

Unit 8: Robust Parameter Design

Source : Chapter 11 (sections 11.1 - 11.6, part of sections 11.7 - 11.8 and 11.9).

- Revisiting two previous experiments.
- Strategies for reducing variation.
- Types of noise factors.
- Variation reduction through robust parameter design.
- Cross array, location-dispersion modeling, response modeling.
- Single arrays vs cross arrays.
- Signal-to-noise ratios and limitations.

Robust Parameter Design

- Statistical/engineering method for product/process improvement (G. Taguchi).
- Two types of factors in a system (product/process):
 - control factors: once chosen, values remain fixed.
 - noise factors: hard-to-control during normal process or usage.
- **Robust Parameter design (RPD or PD)**: choose control factor settings to make response less sensitive (i.e. more robust) to noise variation; exploiting control-by-noise interactions.

A Robust Design Perspective of Layer-growth and Leaf Spring Experiments

- The original AT & T layer growth experiment had 8 control factors, 2 noise factors (location and facet). Goal was to achieve *uniform* thickness around $14.5 \mu\text{m}$ over the noise factors. See Tables 1 and 2.
- The original leaf spring experiment had 4 control factors, 1 noise factor (quench oil temperature). The quench oil temperature is not controllable; with efforts it can be set in two ranges of values 130-150, 150-170. Goal is to achieve *uniform* free height around 8 inches over the range of quench oil temperature. See Tables 3 and 4.
- Must understand the role of *noise factors* in achieving *robustness*.

Layer Growth Experiment: Factors and Levels

Table 1: Factors and Levels, Layer Growth Experiment

Control Factor		Level	
		–	+
<i>A.</i>	susceptor-rotation method	continuous	oscillating
<i>B.</i>	code of wafers	668G4	678D4
<i>C.</i>	deposition temperature(°C)	1210	1220
<i>D.</i>	deposition time	short	long
<i>E.</i>	arsenic flow rate(%)	55	59
<i>F.</i>	hydrochloric acid etch temperature(°C)	1180	1215
<i>G.</i>	hydrochloric acid flow rate(%)	10	14
<i>H.</i>	nozzle position	2	6
Noise Factor		Level	
		–	+
<i>L.</i>	location	bottom	top
<i>M.</i>	facet	1 2	3 4

Layer Growth Experiment: Thickness Data

Table 2: Cross Array and Thickness Data, Layer Growth Experiment

Control Factor								Noise Factor							
<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>	L-Bottom				L-Top			
								<i>M-1</i>	<i>M-2</i>	<i>M-3</i>	<i>M-4</i>	<i>M-1</i>	<i>M-2</i>	<i>M-3</i>	<i>M-4</i>
-	-	-	+	-	-	-	-	14.2908	14.1924	14.2714	14.1876	15.3182	15.4279	15.2657	15.4056
-	-	-	+	+	+	+	+	14.8030	14.7193	14.6960	14.7635	14.9306	14.8954	14.9210	15.1349
-	-	+	-	-	-	+	+	13.8793	13.9213	13.8532	14.0849	14.0121	13.9386	14.2118	14.0789
-	-	+	-	+	+	-	-	13.4054	13.4788	13.5878	13.5167	14.2444	14.2573	14.3951	14.3724
-	+	-	-	-	+	-	+	14.1736	14.0306	14.1398	14.0796	14.1492	14.1654	14.1487	14.2765
-	+	-	-	+	-	+	-	13.2539	13.3338	13.1920	13.4430	14.2204	14.3028	14.2689	14.4104
-	+	+	+	-	+	+	-	14.0623	14.0888	14.1766	14.0528	15.2969	15.5209	15.4200	15.2077
-	+	+	+	+	-	-	+	14.3068	14.4055	14.6780	14.5811	15.0100	15.0618	15.5724	15.4668
+	-	-	-	-	+	+	-	13.7259	13.2934	12.6502	13.2666	14.9039	14.7952	14.1886	14.6254
+	-	-	-	+	-	-	+	13.8953	14.5597	14.4492	13.7064	13.7546	14.3229	14.2224	13.8209
+	-	+	+	-	+	-	+	14.2201	14.3974	15.2757	15.0363	14.1936	14.4295	15.5537	15.2200
+	-	+	+	+	-	+	-	13.5228	13.5828	14.2822	13.8449	14.5640	14.4670	15.2293	15.1099
+	+	-	+	-	-	+	+	14.5335	14.2492	14.6701	15.2799	14.7437	14.1827	14.9695	15.5484
+	+	-	+	+	+	-	-	14.5676	14.0310	13.7099	14.6375	15.8717	15.2239	14.9700	16.0001
+	+	+	-	-	-	-	-	12.9012	12.7071	13.1484	13.8940	14.2537	13.8368	14.1332	15.1681
+	+	+	-	+	+	+	+	13.9532	14.0830	14.1119	13.5963	13.8136	14.0745	14.4313	13.6862

Leaf Spring Experiment

Table 3: Factors and Levels, Leaf Spring Experiment

Control Factor		Level	
		-	+
<i>B.</i>	high heat temperature ($^{\circ}$ F)	1840	1880
<i>C.</i>	heating time (seconds)	23	25
<i>D.</i>	transfer time (seconds)	10	12
<i>E.</i>	hold down time (seconds)	2	3
Noise Factor		Level	
		-	+
<i>Q.</i>	quench oil temperature ($^{\circ}$ F)	130-150	150-170

Table 4: Cross Array and Height Data, Leaf Spring Experiment

Control Factor				Noise Factor					
<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	Q^{-}			Q^{+}		
-	+	+	-	7.78	7.78	7.81	7.50	7.25	7.12
+	+	+	+	8.15	8.18	7.88	7.88	7.88	7.44
-	-	+	+	7.50	7.56	7.50	7.50	7.56	7.50
+	-	+	-	7.59	7.56	7.75	7.63	7.75	7.56
-	+	-	+	7.94	8.00	7.88	7.32	7.44	7.44
+	+	-	-	7.69	8.09	8.06	7.56	7.69	7.62
-	-	-	-	7.56	7.62	7.44	7.18	7.18	7.25
+	-	-	+	7.56	7.81	7.69	7.81	7.50	7.59

Strategies for Variation Reduction

- *Sampling inspection*: passive, sometimes last resort.
- *Control charting and process monitoring*: can remove special causes. If the process is stable, it can be *followed* by using a *designed experiment*.
- *Blocking, covariate adjustment*: passive measures but useful in reducing variability, not for removing root causes.
- *Reducing variation in noise factors*: effective as it may reduce variation in the response but can be expensive. Better approach is to change control factor settings (*cheaper* and *easier* to do) by exploiting control-by-noise interactions, i.e., use robust parameter design!

Types of Noise Factors

1. Variation in process parameters.
 2. Variation in product parameters.
 3. Environmental variation.
 4. Load Factors.
 5. Upstream variation.
 6. Downstream or user conditions.
 7. Unit-to-unit and spatial variation.
 8. Variation over time.
 9. Degradation.
- **Traditional design uses 7 and 8.**

Variation Reduction Through RPD

- Suppose $y = f(x, z)$, x control factors and z noise factors. If x and z interact in their effects on y , then the $var(y)$ can be reduced either by reducing $var(z)$ (i.e. method 4 on p.7) or by changing the x values (i.e., RPD).
- An example:

$$\begin{aligned}y &= \mu + \alpha x_1 + \beta z + \gamma x_2 z + \varepsilon, \\ &= \mu + \alpha x_1 + (\beta + \gamma x_2)z + \varepsilon.\end{aligned}$$

By choosing an appropriate value of x to reduce the coefficient $\beta + \gamma x_2$, the impact of z on y can be reduced. Since β and γ are unknown, this can be achieved by using the control-by-noise interaction plots or other methods to be presented later.

Exploitation of Nonlinearity

- Nonlinearity between y and \mathbf{x} can be exploited for robustness if \mathbf{x}_0 , nominal values of \mathbf{x} , are control factors and deviations of \mathbf{x} around \mathbf{x}_0 are viewed as noise factors (called *internal noise*). Expand $y = f(\mathbf{x})$ around \mathbf{x}_0 ,

$$y \approx f(\mathbf{x}_0) + \sum_i \left. \frac{\partial f}{\partial x_i} \right|_{x_{i0}} (x_i - x_{i0}). \quad (1)$$

This leads to

$$\sigma^2 \approx \sum_i \left(\left. \frac{\partial f}{\partial x_i} \right|_{x_{i0}} \right)^2 \sigma_i^2,$$

where $\sigma^2 = \text{var}(y)$, $\sigma_i^2 = \text{var}(x_i)$, each component x_i has mean x_{i0} and variance σ_i^2 .

- From (1), it can be seen that σ^2 can be reduced by choosing x_{i0} with a smaller slope $\left. \frac{\partial f}{\partial x_i} \right|_{x_{i0}}$. This is demonstrated in Figure 1. Moving the nominal value a to b can reduce $\text{var}(y)$ because the slope at b is more flat. This is a **parameter design** step. On the other hand, reducing the variation of x around a can also reduce $\text{var}(y)$. This is a **tolerance design** step.

Exploitation of Nonlinearity to Reduce Variation

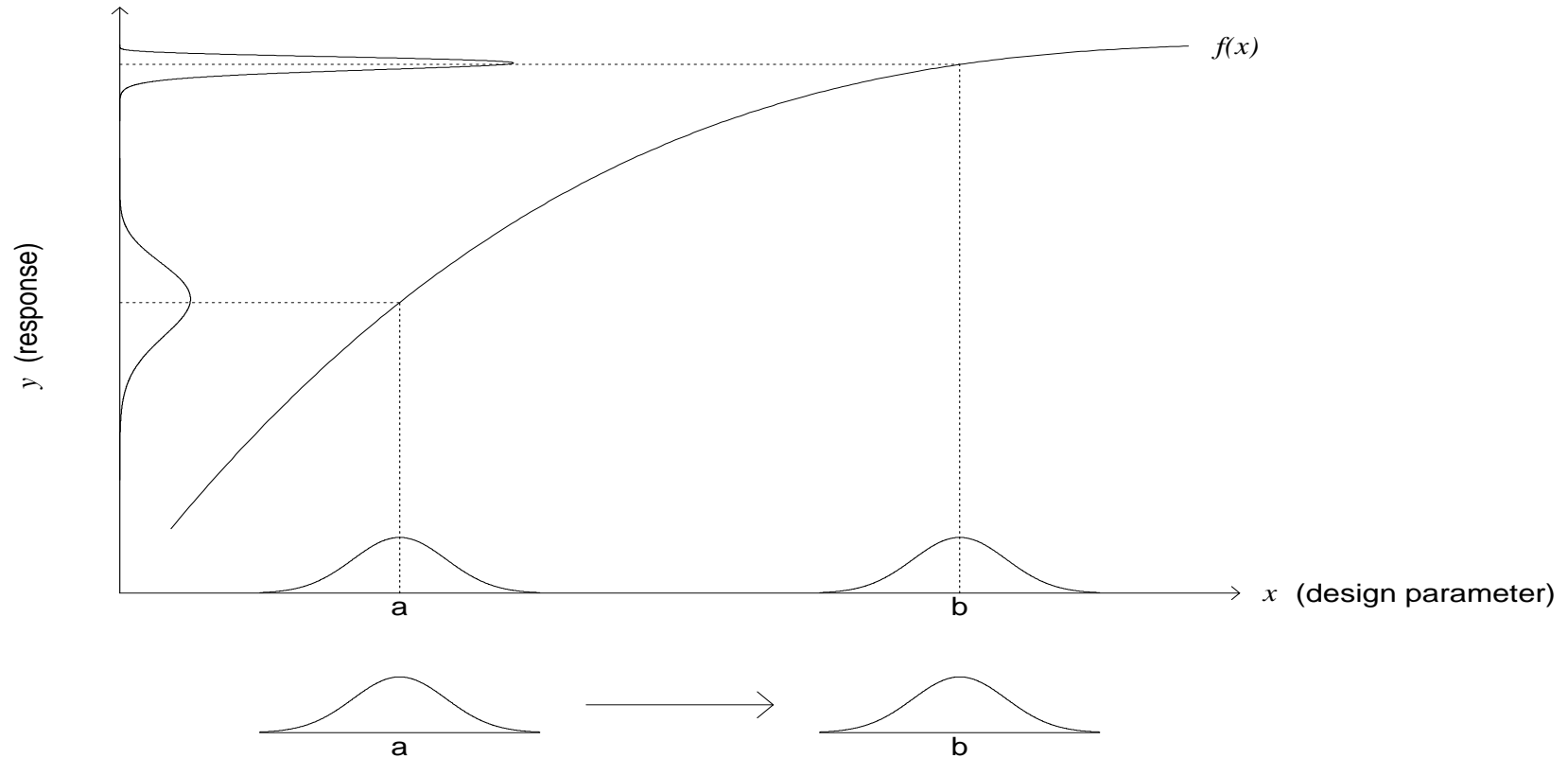


Figure 1: Exploiting the Nonlinearity of $f(x)$ to Reduce Variation

Cross Array and Location-Dispersion Modeling

- Cross array = control array \times noise array,
control array = array for control factors,
noise array = array for noise factors.
- Location-dispersion modeling
 - compute \bar{y}_i, s_i^2 based on the noise settings for the i^{th} control setting,
 - analyze \bar{y}_i (location), and $\ln s_i^2$ (dispersion), identify significant location and dispersion effects.

Two-step Procedures for Parameter Design Optimization

- **Two-Step Procedure for Nominal-the-Best Problem**

(i) select the levels of the dispersion factors to minimize dispersion,

(ii) select the level of the adjustment factor to bring the location on target .

(2)

- **Two-Step Procedure for Larger-the-Better and Smaller-the-Better Problems**

(i) select the levels of the location factors to maximize (or minimize) the location,

(ii) select the levels of the dispersion factors that are not location factors to minimize dispersion.

(3)

Note that the two steps in (3) are in reverse order from those in (2).

Reason: It is usually harder to increase or decrease the response y in the latter problem, so this step should be the first to perform.

Analysis of Layer Growth Experiment

- From the \bar{y}_i and $\ln s_i^2$ columns of Table 5, compute the factorial effects for location and dispersion respectively. (These numbers are not given in the book.) From the half-normal plots of these effects (Figure 2), D is significant for location and H , A for dispersion.

$$\hat{y} = 14.352 + 0.402x_D,$$

$$\hat{z} = -1.822 + 0.619x_A - 0.982x_H.$$

- Two-step procedure:
 - (i) choose A at the “−” level (continuous rotation) and H at the “+” level (nozzle position 6).
 - (ii) By solving

$$\hat{y} = 14.352 + 0.402x_D = 14.5,$$

choose $x_D = 0.368$.

Layer Growth Experiment: Analysis Results

Table 5: Means, Log Variances and SN Ratios, Layer Growth Experiment

Control Factor								\bar{y}_i	$\ln s_i^2$	$\ln \bar{y}_i^2$	$\hat{\eta}_i$
<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>				
-	-	-	+	-	-	-	-	14.79	-1.018	5.389	6.41
-	-	-	+	+	+	+	+	14.86	-3.879	5.397	9.28
-	-	+	-	-	-	+	+	14.00	-4.205	5.278	9.48
-	-	+	-	+	+	-	-	13.91	-1.623	5.265	6.89
-	+	-	-	-	+	-	+	14.15	-5.306	5.299	10.60
-	+	-	-	+	-	+	-	13.80	-1.236	5.250	6.49
-	+	+	+	-	+	+	-	14.73	-0.760	5.380	6.14
-	+	+	+	+	-	-	+	14.89	-1.503	5.401	6.90
+	-	-	-	-	+	+	-	13.93	-0.383	5.268	5.65
+	-	-	-	+	-	-	+	14.09	-2.180	5.291	7.47
+	-	+	+	-	+	-	+	14.79	-1.238	5.388	6.63
+	-	+	+	+	-	+	-	14.33	-0.868	5.324	6.19
+	+	-	+	-	-	+	+	14.77	-1.483	5.386	6.87
+	+	-	+	+	+	-	-	14.88	-0.418	5.400	5.82
+	+	+	-	-	-	-	-	13.76	-0.418	5.243	5.66
+	+	+	-	+	+	+	+	13.97	-2.636	5.274	7.91

Layer Growth Experiment: Plots

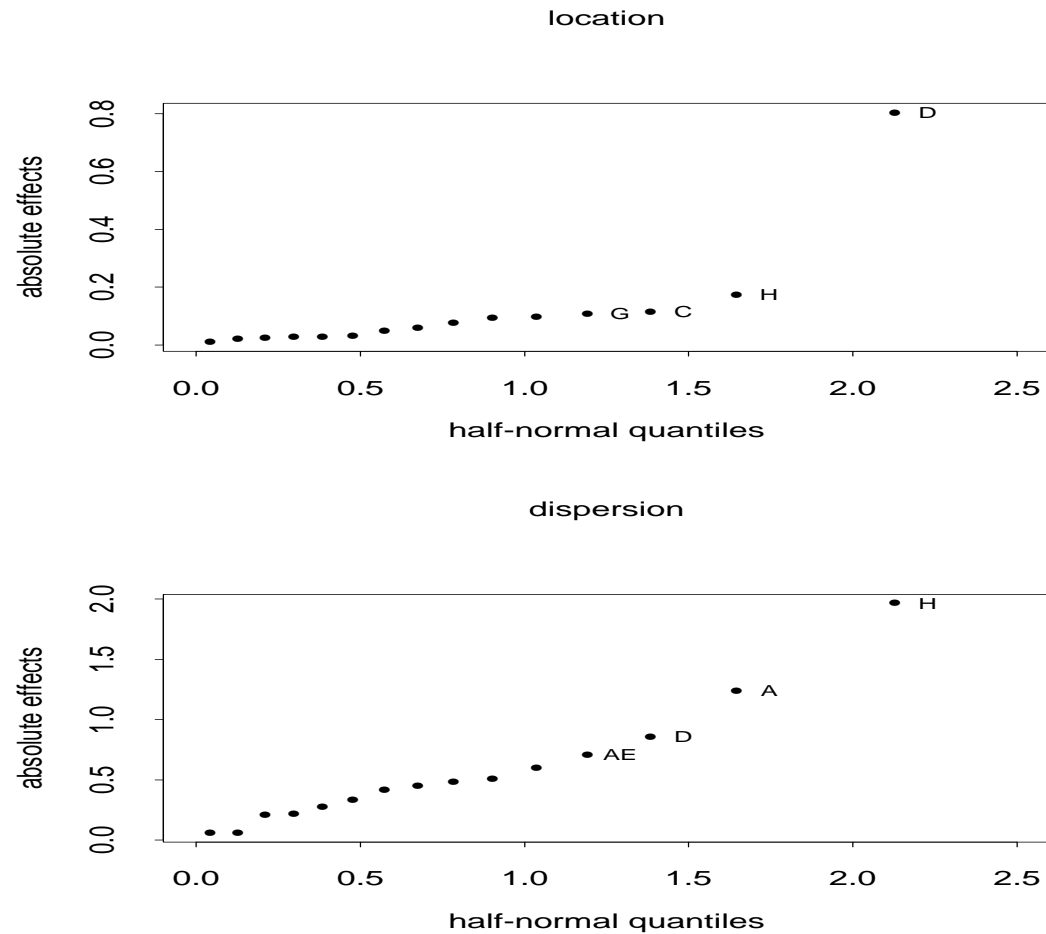


Figure 2: Half-Normal Plots of Location and Dispersion Effects, Layer Growth Experiment

Analysis of Leaf Spring Experiment

- Based on the half-normal plots in Figure 3, B , C and E are significant for location, C is significant for dispersion:

$$\hat{y} = 7.6360 + 0.1106x_B + 0.0881x_C + 0.0519x_E,$$

$$\hat{z} = -3.6886 + 1.0901x_C.$$

- Two-step procedure:

(i) choose C at $-$.

(ii) With $x_C = -1$, $\hat{y} = 7.5479 + 0.1106x_B + 0.0519x_E$.

To achieve $\hat{y} = 8.0$, x_B and x_E must be chosen beyond $+1$, i.e., $x_B = x_E = 2.78$. This is too drastic, and not validated by current data. An alternative is to select $x_B = x_E = x_C = +1$ (not to follow the two-step procedure), then $\hat{y}=7.89$ is closer to 8. (Note that $\hat{y} = 7.71$ with $B_+C_-E_+$.) Reason for the breakdown of the 2-step procedure: its second step cannot achieve the target 8.0.

Leaf Spring Experiment: Analysis Results

Table 6: Means and Log Variances, Leaf Spring Experiment

Control Factor				\bar{y}_i	$\ln s_i^2$
<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>		
–	+	+	–	7.540	-2.4075
+	+	+	+	7.902	-2.6488
–	–	+	+	7.520	-6.9486
+	–	+	–	7.640	-4.8384
–	+	–	+	7.670	-2.3987
+	+	–	–	7.785	-2.9392
–	–	–	–	7.372	-3.2697
+	–	–	+	7.660	-4.0582

Leaf Spring Experiment: Plots

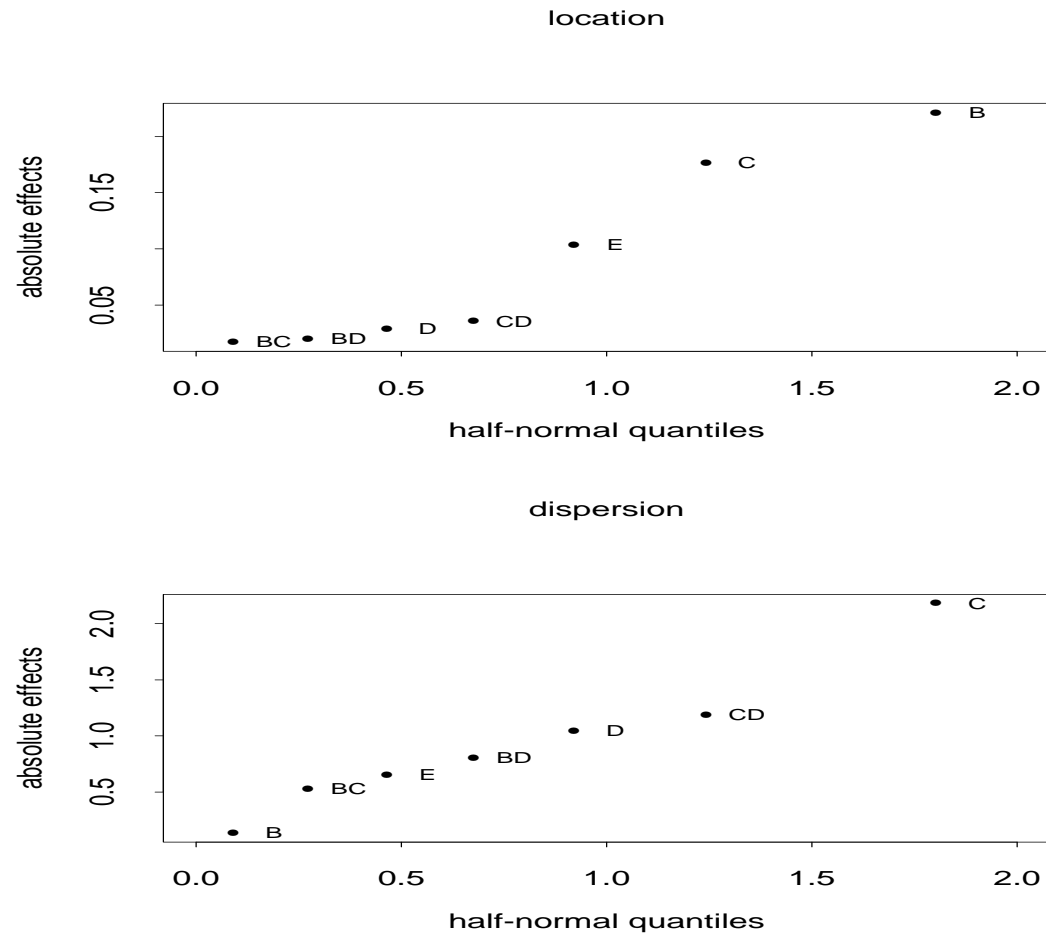


Figure 3: Half-Normal Plots of Location and Dispersion Effects, Leaf Spring Experiment

Response Modeling and Control-by-Noise

Interaction Plots

- Response Model: model y_{ij} directly in terms of control, noise effects and control-by-noise interactions.
 - half normal plot of various effects.
 - regression model fitting, obtaining \hat{y} .
- Make control-by-noise interaction plots for significant effects in \hat{y} , choose **robust** control settings at which y has a flatter relationship with noise.
- Compute $Var(\hat{y})$ with respect to variation in the noise factors. Call $Var(\hat{y})$ the **transmitted variance model**. Use it to identify control factor settings with small transmitted variance.

Half-normal Plot, Layer Growth Experiment

- Define

$$M_l = (M_1 + M_2) - (M_3 + M_4),$$

$$M_q = (M_1 + M_4) - (M_2 + M_3),$$

$$M_c = (M_1 + M_3) - (M_2 + M_4),$$

- From Figure 4, select D , L , HL as the most significant effects.
- How to deal with the next cluster of effects in Figure 4? Use **step-down multiple comparisons**.
- After removing the top three points in Figure 4, make a half-normal plot (Figure 5) on the remaining points. The cluster of next four effects (M_l, H, CM_l, AHM_q) appear to be significant.

Half-normal Plot of Factorial Effects

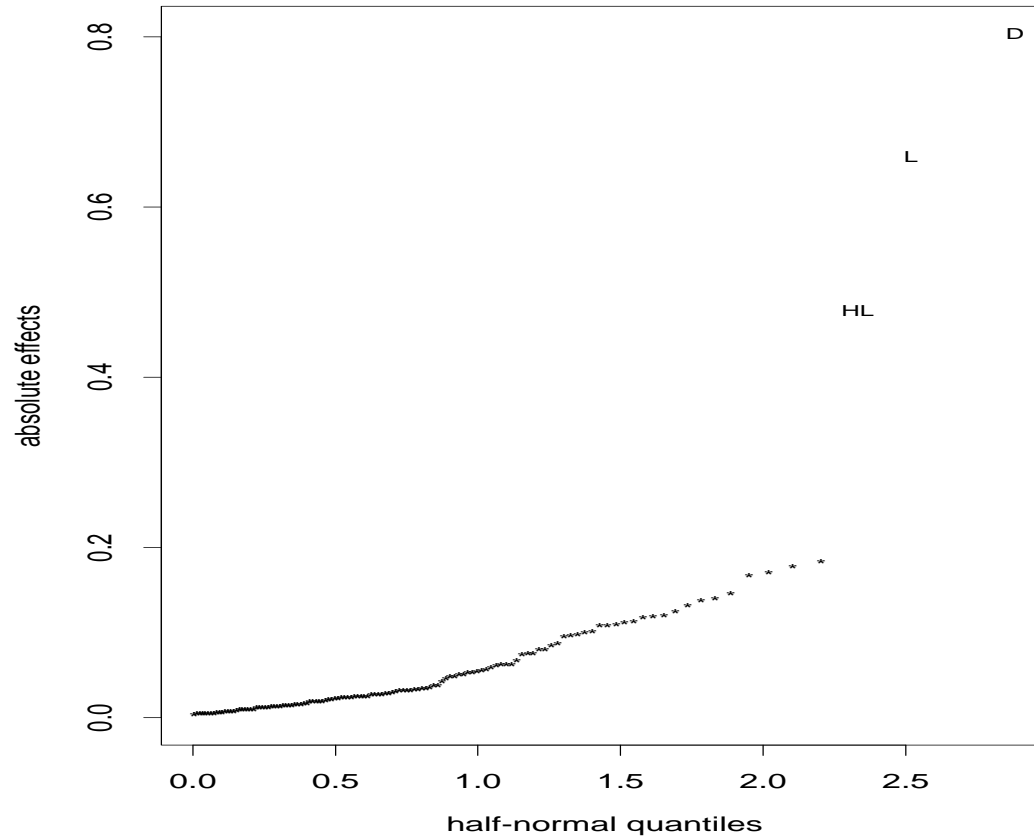


Figure 4: Half-Normal Plot of Response Model Effects, Layer Growth Experiment

Second Half-normal Plot of Factorial Effects

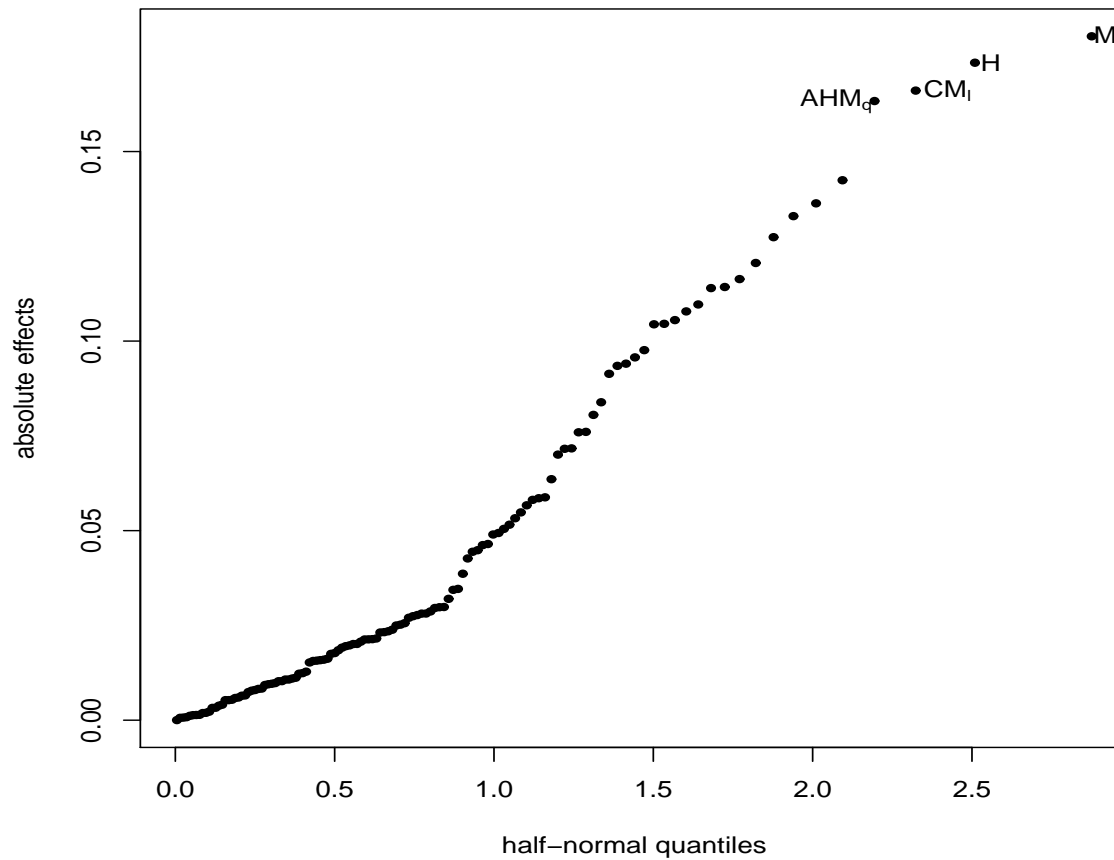


Figure 5: Second Half-Normal Plot of Response Model Effects, Layer Growth Experiment

Control-by-noise Interaction Plots

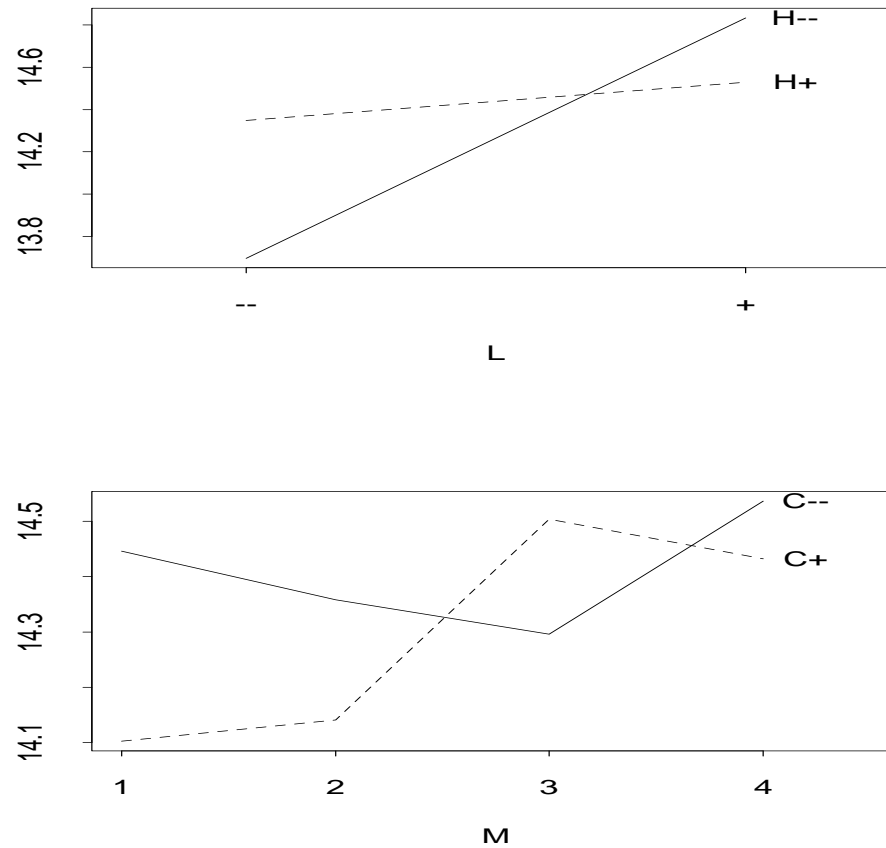


Figure 6: $H \times L$ and $C \times M$ Interaction Plots, Layer Growth Experiment

$A \times H \times M$ Plot

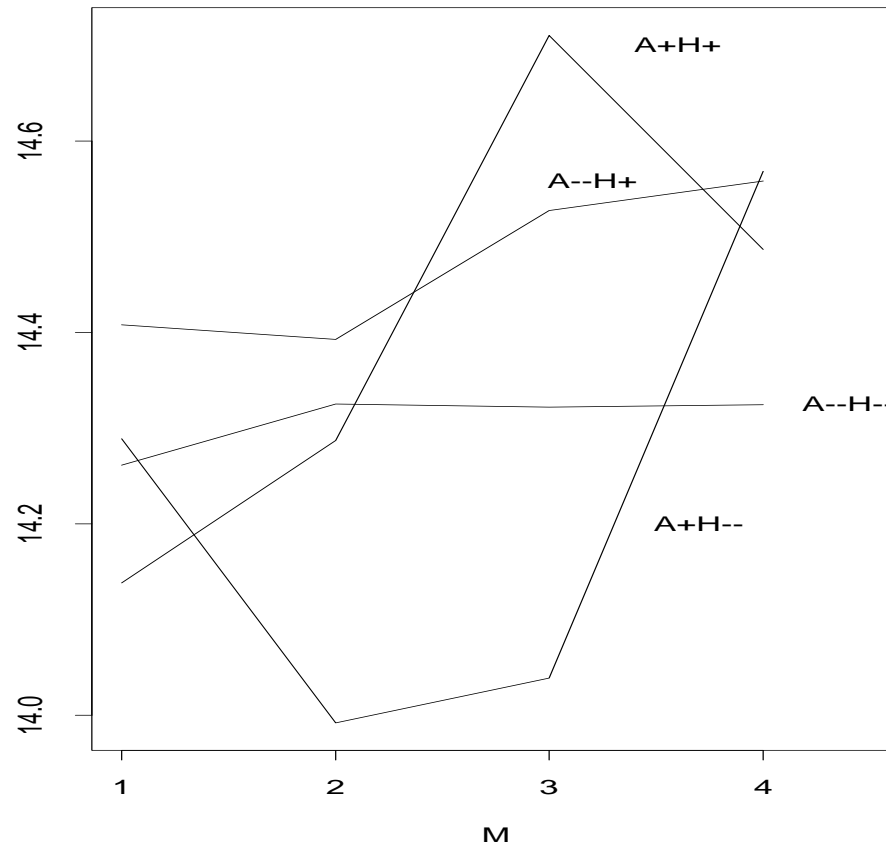


Figure 7: $A \times H \times M$ Interaction Plot, Layer Growth Experiment

Response Modeling, Layer Growth Experiment

- The following model is obtained:

$$\begin{aligned}\hat{y} = & 14.352 + 0.402x_D + 0.087x_H + 0.330x_L - 0.090x_{M_l} \\ & - 0.239x_Hx_L - 0.083x_Cx_{M_l} - 0.082x_Ax_Hx_{M_q}.\end{aligned}\quad (4)$$

- Recommendations:

H: – (position 2) to + (position 6)

A: + (oscillating) to – (continuous)

C: + (1210) to – (1220)

resulting in 37% reduction of thickness standard variation.

Predicted Variance Model

- Assume L , M_l and M_q are random variables, taking -1 and $+1$ with equal probabilities. This leads to

$$\begin{aligned}
 x_L^2 &= x_{M_l}^2 = x_{M_q}^2 = x_A^2 = x_C^2 = x_H^2 = 1, \\
 E(x_L) &= E(x_{M_l}) = E(x_{M_q}) = 0, \\
 Cov(x_L, x_{M_l}) &= Cov(x_L, x_{M_q}) = Cov(x_{M_l}, x_{M_q}) = 0.
 \end{aligned}
 \tag{5}$$

- From (4) and (5), we have

$$\begin{aligned}
 Var(\hat{y}) &= (.330 - .239x_H)^2 Var(x_L) + (-.090 - .083x_C)^2 Var(x_{M_l}) \\
 &\quad + (.082x_Ax_H)^2 Var(x_{M_q}) \\
 &= \text{constant} + (.330 - .239x_H)^2 + (-.090 - .083x_C)^2 \\
 &= \text{constant} - 2(.330)(.239)x_H + 2(.090)(.083)x_C \\
 &= \text{constant} - .158x_H + .015x_C.
 \end{aligned}$$

- Choose $H+$ and $C-$. But factor A is not present here. (Why? See explanation on p. 532).

Estimation Capacity for Cross Arrays

- Example 1. Control array is a 2_{III}^{3-1} design with $\mathbf{I} = \mathbf{ABC}$ and the noise array is a 2_{III}^{3-1} design with $\mathbf{I} = \mathbf{abc}$. The resulting cross array is a 16-run 2_{III}^{6-2} design with $\mathbf{I} = \mathbf{ABC} = \mathbf{abc} = \mathbf{ABCabc}$. Easy to show that all 9 control-by-noise interactions are clear, (but not the 6 main effects). This is indeed a general result stated next.

Theorem: Suppose a 2^{k-p} design d_C is chosen for the control array, a 2^{m-q} design d_N is chosen for the noise array, and a cross array, denoted by $d_C \otimes d_N$, is constructed from d_C and d_N .

- (i) If $\alpha_1, \dots, \alpha_A$ are the estimable factorial effects (among the control factors) in d_C and β_1, \dots, β_B are the estimable factorial effects (among the noise factors) in d_N , then $\alpha_i, \beta_j, \alpha_i\beta_j$ for $i = 1, \dots, A, j = 1, \dots, B$ are estimable in $d_C \otimes d_N$.
- (ii) All the km control-by-noise interactions (i.e., two-factor interactions between a control factor main effect and a noise factor main effect) are clear in $d_C \otimes d_N$.

Cross Arrays or Single Arrays?

- Three control factors A, B, C two noise factors a, b : $2^3 \times 2^2$ design, allowing all main effects and two-factor interactions to be clearly estimated.
- Use a single array with 16 runs for all five factors: a resolution V 2^{5-1} design with $\mathbf{I} = ABCab$ or $\mathbf{I} = -ABCab$, all main effects and two-factor interactions are clear. (See Table 7)
- Single arrays can have smaller runs, but cross arrays are easier to use and interpret.

32-run Cross Array and 16-run Single Arrays

Table 7: 32-Run Cross Array

Runs	<i>A</i>	<i>B</i>	<i>C</i>	<i>a</i>				
				+	+	−	−	
				<i>b</i>	+	−	+	−
1–4	+	+	+	+	●	○	○	●
5–8	+	+	−	−	○	●	●	○
9–12	+	−	+	+	○	●	●	○
13–16	+	−	−	−	●	○	○	●
17–20	−	+	+	+	○	●	●	○
21–24	−	+	−	−	●	○	○	●
25–28	−	−	+	+	●	○	○	●
29–32	−	−	−	−	○	●	●	○

● : $\mathbf{I} = ABCab$, ○ : $\mathbf{I} = -ABCab$,

Comparison of Cross Arrays and Single Arrays

- Example 1 (continued) An alternative is to choose a single array 2_{IV}^{6-2} design with $\mathbf{I} = \mathbf{ABCa} = \mathbf{ABbc} = \mathbf{abcC}$. This is not advisable because no 2fi's are clear and only main effects are clear. (Why? We need to have some clear control-by-noise interactions for robust optimization.) A better one is to use a 2_{III}^{6-2} design with $\mathbf{I} = \mathbf{ABCa} = \mathbf{abc} = \mathbf{ABCbc}$. It has 9 clear effects: $A, B, C, Ab, Ac, Bb, Bc, Cb, Cc$ (3 control main effects and 6 control-by-noise interactions).

Signal-to-Noise Ratio

- Taguchi's SN ratio $\hat{\eta} = \ln \frac{\bar{y}^2}{s^2}$
- Two-step procedure:
 1. Select control factor levels to maximize SN ratio,
 2. Use an adjustment factor to move mean on target.
- **Limitations**
 - maximizing \bar{y}^2 not always desired.
 - little justification outside linear circuitry.
 - statistically justifiable only when $Var(y)$ is proportional to $E(y)^2$
- **Recommendation:** Use SN ratio sparingly. Better to use the location-dispersion modeling or the response modeling. The latter strategies can do whatever SN ratio analysis can achieve.

Half-normal Plot for S/N Ratio Analysis

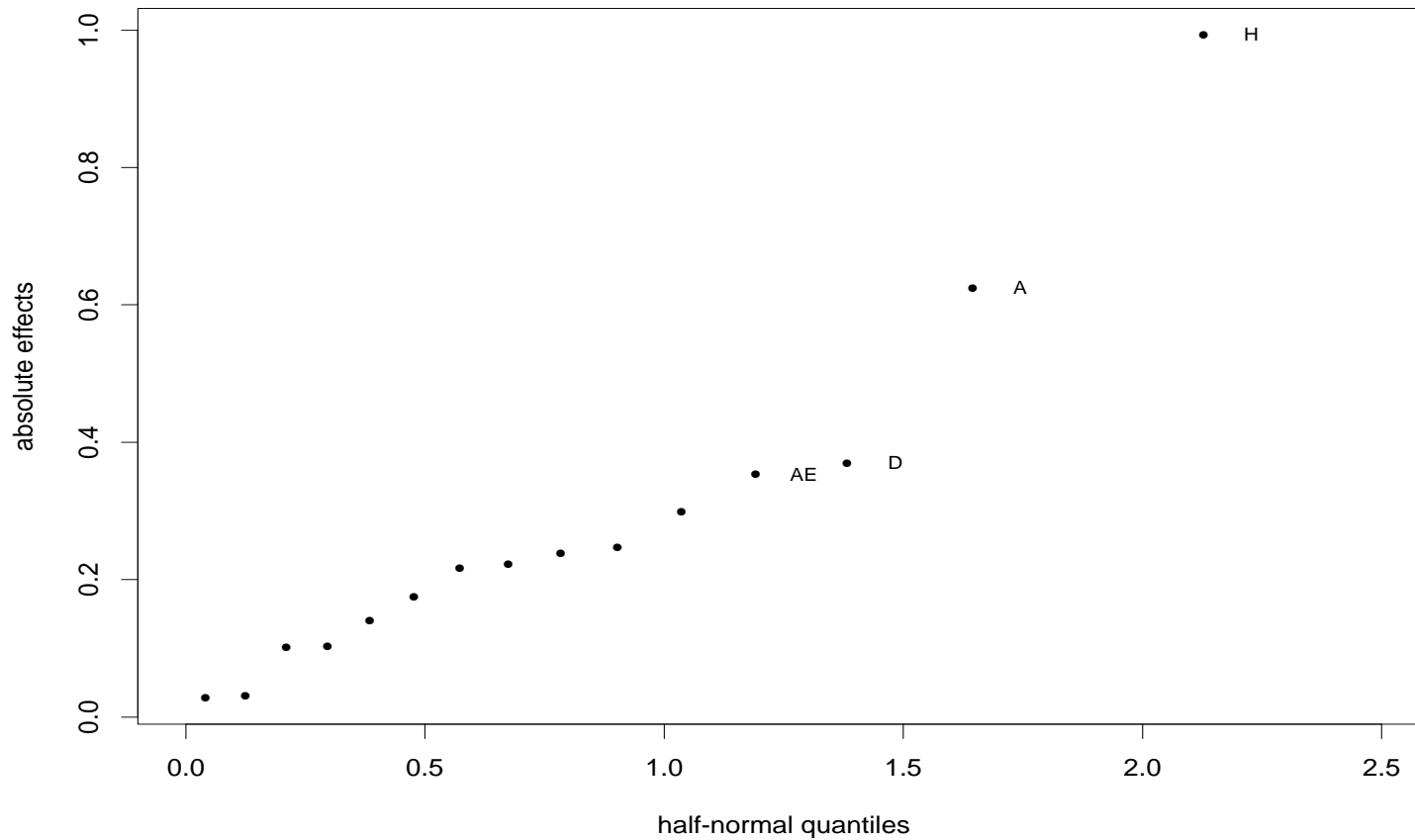


Figure 8: Half-Normal Plots of Effects Based on SN Ratio, Layer Growth Experiment

S/N Ratio Analysis for Layer Growth Experiment

- Based on the $\hat{\eta}_i$ column in Table 5, compute the factorial effects using SN ratio. From Figure 7, the conclusion is similar to location-dispersion analysis. Why? Using

$$\hat{\eta}_i = \ln \bar{y}_i^2 - \ln s_i^2,$$

and from Table 5, the variation among $\ln s_i^2$ is much larger than the variation among $\ln \bar{y}_i^2$; thus maximizing SN ratio is equivalent to minimizing $\ln s_i^2$ in this case.

Comments on Board