MATH 3070 Introduction to Probability and Statistics
Lecture notes
Binomial Distribution

Objectives:

1. Learn the definition of a factorial and how to compute
2. Define a binomial distribution
3. Compute probabilities from a binomial distribution

Factorial

The **factorial** is the product of the first k integers. It is denoted as $k!$. The computation is

$$k! = k \times (k-1) \times (k-2) \ldots \times 2 \times 1$$

There is a special case of the factorial, 0!, which is defined as being 1. This is necessary to make the factorial work in all cases. (See notes on counting theory for a more complete explanation.)

The Binomial Coefficient

Using the concept of the factorial we can derive the **binomial coefficient** which is as follows:

$$\binom{n}{x} = \frac{n!}{x!(n-x)!}$$

where n is a positive integer and x is a nonnegative integer less than or equal to n. This is an example of where the concept of $0! = 1$ is extremely useful and necessary. If this were not the case, there would be instances where the denominator of the term would be zero, and division by zero is not defined.

Definition of a Binomial Experiment

A **binomial experiment** possesses the following properties:

1. The experiment consists of a fixed number n of trials.
2. The result of each trial can be classified into one of two categories: success or failure.
3. The probability p of a success remains constant for each trial.
4. $\text{P}(success) = p$, $\text{P}(failure) = q$, and $p + q = 1$.
5. Each trial of the experiment is independent of the other trials and identical to the previous trials.

Definition of a Binomial Random Variable

The **binomial random variable** indicates the number of successes in n trials of a binomial experiment. The variable may take on any integer value from zero to n.

The Binomial Distribution

Combining all these concepts we can now explore the **binomial distribution**, the probability distribution for the number of success in a sequence of binomial experiments. (Weiss, p. 312)

The probability distribution function for the binomial distribution is

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x} \text{ for } x = 0, 1, 2, \ldots, n$$

Mean and Standard Deviation for a Binomial Distribution

The mean is easily calculated as

$$\mu = n \times p$$

and the standard deviation almost as easily as

$$\sigma = \sqrt{n \times p \times q}$$