ISyE6413

Final Examination December 8, 2004

(Total : 50 points)

Name :

Problem	1	2	3	4	5	Total
Max Points	5	10	10	5	20	50
Your score						

Problem 1 (5 pts)

Compressive strength of cement is measured indirectly in terms of its fineness and the unit of fineness is called Blaine (1 Blaine = 1 cm^2 per gram). The testing method is complicated and there are high chances of error. In a cement plant there are three Blaine testing instruments I_1, I_2, I_3 and the authorities want to test whether there exist differences among them with respect to testing error. One composite sample is collected for this purpose. This sample (rather, a portion of it) is tested externally using a perfectly calibrated instrument in an accredited laboratory and the measured value is assumed to be true. This sample is large enough to be split into as many parts as desired so that each part can be tested separately. There are four testers T_1, T_2, T_3, T_4 who normally do the job, and the manager wants each of them to take part in the experiment.

(a) (3 pts) Suppose the four testers differ with respect to their age and experience, and it is known that experience plays a vital role in Blaine testing. Keeping in mind the basic objective of the experiment what design (with minimum possible number of runs) should be used ? Explain clearly how the experiment should be conducted.

An RBD should be used with tester as the blocking factor. The composite sample should be split into twelve parts. Each tester should be given three parts and asked to test them on each of the three instruments. The testers will not be told in advance that they are testing the same sample; otherwise the results are likely to be biased.

(b) (2 pts) If it is further known that testers T_1 and T_3 are more comfortable in using instrument I_1; T_2 is comfortable with I_2 and T_4 usually prefers I_3, would the above strategy be still appropriate ? Give reasons for your answer.

The RBD will not be appropriate because an interaction between the treatment and block may be inevitable in this case. This time, one has to estimate the interaction and separate it from the true error to conclude about the significance of treatment effect correctly. Each operator-instrument combination should generate two test results to achieve this objective.

Problem 2 (10 pts)

(a) (2 pts) Show that effect hierarchy principle can be used to justify the minimum aberration criterion.

Effect hierarchy principle says that lower order effects are more important. A design with less number of words of a shorter length will alias fewer of the smaller order effects; thus minimum aberration should allow estimation of more effects deemed important by the effect hierarchy principle.

(b) (8 pts) Consider the 2^{6-2} design given by $\mathbf{5} = \mathbf{12}$ and $\mathbf{6} = \mathbf{134}$. The design is arranged in four blocks with $\mathbf{B_1} = \mathbf{13}$ and $\mathbf{B_2} = \mathbf{16}$. Find the treatment and block defining contrast subgroups. Find all the clear effects. Write down all the aliasing relations involving main effects and two factor interactions.

Treatment defining contrast subgroup : $I = 125 = 1346 = 23456$.
Block defining contrast subgroup : $B_1 = 13, B_2 = 16, B_1B_2 = 36$.
Clear main effects : 3, 4, 6.
Clear two-factor interaction effects : 23, 24, 26, 35, 45, 56.
Aliasing relations :

$$B_1 = 13 = 46, B_2 = 16 = 34, B_1B_2 = 36 = 14,$$
$$1 = 25, 2 = 15, 5 = 12.$$

Problem 3 (10 pts)

(a) (3+3 pts) Construct a 9-run design for studying four factors (A,B,C and D) each at three levels (denoted by 0,1 and 2). Give the defining contrast subgroup and the design matrix.

$C = AB, D = AB^2 \Rightarrow I = ABC^2 = AB^2D^2 = ACD = BCD^2$. First two columns of the design matrix constitutes a 3^2 full factorial design. Then the next two columns are obtained by the following relations : $x_C = x_A + x_B$ (mod 3) and $x_D = x_A + 2x_B$ (mod 3).

A	B	C	D
0	0	0	0
0	1	1	2
0	2	2	1
1	0	1	1
1	1	2	0
1	2	0	2
2	0	2	2
2	1	0	1
2	2	1	0

(b) (3 pts) This design is of the form 3^{k-p}. Identify the k and p. What is the resolution of the design ?

3^{4-2}_{III} design. $k = 4, p = 2$, Resolution = III.

(c) (1 pt) Identify the clear effects.

None.

Problem 4 (5 pts)

(a) (2 pts) Suppose an engineer needs to design an experiment using an $OA(N, 2^1 3^4 4^1)$ of strength two. What is the smallest run size N required for this experiment?

72. Least common multiple of 2×3, 2×4, 3×3 and 3×4.

(b) (3 pts) The run size you obtain in (a) is deemed to be too large. Find another OA with a much smaller run size, assuming you are allowed to reduce the number of levels from 4 to 3 or from 3 to 2 for one factor only. What is the run size of the smaller OA? Give the relevant parameters of the OA of your choice but not the actual matrix.

$4 \to 3$.$OA(18, 2^1 3^5)$

Problem 5 (20 pts)
The following questions pertain to an experiment conducted to improve an injection molding process. The goal was to determine process parameter settings for which percent shrinkage would be consistently close to a target value. The control factors are cycle time (A), mold temperature (B), cavity thickness (C), holding pressure (D), injection speed (E), holding time (F) and gate size (G). The noise factors are percentage regrind (a), moisture content (b) and ambient temperature (c). The experiment used a cross array with a 2^{7-4}_{III} design for the control array and a 2^{3-1}_{III} design for the noise array. The design matrix and the percent shrinkage data (in %) are given in Table 1.

(a) (4 pts) Write down the defining relation for the control array (Hint: Express C, E, F, G in terms of A, B, D.)

$I = -ABC = -ADE = -BDF = ABDG = BCDE = ACDF = -CDG = ABEF = -BEG = -AFG = -CEF = ACEG = BCFG = DEFG = -ABCDEFG$.

Table 1: Cross Array and Percent Shrinkage Data, Injection Molding Experiment

							a	−	−	+	+
							b	−	+	−	+
							c	−	+	+	−
Runs	A	B	C	D	E	F	G				
1–4	−	−	−	−	−	−	−	2.2	2.1	2.3	2.3
5–8	−	−	−	+	+	+	+	0.3	2.5	2.7	0.3
9–12	−	+	+	−	−	+	+	0.5	3.1	0.4	2.8
13–16	−	+	+	+	+	−	−	2.0	1.9	1.8	2.0
17–20	+	−	+	−	+	−	+	3.0	3.1	3.0	3.0
21–24	+	−	+	+	−	+	−	2.1	4.2	1.0	3.1
25–28	+	+	−	−	+	+	−	4.0	1.9	4.6	2.2
29–32	+	+	−	+	−	−	+	2.0	1.9	1.9	1.8

(b) (1 pt) Write down the defining relation for the noise array.

$I = -abc$.

(c) (2 pts) What kind of fractional factorial design is the complete cross array design (i.e., express it in s^{k-p} notation)? What is its resolution ?

2^{10-5}_{III} design.

(d) (2 pts) After analyzing the data using the location-dispersion modelling approach, the following models were obtained:

$$\begin{aligned} \ln s^2 &= -2.242 + 2.854 x_F, \\ \bar{y} &= 2.256 + 0.4187 x_A - 0.275 x_D - 0.225 x_G. \end{aligned}$$

What factor setting would you recommend to minimize dispersion ? Which are the adjustment factors ?

$F = -1$ would minimize dispersion. The adjustment factors are A, D, G.

(e) (1+2+2 = 5 pts) Now suppose you would like to use the response modelling approach and re-analyze the data.

(i) How many points should figure in the half normal plot?

The half-normal plot would consist of 31 points, since the total degrees of freedom available from the cross array is 31.

(ii) Which effects do they correspond to ?

These correspond to 7 main effects for control factors, 3 main effects due to noise factors and $7 \times 3 = 21$ interactions between control and noise factors.

(iii) How many of these effects can be clearly estimated ? What type of effects are they ? Explain clearly.

None of the effects are clear in the control array and same is the case for the noise array. Thus, by theorem 10.1, only the control $\times$ noise interactions are clear.

(f) (2+2=4 pts) The following model is now obtained using the response modelling approach.

$$\hat{y} = 2.25 + 0.425x_A - 0.2813x_D - 0.2313x_G + 0.45x_Cx_b - 0.4188x_Ex_b$$

The control $\times$ interaction plots for the significant interactions are given below:

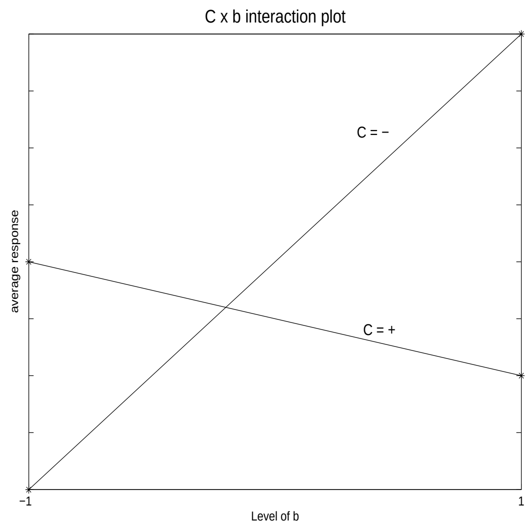

Figure 1 : $C \times b$ interaction plot

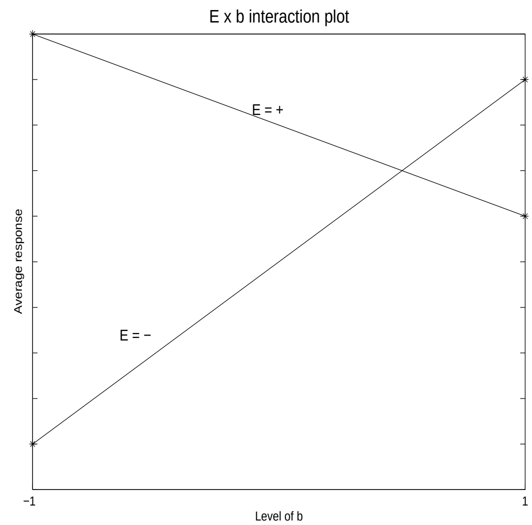

Figure 2 : $E \times b$ interaction plot

(i) Assuming that the noise factor $E(x_b) = 0$ and $Var(x_b) = 1$, obtain the expression for transmitted variance.

The transmitted variance is $Var[(0.45x_C - 0.4188x_E)x_b] = constant - 0.37x_Cx_E$.

(ii) Identify the settings that will minimize the variance in the response using the transmitted variance model and control $\times$ noise interaction plots.

$C = +1, E = +1$.

(g) (2 pts) Note that from the point of view of identifying the factors affecting dispersion and obtaining their best settings, there is a discrepancy between the results obtained using the location-dispersion modelling approach and the response modelling approach. Explain this apparent anomaly. (Hint: Consider the aliasing relations).

$F = -CE$.