Proactive Dynamic DCOPs

Khoi D. Hoang

Ferdinando Fioretto

Ping Hou

Makoto Yokoo

William Yeoh

Roie Zivan

Content

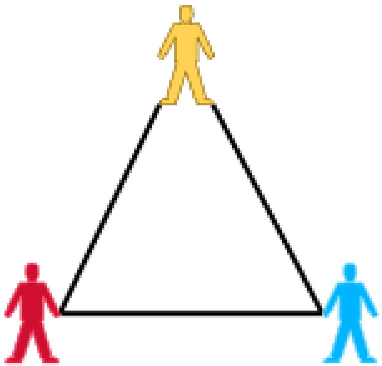
- Distributed Constraint Optimization Problem
- Proactive Dynamic DCOPs
- Algorithms
- Experimental results
- Conclusions

Distributed Meeting Scheduling Problem

Person A	Person B	Utility
8:00	8:00	0
8:00	9:00	invalid
…	…	…
16:00	16:00	0

Person A	Person C	Utility
8:00	8:00	0
8:00	9:00	invalid
…	…	…
16:00	16:00	0

Person A	Utility
8:00	2
9:00	5
…	…
16:00	10



Person B	Person C	Utility
8:00	8:00	0
8:00	9:00	invalid
…	…	…
16:00	16:00	0

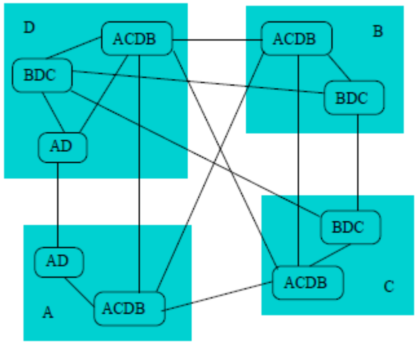
[1] Maheswaran et al., Taking DCOP to the real world: efficient complete solutions for distributed event scheduling, 2004.

Distributed Constraint Optimization Problem (DCOP)

DCOP is a tuple <A, X, D, F, α>

- $A = \{a_1, a_2, \ldots, a_n\}$
- $X = \{x_1, x_2, \ldots, x_m\}$
- $D = \{D_1, D_2, \ldots, D_m\}$
- $F = \{f_1, f_2, \ldots, f_l\}$
- $F(\sigma) = \Sigma f_i$
- $\sigma_{max} = \text{argmax } F(\sigma)$

Person A	Utility
8:00	2
9:00	5
…	…
16:00	10



[2] Modi et al., ADOPT: Asynchronous Distributed Constraint Optimization with Quality Guarantees, 2005
[3] P. Ferreira and A. Bazzan, Distributed Meeting Schedule through Cooperative Mediation: analysing OptAPO' s performance in a complex scenario, 2005

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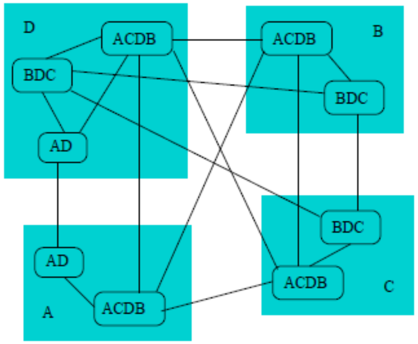
8:00	2
9:00	5
…	…
16:00	10



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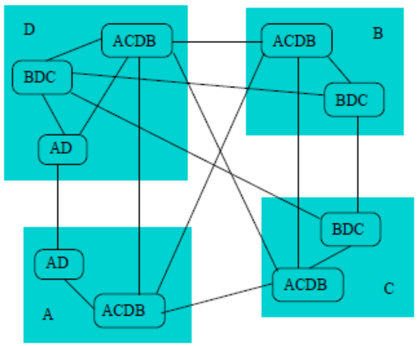
Person A	Utility
8:00	2
9:00	5
…	…
16:00	10



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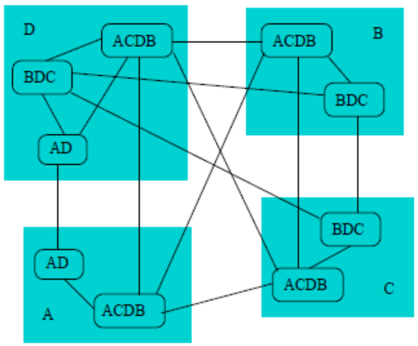
8:00	2
9:00	5
…	…
16:00	10



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Person A	Utility
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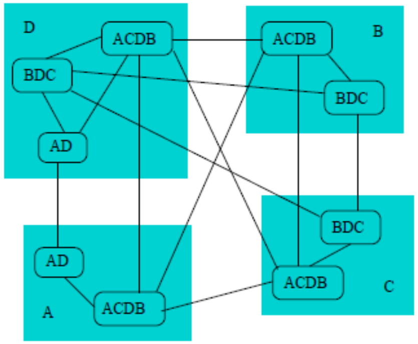

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Proactive Dynamic DCOPs

- Random variables
  - Initial distribution
  - Transition function

Person A	Raining	Utility
8:00	8:00	2
8:00	9:00	9
…	…	…
16:00	16:00	10

Week 0	Raining
	8:00

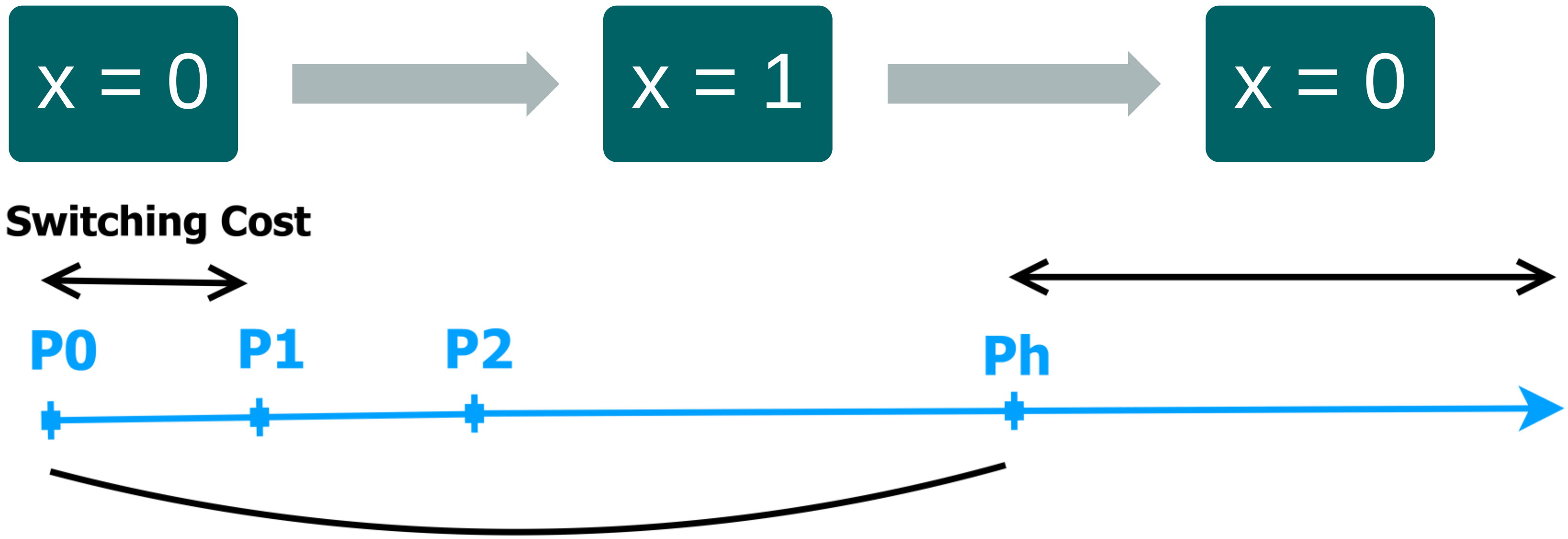
→

Week 1	Raining
	10:00

$$P = \begin{bmatrix} 0.9 & 0.075 & 0.025 \\ 0.15 & 0.8 & 0.05 \\ 0.25 & 0.25 & 0.5 \end{bmatrix}$$

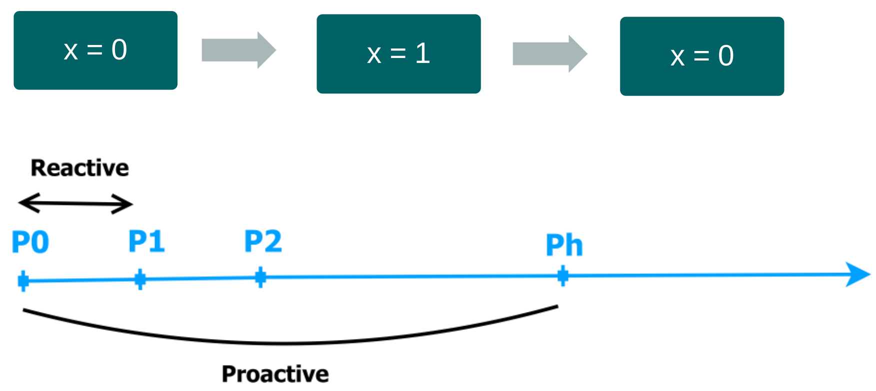
Proactive Dynamic DCOPs

- Switching cost



Proactive Dynamic DCOPs (cont.)

- $Y = \{y_1, y_2, \ldots, y_m\}$
  - Ω: event space
  - p^0: initial distribution
  - T: transition function
- c: switching cost
- h: horizon
- Discounted utility



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Algorithms

- Preprocessing
  - Eliminate random variables
  - Calculate discounted and expected utility
- Exact algorithm
- Approximation algorithm

Preprocessing

- Constraint without random variables

x1	x2	
0	0	a
0	1	b
1	0	c
1	1	d

x1	x2	ts = k
0	0	$\delta^k a$
0	1	$\delta^k b$
1	0	$\delta^k c$
1	1	$\delta^k d$

x1	x2	ts = h
0	0	$\delta^h * 1/(1-\delta) * a$
0	1	$\delta^h * 1/(1-\delta) * b$
1	0	$\delta^h * 1/(1-\delta) * c$
1	1	$\delta^h * 1/(1-\delta) * d$

Preprocessing (cont.)

- Constraint with random variables

x	y	
0	0	a
0	1	b
1	0	c
1	1	d

x	ts = k
0	δ^k [a*prob(y=0) + b*prob(y=1)]
1	δ^k [c*prob(y=0) + d*prob(y=1)]

Preprocessing (cont.)

- Constraint with random variables

x	y	
0	0	a
0	1	b
1	0	c
1	1	d

x	y	ts = h
0	0	u(0,0)
0	1	u(0,1)
1	0	u(1,0)
1	1	u(1,1)

u(0,0)	δ^h * v+ δ[u(0,0)*prob(y=0|y=0) + u(0,1)*prob(y=1|y=0)]
u(0,1)	δ^h * x+ δ[u(0,1)*prob(y=1|y=1) + u(0,0)*prob(y=0|y=1)]

Preprocessing (cont.)

- Constraint with random variables

x	y	ts = h
0	0	u(0,0)
0	1	u(0,1)
1	0	u(1,0)
1	1	u(1,1)

x	ts = h
0	u(0,0)*prob(y=0) + u(0,1)*prob(y=1)
1	u(1,0)*prob(y=0) + u(1,1)*prob(y=1)

- Regular DCOP at every time step

Exact algorithm

- Collapse h+1 DCOPs into a single DCOP
- Use any off-the-shelf exact DCOP algorithm
- Optimal solution

Exact algorithm (cont.)

t=0	x1	x2	Utility
	0	0	u11
	0	1	u12
	1	0	u13
	1	1	u14

t=1	x1	x2	Utility
	0	0	u21
	0	1	u22
	1	0	u23
	1	1	u24

t=2	x1	x2	Utility
	0	0	u31
	0	1	u32
	1	0	u33
	1	1	u34

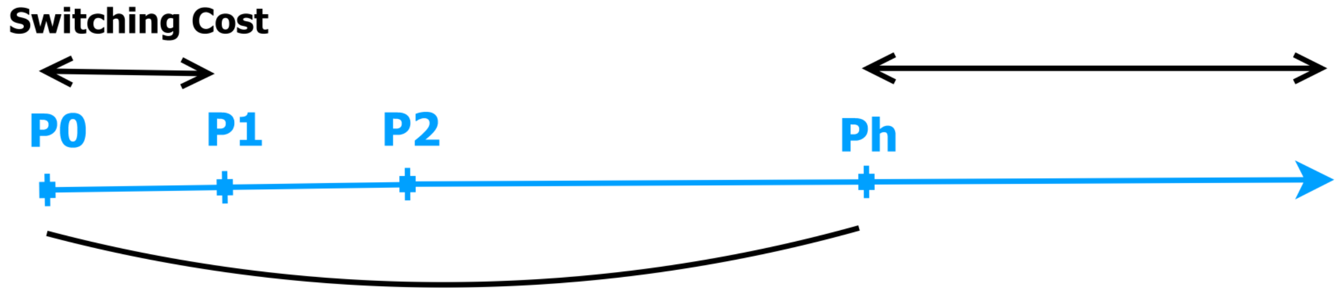
Collapsed table	x1	x2	Aggregated utility
	0,0,0	0,0,0	u11 + u21 + u31
	0,0,0	0,0,1	u11 + u21 + u32
	…	…	…
	1,1,1	1,1,1	u14 + u24 + u34

Approximation algorithm

- Each variable picks a sequence of initial assignments for every time step
- Use any local search approach
- Initial assignments
  - Random
  - Heuristic-based

Approximation algorithm (cont.)

- Initial random assignments
- Heuristic based
  - Solve each DCOP optimally
  - Reuse information at every time step



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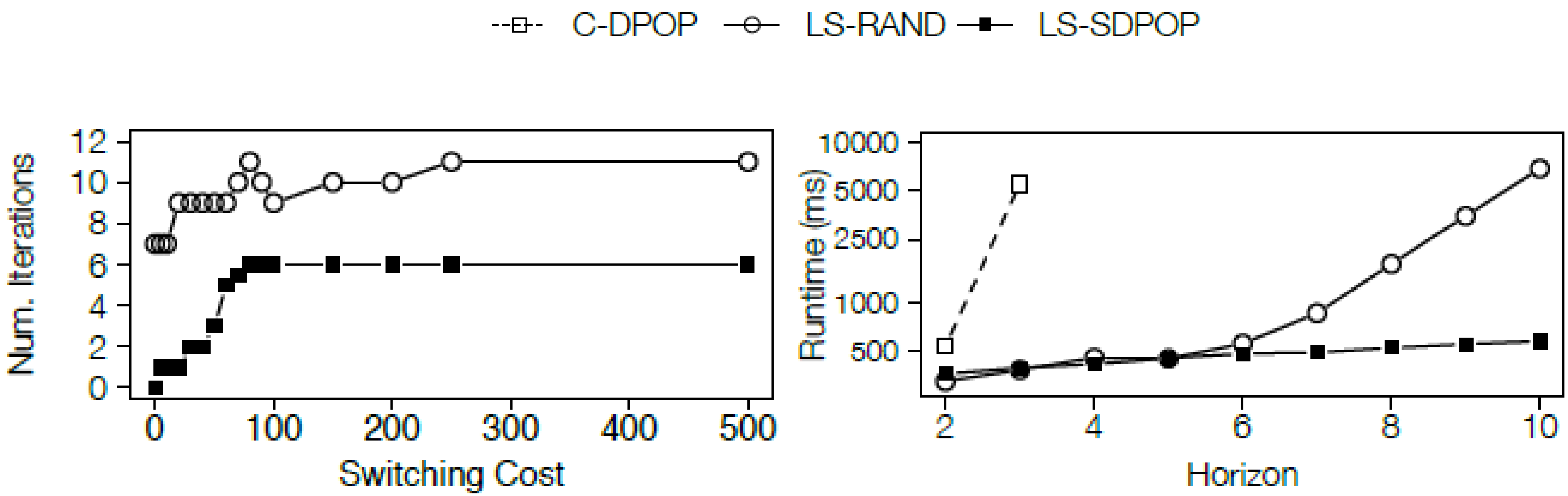
Experimental setup

- Decision variable: 12
- Random variables: 3
- Domain size: 3
- Horizon: 3
- Constraint density: 0.5
- Real distributed system, actual runtime

Experimental results

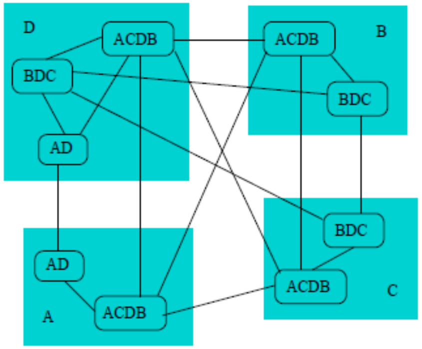


Experimental results (cont.)

|A|	C-DPOP		LS-SDPOP			LS-RAND	
	time (ms)	ρ	time (ms)		ρ	time (ms)	ρ
2	223	1.001	197	(207)	1.003	203	1.019
4	489	1.000	255	(307)	1.009	273	1.037
6	5547	1.000	382	(456)	1.011	385	1.045
8	—		739	(838)	1.001	556	1.034
12	—		4821	(7091)	1.003	1092	1.031
16	—		264897	(595245)	1.033	2203	1.015

Result for DisMSP

- 4 starting times, 2 locations
- 1 meeting per week



|A|	C-DPOP		LS-SDPOP		LS-RAND	
	time (ms)	% SAT	time (ms)	% SAT	time (ms)	% SAT
2	509	100	262	100	271	100
4	4786	100	367	100	399	100
6	—		2651	96	718	93
8	—		71726	96	3249	86
10	—		—		9723	86
12	—		—		15370	86

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Conclusions

- DCOP can model static problems
- Proactive Dynamic DCOP:
  - Prior information on changes of random variables
  - Initial distribution, transition function
  - Switching cost
- Exact algorithm and approximation algorithms

Thank you