# Proactive Dynamic DCOPs

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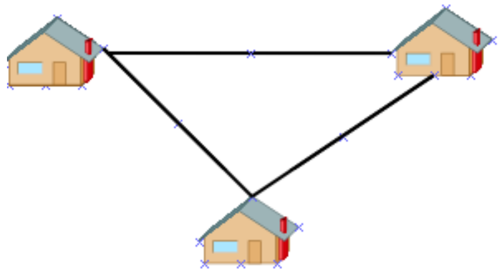
# Content

- **Electric Vehicle Charging Schedule Problem**
- Distributed Constraint Optimization Problems (DCOPs)
- Proactive Dynamic DCOPs
- Algorithms
- Experimental results
- Conclusion

# Electric Vehicle Charging Schedule Problem



- Each house has a charging station for its vehicles
- Each schedule contains a set of **charging times**
- Each vehicle has an fixed **starting time**
- Neighboring houses are connected via transmission lines
- Each transmission line has thermal capacity which limits total amount of energy on the line at a time
- Each house has:
  - Background load
  - Maximal energy usage limit at a time

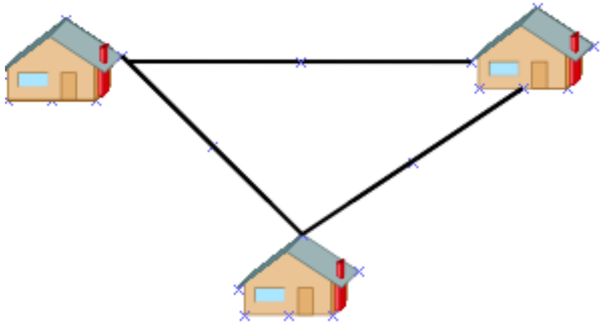
# Electric Vehicle Charging Schedule Problem (cont.)

- Each pair of neighboring vehicles has different preferred charging times
- The goal is to find *best* schedules for all vehicles

| Vehicle A | Vehicle B | Utility |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 3 |
| … | … | … |
| 23 | 23 | 2 |

| Vehicle C | Vehicle A | Utility |
|---|---|---|
| 0 | 0 | 2 |
| 0 | 1 | 10 |
| … | … | … |
| 23 | 23 | 5 |

| Vehicle B | Vehicle C | Utility |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 7 |
| … | … | … |
| 23 | 23 | 0 |



# Why distributed approach?

- Knowledge about neighbors only (privacy concern)
- Take advantage of parallelism
- Remove single point of failure

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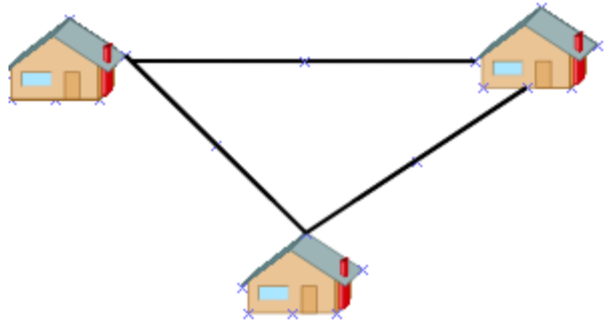
# Distributed Constraint Optimization Problems[1]

- DCOP is a tuple $\langle A, X, D, F, \alpha \rangle$
- $A = \{a_i\}_{i=1}^{p}$ is a set of agents
- $X = \{x_i\}_{i=1}^{n}$ is a set of variables
- $D = \{D_x\}_{x \in X}$ is a set of finite domains
- $F = \{f_i\}_{i=1}^{m}$ is a set of utility functions, where:

  $f_i : \times_{x \in X^{f_i}} D_x \rightarrow \mathbb{R}^+ \cup \{\perp\}$
- $F(\sigma) = \sum_{f \in F, x^f \subseteq x_\sigma} f(\sigma)$ is the sum of utilities across all utility functions
- $x^* = \arg\max_x F(x)$ is the optimal solution
- $\alpha$ is a mapping function

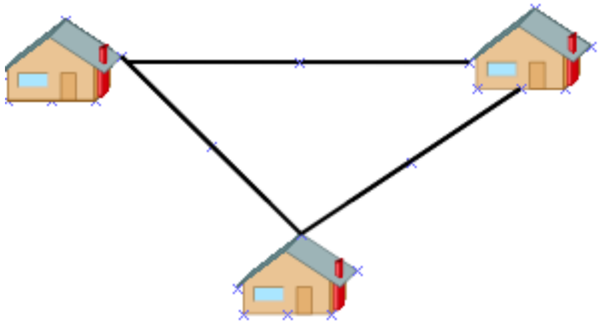
| Vehicle A | Vehicle B | Utility |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 3 |
| … | … | … |
| 23 | 23 | 2 |



# Dynamic DCOPs[2]

- DCOPs only model static problems
- In real-world applications, agents often act in dynamic environments
- Stochastic events are composed of:
  - Increase/decrease in utilities (e.g. changes in preferred time)
  - Addition/removal of variables (e.g. add more vehicle)
  - Change in values of variables (e.g. some charging times are no longer valid)

| Vehicle A | Vehicle B | Utility |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 3 |
| … | … | … |
| 23 | 23 | 2 |



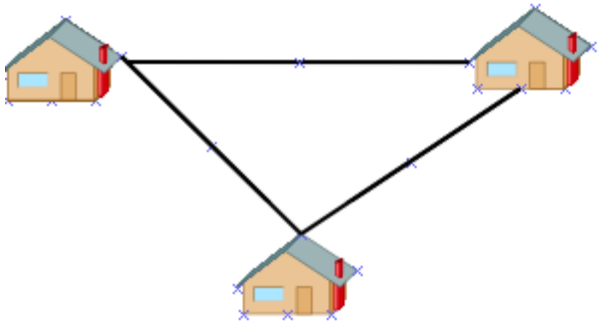
# Dynamic DCOPs (cont.)

- Dynamic DCOPs is a sequence of static DCOPs $\langle p_1, p_2, \ldots, p_n \rangle$ where each followed DCOP changes based on stochastic events:
  - Increase or decrease in value of cost functions
  - Addition or removal of variables
  - Changes in values of variables
- The goal is to find utility-maximal solution for each DCOP in the sequence
- No harder than solving each DCOP separately

# Proactive Dynamic DCOPs

- Dynamic DCOPs does not consider future changes
- Proactive Dynamic DCOPs:
  - Take advantage of future changes
  - Find a solution requires little or no change despite future changes
  - Consider changes of solution after time steps as switching cost
- Fix the solution after a finite time step

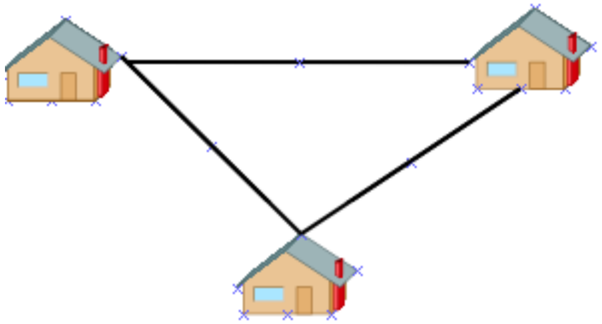
| Vehicle A | Vehicle B | Utility |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 3 |
| … | … | … |
| 23 | 23 | 2 |



# Dynamic Distributed Electric Vehicle Charging Schedule Problem

- Each vehicle has a flexible starting time
- Each vehicle has initial probabilities and transition function for its starting time
- Changes in the charging schedule incur costs

| Vehicle A | Vehicle B | Utility |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 3 |
| … | … | … |
| 23 | 23 | 2 |



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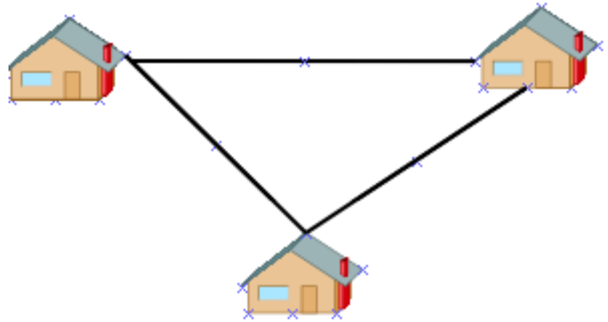
# Proactive Dynamic DCOPs

| Vehicle A | Vehicle B | Utility |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 3 |
| … | … | … |
| 23 | 23 | 2 |



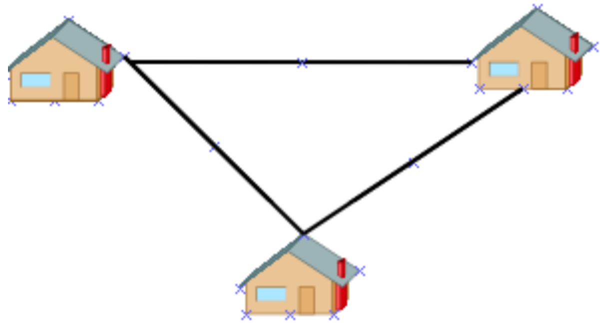
- A Proactive Dynamic DCOP (PD-DCOP) is a tuple

  $$\langle A, X, D, F, T, \gamma, h, c, p_Y^0, \alpha \rangle$$

- $A = \{a_i\}_{i=1}^{p}$ is a set of agents
- $X = \{x_i\}_{i=1}^{n}$ is a set of variables
  - $Y \subseteq X$ is a set of random variables
- $D = \{D_x\}_{x \in X}$ is a set of finite domains
  - $\Omega = \{\Omega_y\}_{y \in Y} \subseteq D$ is a set of event spaces for random events
- $F = \{f_i\}_{i=1}^{m}$ is a set of utility functions
- $h \in \bullet$ is a finite horizon

# Proactive Dynamic DCOPs (cont.)

| Vehicle A | Vehicle B | Utility |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 3 |
| … | … | … |
| 23 | 23 | 2 |



- A Proactive Dynamic DCOP (PD-DCOP) is a tuple

$$\langle A, X, D, F, T, \gamma, h, c, p_Y^0, \alpha \rangle$$

- $T = \{T_y\}_{y \in Y}$ is set of transition functions
  - $T_y : \Omega_y \times \Omega_y \rightarrow [0,1] \subseteq °$ for $y \in Y$
- $c \in °$ is a switching cost
- $\gamma \in [0,1)$ is a discount factor
- $p_Y^0 = \{p_y^0\}_{y \in Y}$ is a set of initial probability distributions
- $\alpha$ is a mapping function

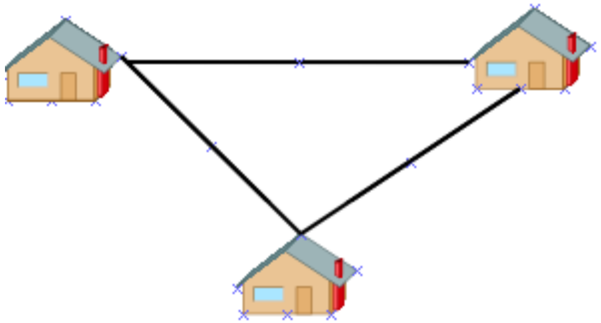
# Proactive Dynamic DCOPs (cont.)

| Vehicle A | Vehicle B | Utility |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 3 |
| … | … | … |
| 23 | 23 | 2 |

- A Proactive Dynamic DCOP (PD-DCOP) is a tuple

$$\langle A, X, D, F, T, \gamma, h, c, p_Y^0, \alpha \rangle$$

- The goal is to find a sequence of h+1 assignments:



$$\mathbf{x}^* = \operatorname*{argmax}_{\mathbf{x}=\langle \mathbf{x}^0,\ldots,\mathbf{x}^h\rangle \in \Sigma^{h+1}} \mathcal{F}^h(\mathbf{x}) \quad (1)$$

Objective functions

$$\mathcal{F}^h(\mathbf{x}) = \sum_{t=0}^{h-1} \gamma^t \left[\mathcal{F}_x^t(\mathbf{x}^t) + \mathcal{F}_y^t(\mathbf{x}^t)\right] \quad (2)$$

Sum of utility functions over first h time steps

$$- \sum_{t=0}^{h-1} \gamma^t \left[c \cdot \Delta(\mathbf{x}^t, \mathbf{x}^{t+1})\right] \quad (3)$$

Switching cost

$$+ \tilde{\mathcal{F}}_x(\mathbf{x}^h) + \tilde{\mathcal{F}}_y(\mathbf{x}^h) \quad (4)$$

Long-term utility at last time step

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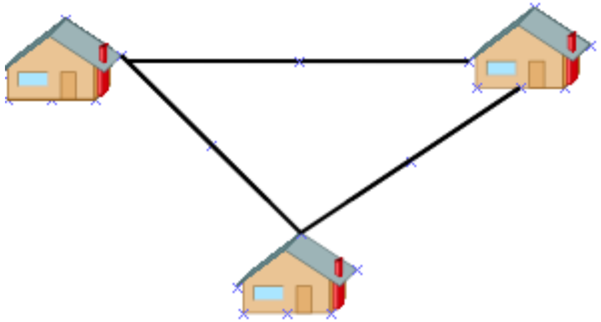
# Proactive Dynamic DCOPs (cont.)

- A Proactive Dynamic DCOP (PD-DCOP) is a tuple

$$\left\langle A, X, D, F, T, \gamma, h, c, p_Y^0, \alpha \right\rangle$$

| Vehicle A | Vehicle B | Utility |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 3 |
| … | … | … |
| 23 | 23 | 2 |



$$\mathcal{F}_x^t(\mathbf{x}) = \sum_{f_i \in \mathbf{F} \setminus \mathbf{F}_Y} f_i(\mathbf{x}_i) \quad (5)$$

Sum of constraints w/o random variables

$$\mathcal{F}_y^t(\mathbf{x}) = \sum_{f_i \in \mathbf{F}_Y} \sum_{\omega \in \Omega_{y_i}} f_i(\mathbf{x}_i|_{y_i=\omega}) \cdot p_{y_i}^t(\omega) \quad (6)$$

Sum of constraints with random variables

$$p_{y_i}^t(\omega) = \sum_{\omega' \in \Omega_{y_i}} T_{y_i}(\omega', \omega) \cdot p_{y_i}^{t-1}(\omega'). \quad (7)$$

Probability of random variable taking a value

# Proactive Dynamic DCOPs (cont.)

- A Proactive Dynamic DCOP (PD-DCOP) is a tuple

$$\langle A, X, D, F, T, \gamma, h, c, p_Y^0, \alpha \rangle$$

$$\tilde{\mathcal{F}}_x(\mathrm{x}) = \frac{\gamma^h}{1-\gamma}\mathcal{F}_x^h(\mathrm{x}) \quad (8)$$

Long-term utility w/o random variables

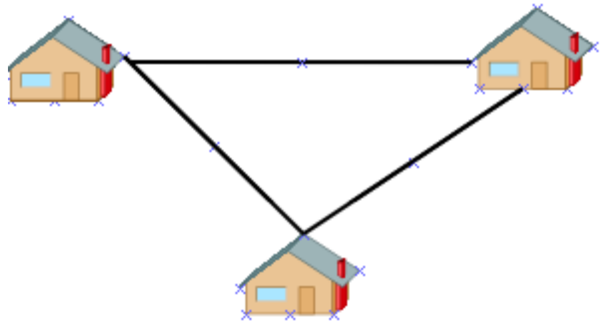
$$\tilde{\mathcal{F}}_y(\mathrm{x}) = \sum_{f_i \in \mathbf{F_Y}} \sum_{\omega \in \Omega_{y_i}} \tilde{f}_i(\mathrm{x}_i|_{y_i=\omega}) \cdot p_{y_i}^h(\omega) \quad (9)$$

Long-term utility with random variables

$$\tilde{f}_i(\mathrm{x}_i|_{y_i=\omega}) = \gamma^h \cdot f_i(\mathrm{x}_i|_{y_i=\omega}) + \gamma \sum_{\omega' \in \Omega_{y_i}} T_{y_i}(\omega, \omega') \cdot \tilde{f}_i(\mathrm{x}_i|_{y_i=\omega'}) \quad (10)$$

Long-term expected utility (Bellman equation)

| Vehicle A | Vehicle B | Utility |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 3 |
| … | … | … |
| 23 | 23 | 2 |



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# PD-DCOPs algorithms

- Exact algorithm:
  - Collapse h+1 DCOPs into a single DCOP
  - Use any off-the-shelf exact DCOP algorithm
- Approximation algorithm:
  - Start with initial assignments: random or heuristics
  - Use local search approach
  - Reuse information and heuristic building pseudo-tree

# Exact algorithm

| t=0 | x1 | x2 | Utility |
|---|---|---|---|
| | 0 | 0 | u11 |
| | 0 | 1 | u12 |
| | 1 | 0 | u13 |
| | 1 | 1 | u14 |

| t=1 | x1 | x2 | Utility |
|---|---|---|---|
| | 0 | 0 | u21 |
| | 0 | 1 | U22 |
| | 1 | 0 | u23 |
| | 1 | 1 | u24 |

| t=2 | x1 | x2 | Utility |
|---|---|---|---|
| | 0 | 0 | u31 |
| | 0 | 1 | u32 |
| | 1 | 0 | u33 |
| | 1 | 1 | u34 |

| Collapsed table | x1 | x2 | Aggregated utility |
|---|---|---|---|
| | 0,0,0 | 0,0,0 | u11 + u21 + u31 |
| | 0,0,0 | 0,0,1 | u11 + u21 + u32 |
| | … | … | … |
| | 1,1,1 | 1,1,1 | u14 + u24 + u34 |

# Exact algorithm (cont.)

| Collapsed table | x1 | x2 | Aggregated utility |
|---|---|---|---|
| | 0,0,0 | 0,0,0 | u11 + u21 + u31 |
| | 0,0,0 | 0,0,1 | u11 + u21 + u32 |
| | … | … | … |
| | 1,1,1 | 1,1,1 | u14 + u24 + u34 |

- After collapsing all the utility tables, we can use any off-the-shelf DCOP algorithms

# Approximation algorithm

- Each variable pick a series of its assignments for every time step:
  - Pick values randomly
  - Solve regular DCOP at every time step
- Then use any local-search algorithm to solve PD-DCOPs

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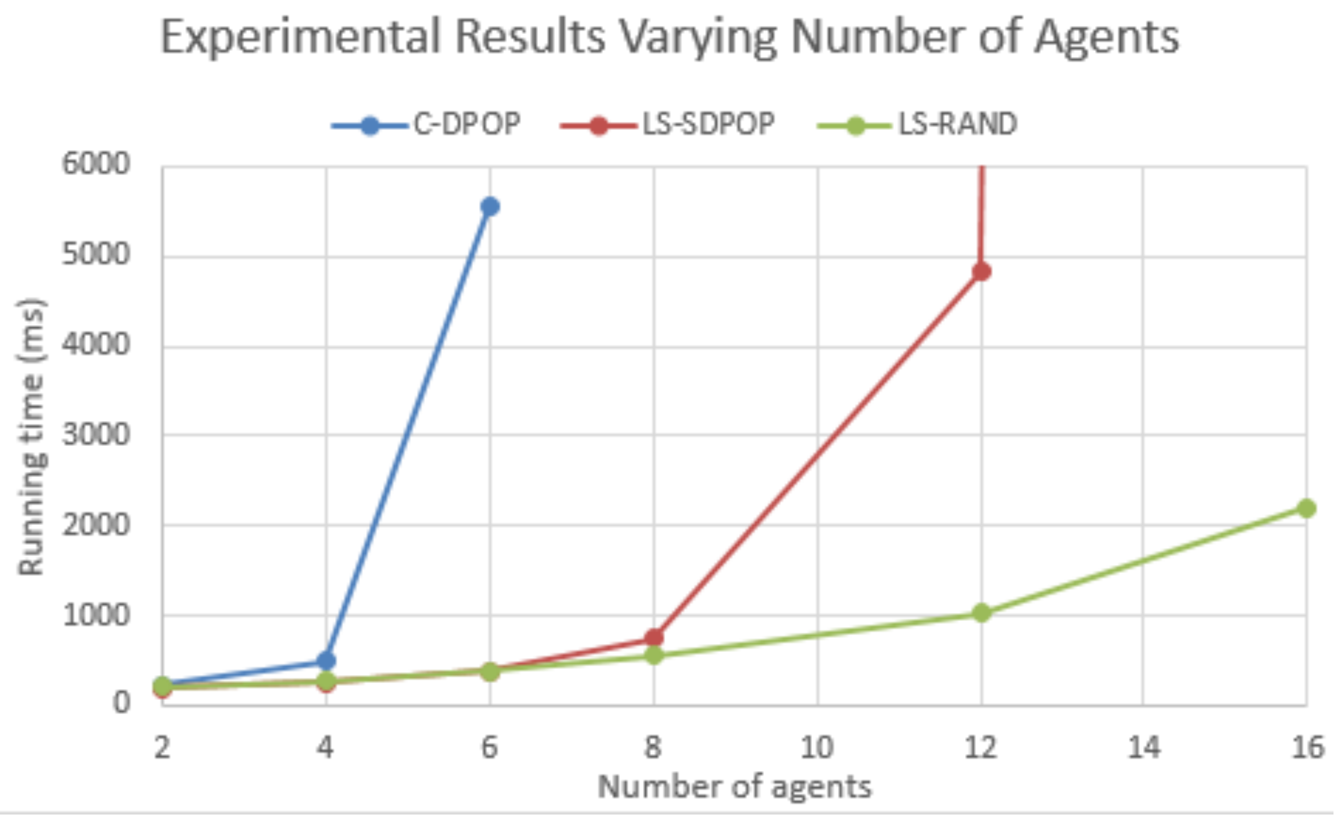
# Experimental results

| \|A\| | C-DPOP | | LS-SDPOP | | | LS-RAND | |
|---|---|---|---|---|---|---|---|
| | time (ms) | $\rho$ | time (ms) | | $\rho$ | time (ms) | $\rho$ |
| 2 | 223 | 1.001 | 197.5 | (207.7) | 1.003 | 203.7 | 1.019 |
| 4 | 489 | 1.000 | 255.7 | (307.3) | 1.009 | 273.4 | 1.037 |
| 6 | 5547 | 1.000 | 382.3 | (456.3) | 1.011 | 385.9 | 1.045 |
| 8 | — | | 739.2 | (838.1) | 1.001 | 556.0 | 1.034 |
| 12 | — | | 4821.6 | (7091.1) | 1.003 | 1092.9 | 1.031 |
| 16 | — | | 264897 | (595245) | 1.033 | 2203.0 | 1.015 |

Table 1: Experimental Results Varying Number of Agents

# Experimental results (cont.)



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# Conclusions

- DCOPs can model some distributed constraint problems
- DCOPs can only static problems, it cannot deal with stochastic events
- Dynamic DCOPs can model series of DCOPs with changes
- PD-DCOP can deal with:
    - Changes in random variables’ values
    - Take advantage of information (initial probabilities, transition functions)
    - Decision variables incur switching costs
- Exact algorithms and approximation algorithms

# References


- [1] Yeoh, W., and Yokoo, M. 2012. Distributed problem solving. *AI Magazine* 33(3):53–65.
- [2] Lass, R.; Sultanik, E.; and Regli, W. 2008. Dynamic distributed constraint reasoning. *In Proceedings of the AAAI Conference on Artificial Intelligence (AAAI)*, 1466–1469.

# Thank you