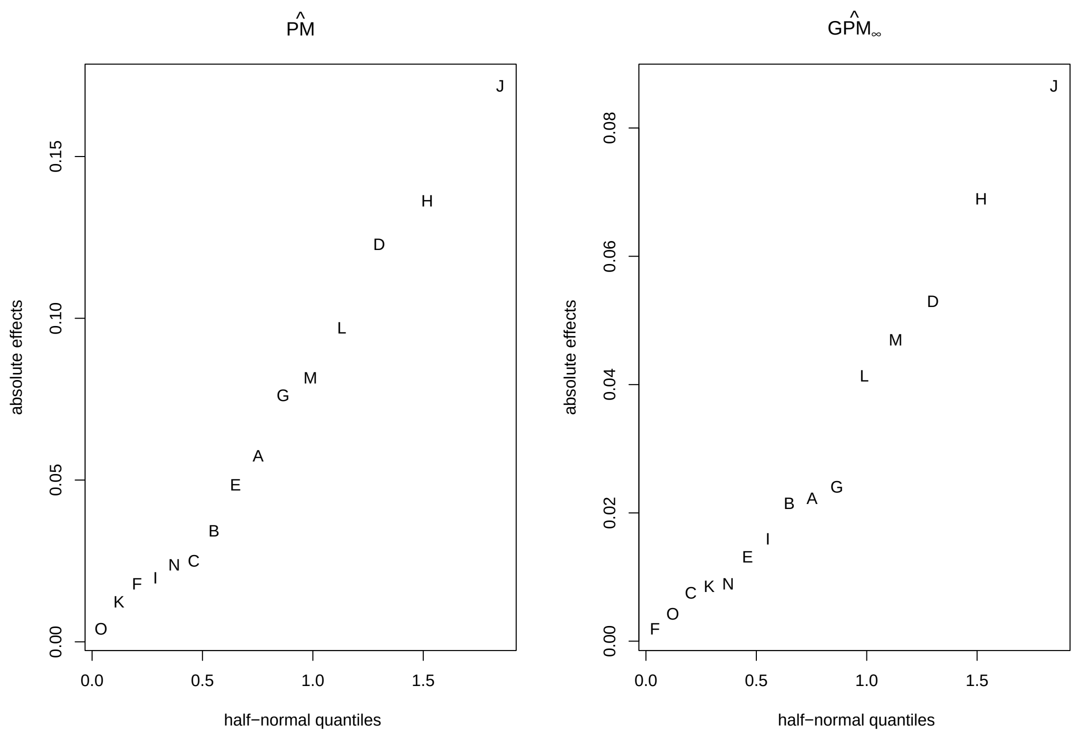

Figure 6: Half-normal plot of $P\hat{M}$ and $G\hat{P}M_\infty$

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Key Words: *Optimization, Performance Measure, Robust Parameter Design, Signal-to-Noise Ratio, Sequential Design.*