

## HOMWORK 3 SOLUTIONS

### 2.6

#### **ANOVA: Strength versus Method, Girder**

Factor	Type	Levels	Values
Method	fixed	10	A, C, G, K, L, O, PC, S, T, Z
Girder	fixed	9	S1/1, S1/2, S2/1, S2/2, S3/1, S3/2, S4/1, S4/2, S5/1

Analysis of Variance for Strength

Source	DF	SS	MS	F	P
Method	9	1.82549	0.20283	23.64	0.000
Girder	8	0.49958	0.06245	7.28	0.000
Error	72	0.61769	0.00858		
Total	89	2.94276			

S = 0.0926230    R-Sq = 79.01%    R-Sq(adj) = 74.05%

Comparisons tests:

The Tukey t-statistic absolute values:

Aarau vs Lehigh : 6.2132  
Prague-Cardiff vs Stockholm : 2.423

$\frac{1}{\sqrt{2}} q_{10,72,0.05} = \frac{4.64}{1.414} = 3.28$ . By comparing 3.28 with the  $t$  values, we find that the Aarau vs Leigh difference is significant at 5% level but not the Prague-Cardiff vs Stockholm difference.

In the stated problem, since you were asked to perform two tests separately, usual paired t-tests would suffice.

#### **Paired T-Test and CI: Aarau, Lehigh**

Paired T for A - L

	N	Mean	StDev	SE Mean
A	9	0.79489	0.05510	0.01837
L	9	1.06622	0.04938	0.01646
Difference	9	-0.271333	0.056276	0.018759

95% CI for mean difference: (-0.314591, -0.228076)  
T-Test of mean difference = 0 (vs not = 0): T-Value = -14.46    P-Value = 0.000

**Conclusion: Significant difference at 1% level.**

#### **Paired T-Test and CI: Prague-Cardiff, Stockholm**

Paired T for PC - S

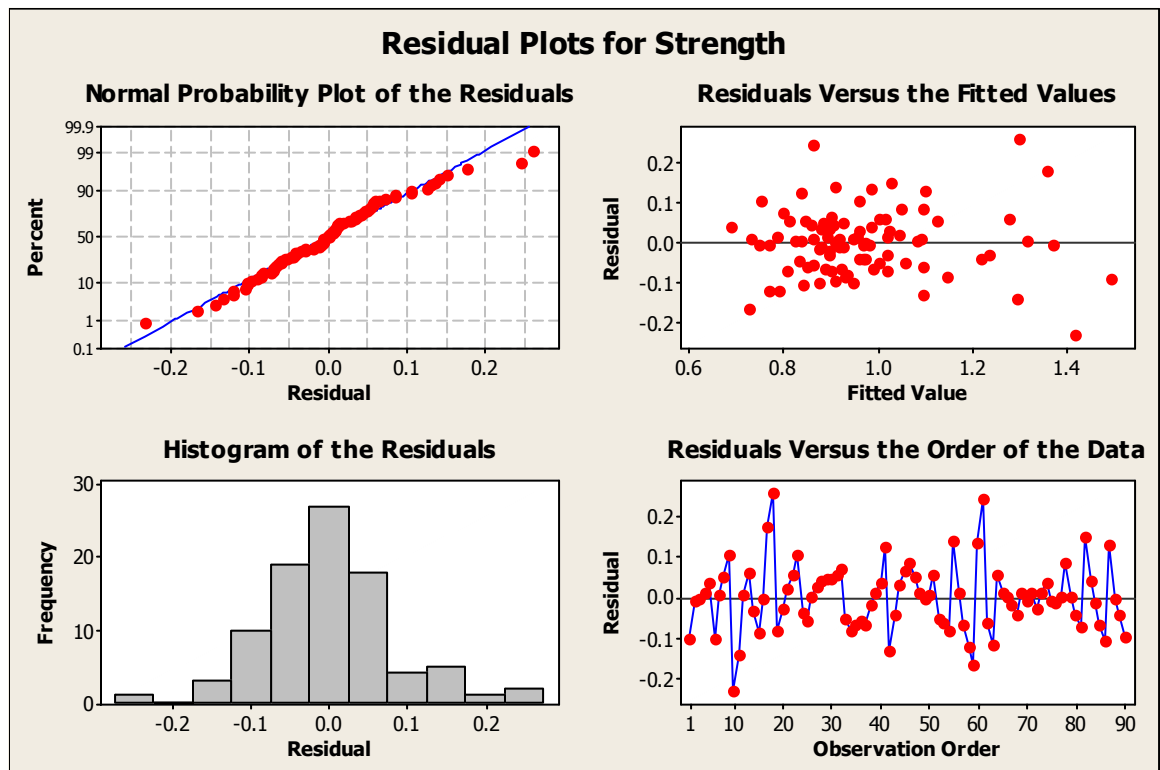
	N	Mean	StDev	SE Mean
PC	9	0.832556	0.210295	0.070098
S	9	0.938444	0.101402	0.033801
Difference	9	-0.105889	0.128279	0.042760

95% CI for mean difference: (-0.204493, -0.007285)

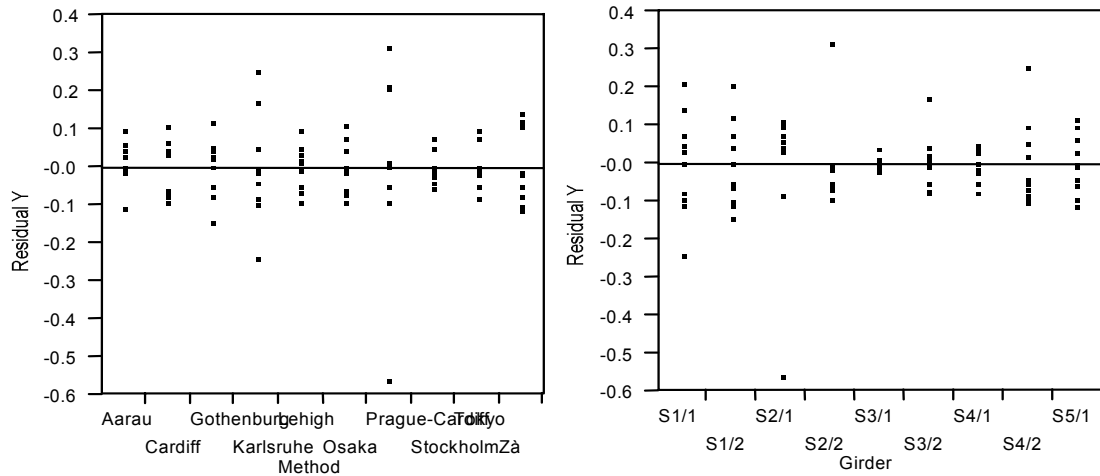
T-Test of mean difference = 0 (vs not = 0): T-Value = -2.48 P-Value = 0.038

**Conclusion: Not significant at 1% level; significant at 5% level.**

Here the result differs from the Tukey test because of using different variance estimate.



There is no significant pattern in the  $r_i$  vs.  $\hat{y}_i$  plot. So  $r$  and  $\hat{y}_i$  can be considered independent. The normal probability plot appears to be a straight line, thereby satisfying normality assumption. The  $r_i$  vs.  $x_i$  plots are shown below, including the plot of residual vs. the treatment factor and the plot of residual vs. the block factor.



Neither the residual vs. treatment plot nor the residual vs. block plot shows systematic patterns. There are some unequal spreads of the residuals over different treatments and blocks, which suggests that the error variance may be affected by the changes of the treatment factor and the block factor. There are, however, a couple of outliers on these plots.

## 2.8

- (a) This is a paired comparison design because a pair of catalysts is compared in each of the six batches. Each batch can be considered as a block of size 2. In this design, the two catalysts are compared on two portions of the same batch, this eliminates a source of variation due to the difference among different batches, which were known to be quite different from one another.
- (b) Use paired t-test,

	1	2	3	4	5	6
A	9	19	28	22	18	8
B	10	22	30	21	23	12
$d_i$	1	3	2	-1	5	4

$$\bar{d} = \frac{1}{N} \sum_{i=1}^N d_i = \frac{14}{6} = 2.33, \quad s_d = \left\{ \frac{1}{N-1} \sum_{i=1}^N (d_i - \bar{d})^2 \right\}^{1/2} = 2.16.$$

With  $\alpha=0.05$ ,  $t_{paired} = \sqrt{N}\bar{d} / s_d = \sqrt{6} \cdot 2.33 / 2.16 = 2.64$  and  $t_{N-1, \alpha/2} = t_{5, 0.025} = 2.57$ . Since  $2.64 = |t_{paired}| > t_{N-1, \alpha/2} = 2.57$ , the two treatments have difference at level 0.05.

- (c) The 95% percent CI for the difference between A and B is  $\bar{d} \pm t_{N-1, \alpha/2} s_d / \sqrt{N} = 2.33 \pm 2.57 * 2.16 / \sqrt{6} = 2.33 \pm 2.27$ , where  $\alpha=1-0.95=0.05$ .

2.14 The  $\ln s_{ij}^2$  which is considered as output are calculated as below:

P&O	C&W	HT	M/B
1.607213	2.883558	2.506886	M
4.878162	2.18979	3.784442	B

### ANOVA: Torque versus Medium, Plating

Factor	Type	Levels	Values
Medium	fixed	2	B, M
Plating	fixed	3	CW, HT, P&O

Analysis of Variance for Torque

Source	DF	SS	MS	F	P
Medium	1	2.476	2.476	1.26	0.378
Plating	2	0.586	0.293	0.15	0.870
Error	2	3.930	1.965		
Total	5	6.992			

S = 1.40175    R-Sq = 43.80%    R-Sq(adj) = 0.00%

Compared with Figure 2.1 of the text, the p-value for plating is 0.870 and the p-value for test is 0.378. From these p-values, the test factor has more significant effect than the plating factor but both effects are not quite significant. From Figure 2.1 of the text, the Box-Whisker plots corresponding to bolt tends to have wider spread than the Box-Whisker plots corresponding to mandrel, which shows some effects of the test factor. And we see a large spread for the bolt-P&O treatment combination in Figure 2.1, which suggests that there may be significant interaction effect between test factor and plating factor. However, we do not see any of significant effects from the ANOVA table. The reason is that the two-way layout analysis for this design cannot capture the interaction effect on  $\ln s_{ij}^2$  due to the lack of replicates and degrees of freedom.

### 2.20 ANOVA: cycles versus length, amplitude, load

Factor	Type	Levels	Values
Length	fixed	3	0, 1, 2
amplitude	fixed	3	0, 1, 2
load	fixed	3	0, 1, 2

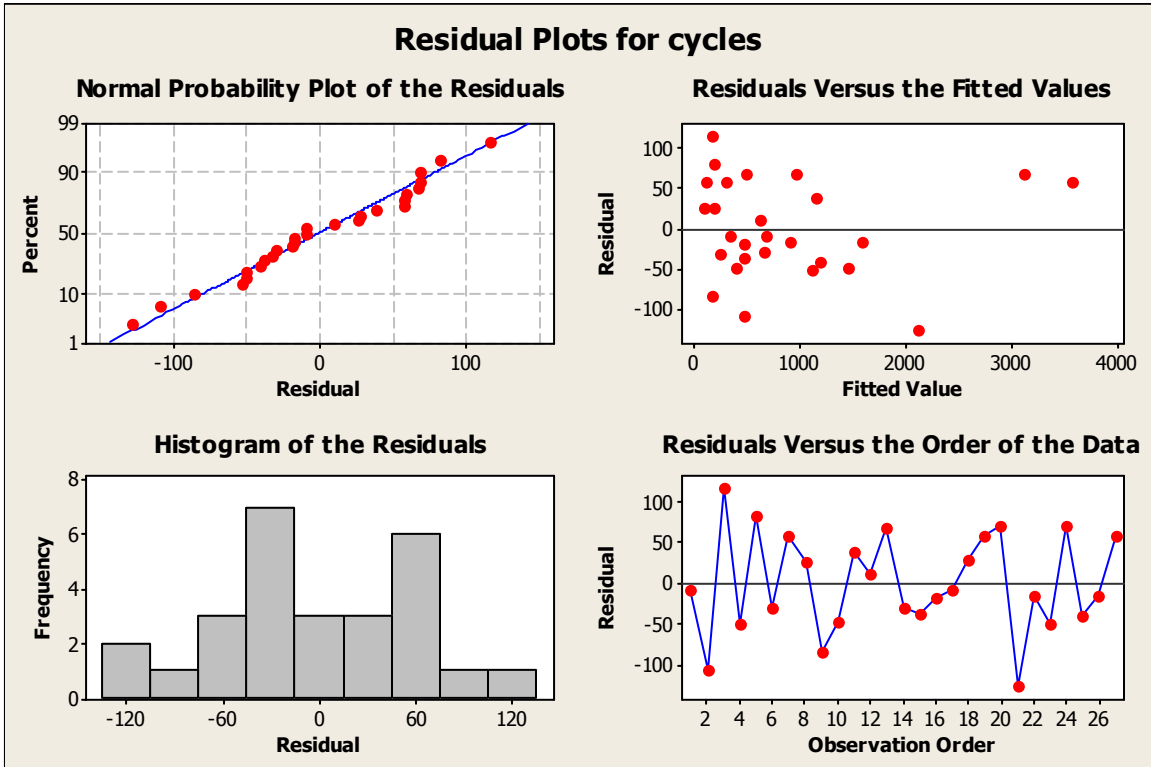
Analysis of Variance for cycles

Source	DF	SS	MS	F	P
length	2	8182571	4091285	337.33	0.000
amplitude	2	5625137	2812568	231.90	0.000
load	2	1752508	876254	72.25	0.000
length*amplitude	4	3555391	888848	73.29	0.000
length*load	4	732956	183239	15.11	0.001
amplitude*load	4	283865	70966	5.85	0.017

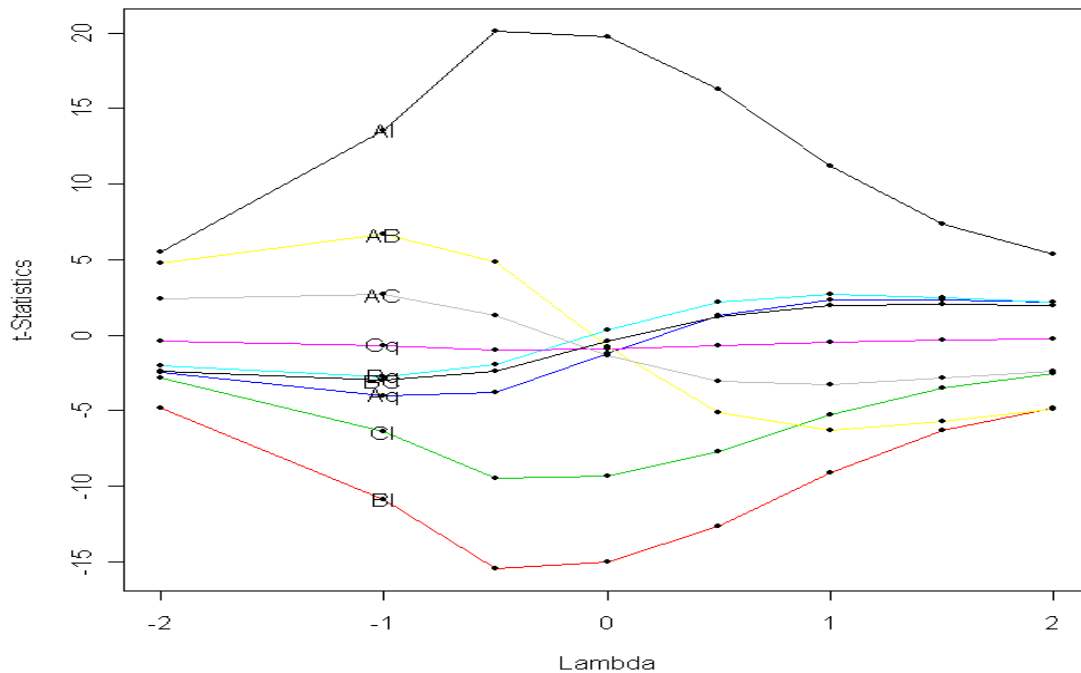
Error	8	97028	12129
Total	26	20229456	

S = 110.130    R-Sq = 99.52%    R-Sq(adj) = 98.44%

### Residual Plots for cycles



From the p-values for all the six effects, all the main effects and the two factor interaction effects are significant. And the most significant effects are all the three main effects and the length\*amplitude interaction. However, note that the 'error' cannot be separated from the three factor interaction. The poor normal probability plot and histogram suggests the need for transformation of the response. Using the system of linear and quadratic contrasts, we obtain the following scaled lambda plot:



Log transformation looks reasonable. It separates the linear effects from other effects very well.

Regression output :

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	12.66933	0.07461	169.816	< 2e-16	***
A1	1.66477	0.09137	18.219	1.37e-12	***
B1	-1.26198	0.09137	-13.811	1.14e-10	***
C1	-0.78499	0.09137	-8.591	1.36e-07	***
Aq	-0.05713	0.05275	-1.083	0.294	
Bq	0.01615	0.05275	0.306	0.763	
Cq	-0.04497	0.05275	-0.852	0.406	
AB	-0.07648	0.11191	-0.683	0.504	
AC	-0.13683	0.11191	-1.223	0.238	
BC	-0.04167	0.11191	-0.372	0.714	

We find that the linear effects are significant. Let us do the usual one way ANOVA

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
factor (A)	2	50.062	25.031	301.3674	2.944e-08	***
factor (B)	2	28.681	14.340	172.6542	2.629e-07	***
factor (C)	2	11.201	5.600	67.4276	9.835e-06	***
factor (A) : factor (B)	4	1.605	0.401	4.8301	0.02815	*
factor (A) : factor (C)	4	0.543	0.136	1.6346	0.25661	
factor (B) : factor (C)	4	0.058	0.015	0.1754	0.94488	

Residuals                      8   0.664   0.083

Here also we find the main effects to be significant. Along with that AB interaction also becomes marginally significant.