Robust Planning for an Open-Pit Mining Problem under Ore-Grade Uncertainty

Daniel Espinoza[1] **Guido Lagos**[2] Eduardo Moreno[3]
Juan Pablo Vielma[4]

[1]Department of Industrial Engineering, Universidad de Chile

[2]H. Milton Stewart School of Industrial and Systems Engineering, Georgia Tech

[3]School of Engineering, Universidad Adolfo Ibáñez

[4]Sloan School of Management, MIT

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1 Open-pit production planning problem

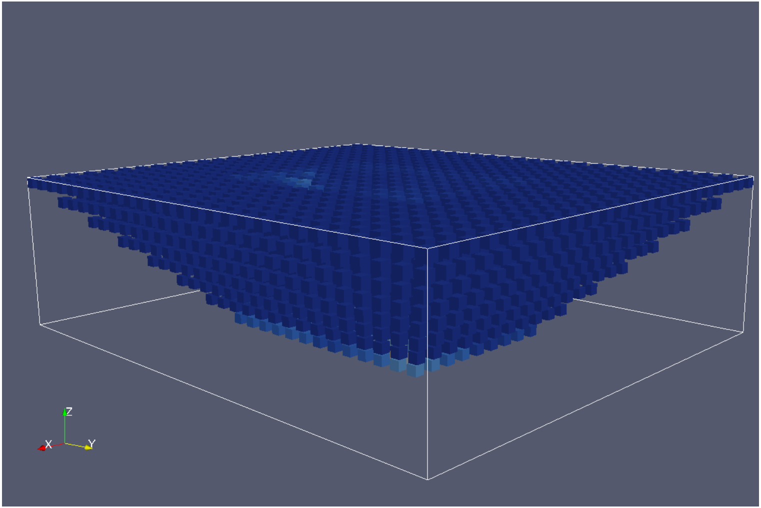
2 Stochastic programming models
- Minimization of VaR
- Minimization of CVaR
- MCH & MCH-ϵ models

3 Computational results

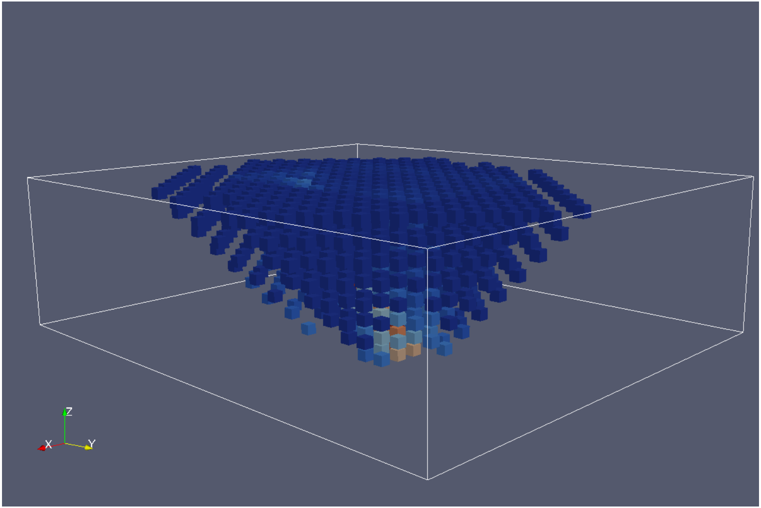
4 Conclusions

Chuquicamata, Chile

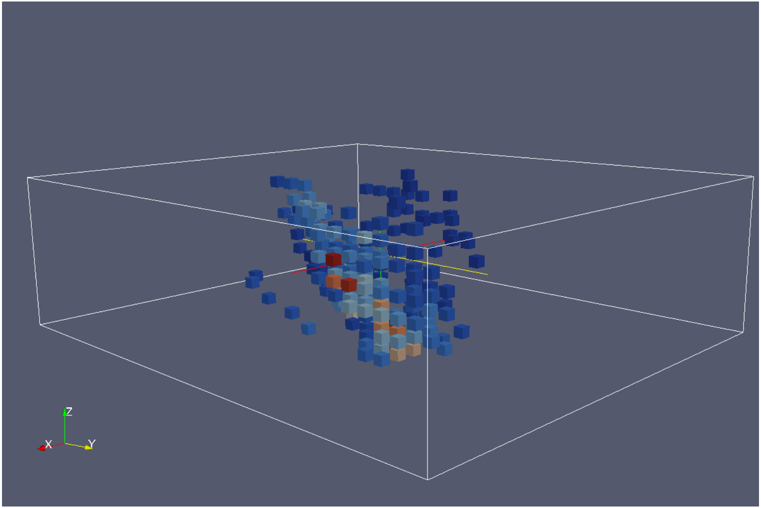
Block model. Color $\sim$ amount of mineral (*ore grade*)



Which blocks to extract?



Between extracted ones, which to process?



Problem without uncertainty

Decisions: for each block $b \in \mathcal{B}$,

extraction decision $x_b^e \in \{0,1\}$, processing decision $x_b^p \in \{0,1\}$

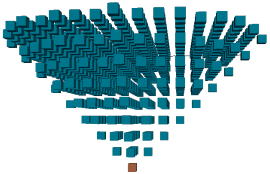
Objective: minimize loss

$$L\left((x^e, x^p), \rho\right) = \underbrace{(v^e)^\mathsf{T} x^e}_{\text{ext. cost}} + \underbrace{(v^p)^\mathsf{T} x^p}_{\text{proc. cost}} - \underbrace{(W^p \rho)^\mathsf{T} x^p}_{\text{proc. profit}}$$

Constraints:

- Precedence constraints
- Extraction & processing capacity

$\Rightarrow$ **Precedence-constrained Knapsack problem** (NP-hard)



UNCERTAINTY IN ORE GRADE ρ

Why? High costs & operation is done only once $\Longrightarrow$ HIGHLY RISKY

- Production plan $(x^e, x^p) \Longrightarrow$ random loss $L((x^e, x^p), \rho)$
- **Objective of our work:**

  Assess different risk-averse approaches

- **Basic hypothesis:** we can obtain an iid sample, as large as we want, of the ore grades vector $\rho \in \mathbb{R}_+^{\mathcal{B}}$

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Minimization of Value-at-Risk (VaR)

- Given $\epsilon \in (0,1)$, for L loss r.v.:

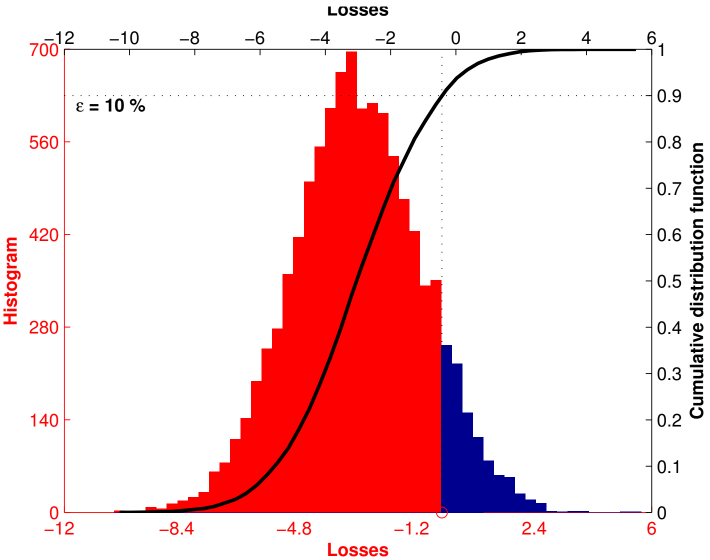
$$\text{VaR}_\epsilon(L) \quad : \quad \text{“}1-\epsilon \quad \text{percentile of losses”}$$

  Is a non-convex risk measure.

- **Model: given risk level ϵ,**

$$\min_{(x^e,x^p)\in X} \ \mathbf{VaR}_\epsilon \left[L\left((x^e,x^p),\rho\right)\right]$$

- **SAA approximation**: take iid sample $\rho^1,\ldots,\rho^N \Longrightarrow$ approximate $\mathbb{P}$ with $\mathbb{P}_N := \frac{1}{N}\sum_i \mathbf{1}_{\{\rho=\rho^i\}}$
  - $\rightarrow$ Consistency: a.s. convergence in objective value and optimal solution set under mild assumptions


Losses
−12
−10
−8
−6
−4
−2
0
2
4
6
700
560
420
280
140
0
Histogram
ε = 10 %
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Cumulative distribution function
−12
−8.4
−4.8
−1.2
2.4
6
Losses

Minimization of Conditional Value-at-Risk (CVaR)

- Given $\epsilon \in (0,1]$, for L atom-less loss r.v.:

$$\text{CVaR}_\epsilon(L) \quad : \quad \text{"mean of } \epsilon \text{ worst losses"}$$

  Is distortion risk measure: coherent, law-invariant & co-monotonic.

- **Model: given risk level ϵ,**

$$\min_{(x^e,x^p)\in X} \ \textbf{CVaR}_\epsilon \left[L\left((x^e, x^p), \rho\right)\right]$$

- SAA approximation: approximate $\mathbb{E}$ using iid sample $\rho^1, \ldots, \rho^N$. Consistency under mild hypothesis.

Modulated Convex-Hull (MCH) & MCH-ϵ models

- **Robust model**: given risk level $\epsilon \in [0, 1]$ and iid sample $\rho^1, \dots, \rho^N$,

$$\min_{(x^e, x^p) \in X} \max_{\rho \in \mathcal{U}_\epsilon} L\left((x^e, x^p), \rho\right)$$

- In $(\Omega_N, \mathcal{P}, \mathbb{P}_N)$, equivalent to minimizing the risk measure

$$\epsilon\, \mathbb{E}(\cdot) + (1 - \epsilon) \underbrace{\text{Worst-Case}(\cdot)}_{=\text{CVaR}_{1/N}(\cdot)}$$

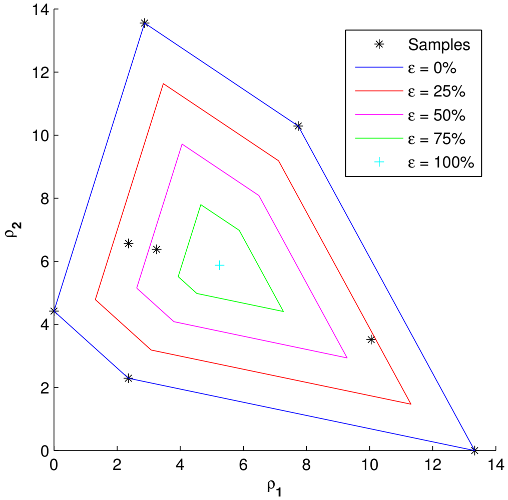
  of losses $L\left((x^e, x^p), \rho\right)$

- **MCH-ϵ model**: minimize risk measure

$$\epsilon\, \mathbb{E}(\cdot) + (1 - \epsilon)\, \text{CVaR}_\epsilon(\cdot)$$

  of losses, which allows to perform a convergence analysis

Example: $\mathcal{U}_\epsilon$ for $N = 8$ samples of ρ in mine with 2 blocks



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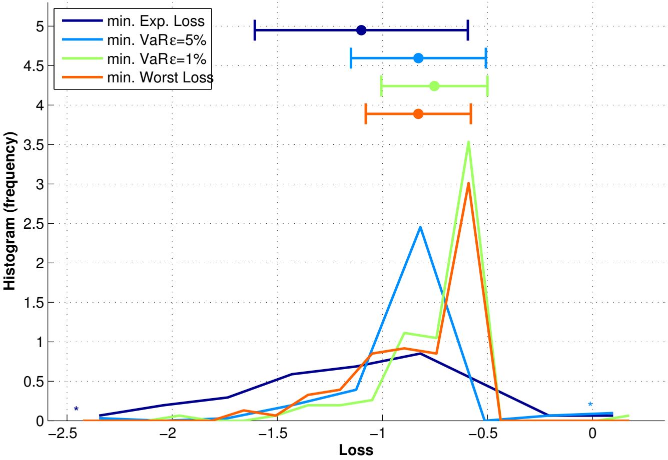
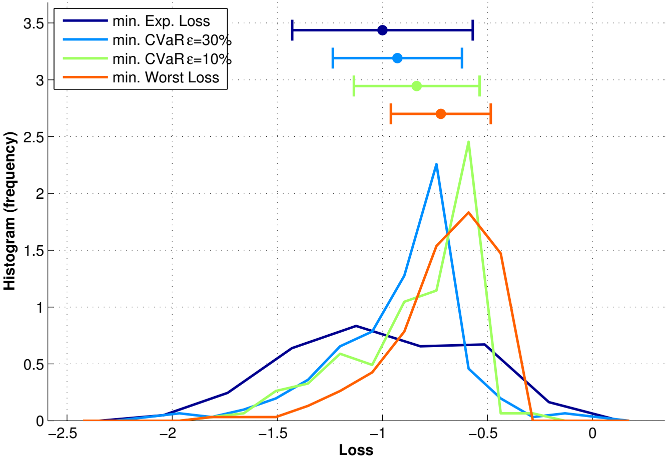
- Vein-type mine with $\approx 20K$ blocks, $20K$ scenarios (*TBsim* algorithm)
- Solve SAA approximation of each model (VaR, CVaR, MCH, MCH-ϵ)
  - taking $N = 50,\ 100,\ 200,\ 400,\ \ldots$ samples
  - for several risk levels ϵ
- ! Also solve *minimization of worst loss* and *minimization of expected loss*
- **SAA approximation $\Longrightarrow$ repetitions algorithm:**
  - **Find statistically optimal solution**
  - **Estimate optimality gap to “true” problem**

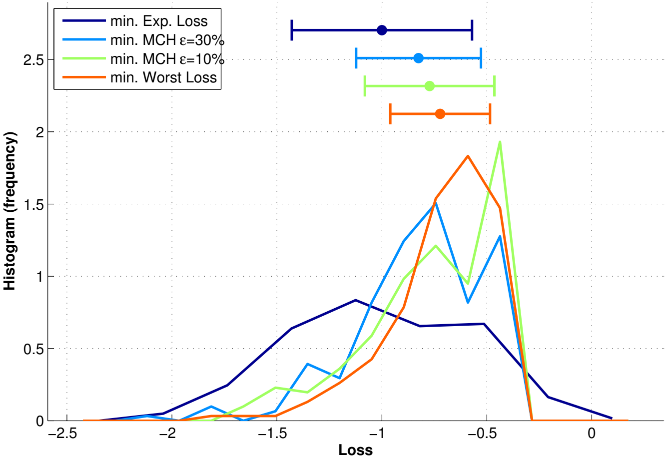
VaR, $N = 100$, in-sample



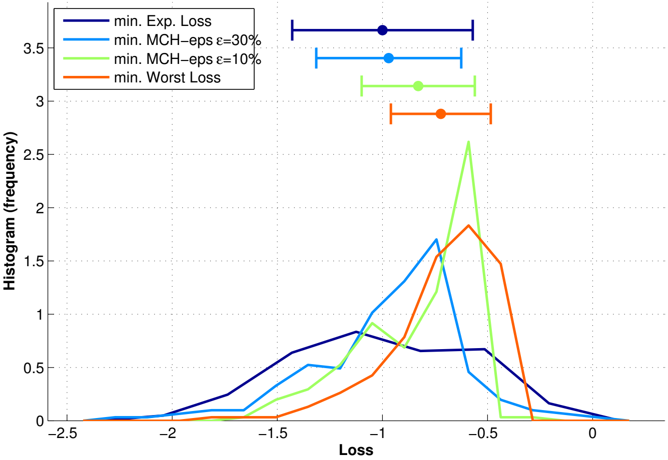
CVaR, $N = 200$, in-sample



MCH, $N = 200$, in-sample

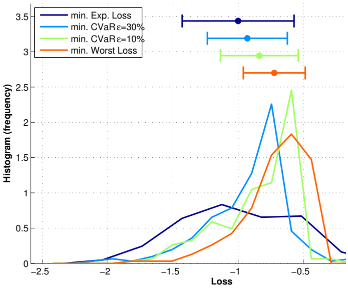


MCH-ϵ, $N = 200$, in-sample

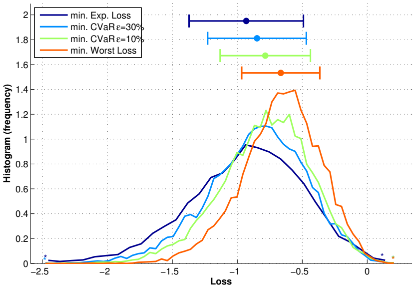


CVaR, $N = 200$, in-sample vs. out-of-sample

(a) In-Sample histogram



(b) Out-of-Sample histogram



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- VaR model: low risk aversion $\Longrightarrow$ inadequate for our problem
- CVaR, MCH & MCH-ϵ models: risk averse performance, controllable with parameter ϵ
- However behavior is not as clear when testing plan out-of-sample
- Slow statistical convergence of VaR, CVaR & MCH-ϵ models: HIGH DIMENSIONALITY!
- **Two-stage variant (extract $\rightarrow$ see ρ $\rightarrow$ process): no losses / fast convergence / not much difference between models**