ISyE6413
Second Midterm Examination November 10, 2004
(Total : 50 points)

Problem 1 (4 pts)

A taste panel will convene to compare six different brands of ice cream. The panel is composed of 10 persons who are expert tasters. The maximum number of different brands that an individual taster will taste is 3. What experimental design would be the best to use in this situation? Give the parameters of the design.

BIBD. Block size $k = 3$, Number of treatment $t = 6$, Number of blocks $b = 10$, Each treatment is replicated $r = 5$ times, such that each pair of treatments appear in the *same* number (denoted by $\lambda = 2$) of blocks.

Problem 2 (5 pts)

Consider a **symmetric** Latin Square Design of order n where n is a **odd integer**. Prove that each of the Latin Letters $(A, B, C, \ldots)$ appear on the main diagonal of the design.

Clarification : Consider the following two Latin Squares of order 3 and 4.

$$\begin{array}{ccc} A & B & C \\ B & C & A \\ C & A & B \end{array} \qquad \begin{array}{cccc} A & B & C & D \\ B & A & D & C \\ C & D & A & B \\ D & C & B & A \end{array}$$

Both of these are symmetric Latin Squares with the main diagonals (A, C, B) and (A, A, A, A), respectively. Note that, 3 is an odd number and we have each of the Latin Letters A, B and C to appear on the main diagonal of the design. However, this is not the case for the Latin Square of order 4. (Why ? 4 is not an odd number.) The following two Latin Squares are not symmetric :

$$\begin{array}{ccc} A & B & C \\ C & A & B \\ B & C & A \end{array} \qquad \begin{array}{cccc} A & B & C & D \\ C & D & A & B \\ D & C & B & A \\ B & A & D & C \end{array}$$

Here each of the Latin Letters appear on the main diagonal of the Latin Square Design of order 4, not on that of order 3. (Why ? These are not symmetric designs.)

If not, say A does not appear on the main diagonal of the Latin Square design, and it appears k times in the upper triangular part of the design matrix. By **symmetry** of Latin Square, A appears $k + k = 2k$ times in the Latin Square, but $2k$ is an **even** integer when n is **odd**, hence $n \neq 2k$, a contradiction to the definition of Latin Square.

Problem 3 (16 pts)

Speedometer cables can be noisy because of shrinkage in the plastic casing material, so an experiment was conducted to find out what caused shrinkage. The engineers identified four factors at two levels: A = wire braid type, B = braiding tension, C = wire diameter, D = line speed. Response y is percentage shrinkage per specimen. A full factorial design with 16 runs was used. Full data, collapsed data and output of analysis are given below. Note that the fit given here is a regression output and the plot is a half normal plot.

Data

Run	A	B	C	D	y
1	-1	-1	-1	-1	0.2750
2	-1	-1	-1	1	0.1700
3	-1	-1	1	-1	0.0875
4	-1	-1	1	1	0.1750
5	-1	1	-1	-1	0.1750
6	-1	1	-1	1	0.2250
7	-1	1	1	-1	0.1250
8	-1	1	1	1	0.1200
9	1	-1	-1	-1	0.5350
10	1	-1	-1	1	0.4550
11	1	-1	1	-1	0.1950
12	1	-1	1	1	0.1450
13	1	1	-1	-1	0.5750
14	1	1	-1	1	0.4850
15	1	1	1	-1	0.3425
16	1	1	1	1	0.5825

Collapsed data with factors A and C

Run	A	C	Average	Variance	Log Variance
1	-	-	0.211250	0.002423	-6.02278
2	-	+	0.126875	0.001306	-6.64099
3	+	-	0.512500	0.002825	-5.86925
4	+	+	0.316250	0.038535	-3.25618

Regression output

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Call:
lm(formula = y ~ A + B + C + D + AB + AC + AD + BC + BD + CD
                   +  ABC + ABD + BCD)

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.2917188  0.0261181  11.169  0.00792 **
A            0.1226563  0.0261181   4.696  0.04247 *
B            0.0370312  0.0261181   1.418  0.29199
C           -0.0701563  0.0261181  -2.686  0.11515
D            0.0029687  0.0261181   0.114  0.91988
AB           0.0448438  0.0261181   1.717  0.22812
AC          -0.0279688  0.0261181  -1.071  0.39633
AD          -0.0004687  0.0261181  -0.018  0.98731
BC           0.0339063  0.0261181   1.298  0.32376
BD           0.0214063  0.0261181   0.820  0.49858
CD           0.0310938  0.0261181   1.191  0.35599
ABC          0.0304688  0.0261181   1.167  0.36367
ABD          0.0135937  0.0261181   0.520  0.65462
BCD          0.0032812  0.0261181   0.126  0.91151
```

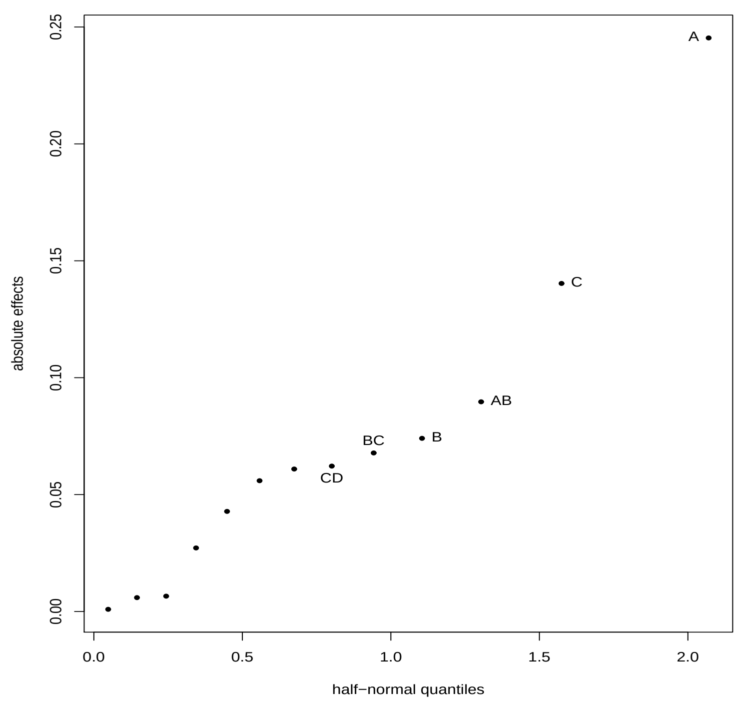

Figure 1 : Half Normal Plot

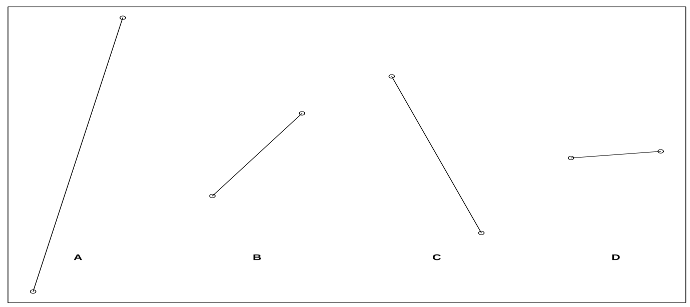

Figure 2 : Main Effects Plot

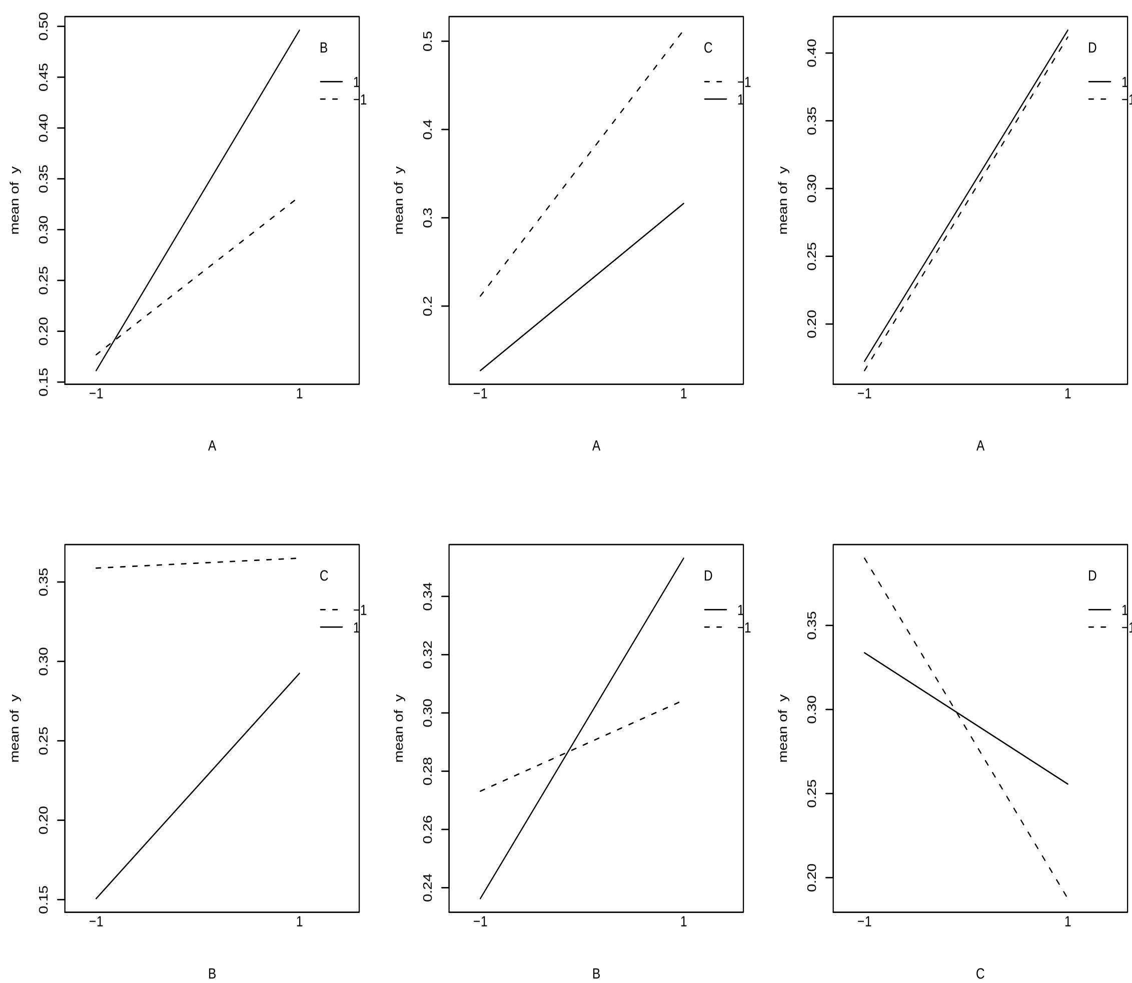

Figure 3 : Interaction Effects Plot

(a) (1 pt) Based on the output, what effects seem to be significant? (You need to choose at least two.)

A and C.

(b) (1 pt) Compute the factorial effect for B.

$2 \times 0.0370312 = 0.0740624$.

(b) (2 pts) Based on the effect estimates and plots, determine the optimal setting of the two most important factors. (Hint: what shrinkage values are considered optimal?)

$A = -1$ and $C = +1$.

(c) (1 pt) Is the setting you choose supported by the information in the original data (first table)?

Yes. The two lowest values of the response correspond to A- and C+.

(d) (3+1 pts) The second table gives the data collapsed onto factors A and C. It is a 2^2 design with 4 replicates. Compute the three factorial effects for the log variance (last column of table). Which one is the largest?

ME(A) = 1.76917, ME(C) = 0.99743, INT(A,C)=1.61564. ME(A) is largest.

(e) (2 pts) Identify the optimal factor setting in terms of minimizing the variance.

$A = -1$ and $C = +1$.

(f) (2+1 pts) Based on your findings in (b) and (e), what is your overall recommendation for choosing the factor setting? Any conflict?

$A = -1$ and $C = +1$, choose B and D to accommodate any other economic or engineering criteria. No Conflict.

(g) (1 pt) Look at the $B \times D$ and $D \times B$ interaction plots - one is synergistic and another is antagonistic. Any contradiction ?

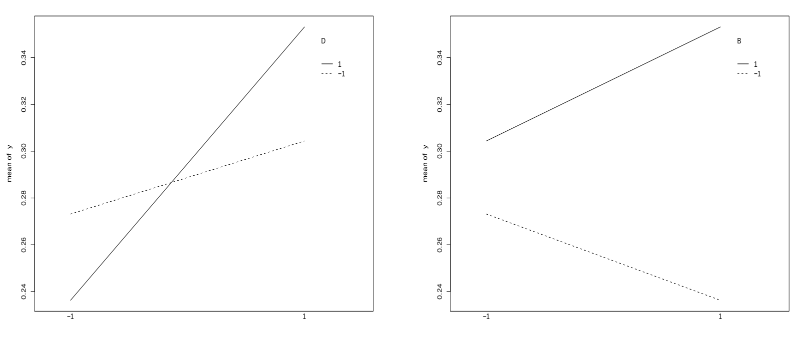

Figure 4 : Interaction Effects Plot for B and D

No contradiction, but this suggests that underlying response surface is more complicated.

Problem 4 (12 pts)

(a) Consider the 2^{7-3} design with 5 = 123, 6 = 124, 7=1234. Find its defining contrast subgroup.

$I = 457 = 367 = 1235 = 1246 = 3456 = 12347 = 12567$.

(b) Based on (a), find its resolution and all clear main effects and clear 2fi's.

III. Clear main effects = 1 and 2. Clear 2fi = 17 and 27.

(c) For this design, if we further know that any 2fi involving factor 3 is negligible, which 2fi's are estimable under the usual assumptions that three-factor and higher interactions are negligible?

15, 25 and 56.

Problem 5 (7 pts)

(a) (3 pts) What is the resolution of the most economical design that can estimate the main effects of five factors (each with two levels) and all their two-factor interactions? Explain why this should be the resolution.

V. All main effects and two factor interactions need to be clear. So no letter in the defining contrast subgroup can have length 4 or less.

(b) (2 pts) Find the design (*Hints: Use Appendix 4A*).

From table 4A.2, the 16 run design $2_V^{5-1} : 5 = 1234$ can be used.

(c) (2 pts) If it is known that the level combination $(+,+,+,+,+)$ of the five factors can lead to disastrous results (e.g., explosion, burn-out), specify how the design should be chosen.

To avoid (+, +, +, +, +) choose the generator as 5 = -1234.

Problem 6 (6 pts)

Consider a 2^{k-p} design. Let A_i be the number of defining words of length i and $W = (A_3, A_4, \ldots)$ is the wordlength pattern.

(a) For a 2^{7-2} design, how many entries are in W? Explain clearly.

7-2=5. $I = ABCDEFG$ is the word of maximum length one can have, so $W = (A_3, \ldots, A_7)$.

(b) Consider two designs d_1 and d_2 with wordlength patterns $W(d_1) = (0,1,2,0,0)$ and $W(d_2) = (0,2,0,1,1)$. Can you tell which one is better ? (*Hints : think carefully*).

No. One has 3 members in the defining contrast subgroup and another has 4. These two designs are not comparable. Actually design d_2 does not exist. Number of elements in the defining contrast subgroup must be of the form $2^m - 1$ (excluding the Identity element).

(c) In a resolution *IV* design, can all two-factor interactions be clear ? Explain.

No. If all of them are clear, then no two-factor interaction is aliased with any other two-factor interaction. So the minimum length of an word in the defining contrast subgroup is 2+2+1 = 5, which in turn results into a resolution *V* design.