

Valuation of Investment and Opportunity-to-Invest in Power
Generation Assets with Spikes in Electricity Price

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Abstract

We address the problem of valuing electricity generation capacity and the opportunities to invest in power generation assets in the deregulated electric power industry. The spark spread option-based valuation framework is extended to take into consideration the electricity price spikes. This framework provides a valuable tool for merchant power plant owners to perform hedging and risk management. With jumps in the value process of power generation capacity, we demonstrate how to determine the value of an opportunity to invest in acquiring the generation capacity and the threshold value above which a firm should invest. We illustrate the implications of price spikes on the value of electricity generating capacity and the investment timing decisions on when to invest in such capacity.

1 Introduction

Restructuring of the electricity supply industries in the United States has been spread out to over twenty states¹ since Federal Energy Regulatory Commission (FERC) issued its Order 888 and 889 in 1996². By breaking up the traditionally vertically integrated generation, transmission, and distribution sectors of an entire electricity industry, legislators hope to introduce market competition into the generation sector and induce efficient mid- to long-term investment in generation capacity through competitive electricity wholesale markets.

To foster a competitive environment for power markets, policymakers felt that it is necessary to dilute the concentration in ownerships of generation assets by large investor owned utility (IOU) companies. With an intention to encourage competition among generating companies and preventing dominating firms from exercising market power, state regulators have either mandated or created financial incentives for the divestiture of generating capacity by the IOUs. These efforts generated a big wave of selling and buying of generation assets among utility and non-utility companies throughout the nation. By the end of 2000, it is estimated that approximately 16 percent of all US utility-owned power generating capacity had been acquired by unregulated, independent power producers (IPPs) ([10]). For instance, in the six New England states, the generating capability ownership shares had changed dramatically from 21,281 megawatts (utilities) vs. 4,809 megawatts (non-utilities) in 1997 to 8,304 megawatts (utilities) vs. 18,358 megawatts (non-utilities) in 1999 ([10]).

In the transactions of transferring ownerships of the divested power plants, public auctions are typically conducted. Establishing the market value of these generation assets has become an

¹As of December 2001, there are 24 states which have deregulated or are in the process of deregulating their electricity supply industries.

²Joskow (2001) provides a detailed discussion on the evolution of industry restructuring and regulatory reforms in the U.S. electricity sector.

important problem for public utility commissions and private firms such as IOUs and IPPs who sell or buy the assets. It is obvious that private firms are highly interested in getting more accurate market assessment on the valuation of divested assets since they need the market guidance for making offers or evaluating bids. To the less obvious end, the interests of public organizations on obtaining market valuation of divested assets come from a fact that the sales proceeds to the IOUs are used to offset the ratepayer's liability to so-called "stranded assets" owned by the IOUs, which are legacies of uneconomic investments made under the old regulatory regime.

The needs for market-based valuation arise not only from the divestiture process of existing generation assets but also from the decision-making processes for building and financing new generation assets. To ensure the security and reliability of a bulk power system and avoid system blackouts, some extra generating capacity reserves are required for buffering the unexpected increases in demands and losses in generating supply due to events like forced outages of equipments. Historically, the nationwide capacity margin (defined as one less the percentage of aggregate system load with respect to total system generating capacity) of the US utilities averaged between 25 and 30 percent during the period from 1978 to 1992, gradually declined to less than 15 percent in 1998, and then reversed back slightly to 15.6 percent³ in 2001 ([11]). As aggregate demands are predicted to grow steadily, new generation capacity has to be added to the system sooner or later in order to maintain a viable power system. In the aftermath of restructuring, IOUs are no longer responsible for long-term capacity planning guided by the capacity margin calculations for the purpose of providing generation adequacy. Instead, the decisions on capacity additions are mostly left in the invisible hands of the electricity markets. Investors and financial companies need to rely more

³The capacity margins vary in the Eastern, Western, and Texas grids in 2001. The largest grid (Eastern) has the lowest margin of 13.9 percent with 501 gigawatts (GW) demand (75 percent of national aggregate demand) and 582 GW supply capacity. The Western grid has a margin of 18.6 percent with 114.8 GW demand and 141 GW capacity ([11]).

and more on market signals to evaluate the investment opportunities in building new generating capacity.

Although the power markets have yet been able to determine the appropriate percentage level of generating capacity margin for guaranteeing electricity supply at all times, they have surely signaled the urgent needs for investments in new capacity or more active demand management through the tremendous power price spikes across the nation. Figure 1 plots the historical electricity prices in two regions, California Power Exchange (Cal-PX: Western grid) and a hub in Pennsylvania-New Jersey-Maryland (PJM: Eastern grid), during the time period of April 1, 1998 to August 31, 2000. The data reveals enormous amount of jumps and spikes in power prices. The widely reported

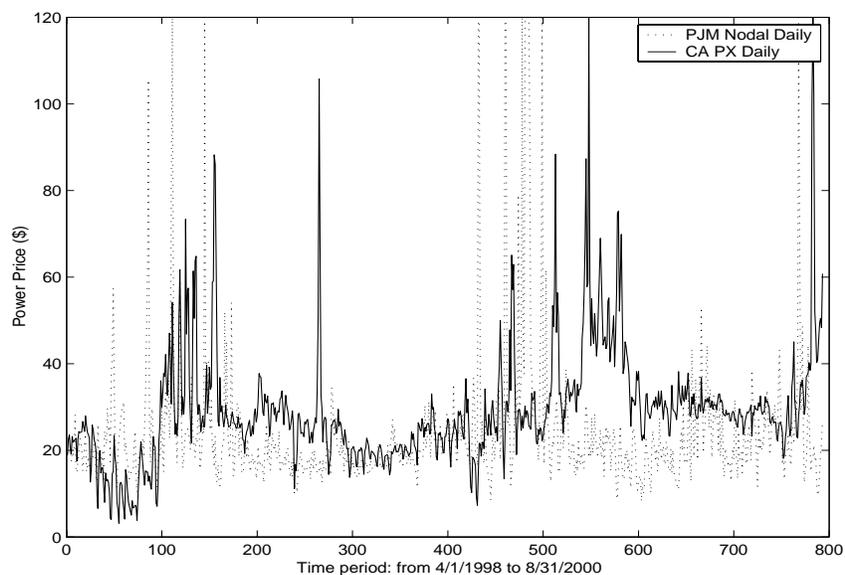


Figure 1: Historical Daily Electricity Spot Prices in California (Cal-PX) and PJM

capacity shortage problem in the California market led to abnormally high power prices and the rolling blackouts in the year of 2000. The state of New York may face similar problems soon since, in a report issued in 2001, the New York ISO predicts demand growth in the next few years to be between 1.2 and 1.4 percent annually and recommends 8600 megawatts (MW) of new generation be built by 2005 in order to meet the increasing demand with the in-state supply but only 450 MW

of new capacity completed the licensing and siting requirements in 2001 ([10]).

As the market-based valuation has become the norm for valuing both existing and new power generating capacity in a deregulated environment (see Risk publications [15] for more discussions), it is imperative to understand its key ingredients. The market-based valuation is based on the ability of replicating the cash flow generated by a power plant with market-traded financial instruments on electricity and some generating fuel subject to market frictions and operational constraints. To prevent arbitrage opportunities, the market value of a power plant shall be comparable to the present value of future profit stream tied to the physical asset. Thus getting an accurate characterization of future profit stream of generating capacity is crucial to market-based capacity valuation.

For a fossil fuel fired power plant, accurate power and fuel price models are central to projecting its future profit flows. When modeling power price, the most important aspect is to capture the jumps and spikes since intuitively the price spikes would be one of the few key factors affecting the value of the merchant power plants, especially those inefficient ones. By explicitly modeling price spikes, we can examine the sensitivity of capacity value to the characteristics of price jumps such as the frequency and the average jump size.

Jumps and spikes in power price also have significant impacts on the values of an opportunity to invest in power generating capacity (i.e. to build a power plant) and on the optimal timing of making such investment. For instance, as we shall see in section 3, a non-foreseeable downward jump in the capacity value process reduces the value of an investment opportunity for acquiring the capacity and shorten the expected waiting time to invest. On the other hand, a foreseeable downward jump in a regime-switching type of value process (as defined in section 3) would restore some of the value of the investment opportunity and make it advantageous to wait longer. Moreover, when

the capacity value process is of the aforementioned regime-switching type, investment in capacity should never occur in the “low” state regardless the investment value; while in the “high” state, investment is made when the capacity value exceeds a certain threshold value.

The remainder of the paper is organized as follows. In the next section, we describe a realistic power price model with jumps and present the spark spread capacity option valuation model based on it. The sensitivity of generating capacity value with respect to various power price characteristics are demonstrated. In section 3, we determine the value of an investment opportunity to acquire some generating capacity when the capacity value evolves according to some jump-diffusion process. We illustrate the threshold capacity value above which a firm should invest and how jumps and spikes in the capacity value process affect the value of the investment opportunity and the investment timing decision. We conclude in section 4 by summarizing several implications of power price spikes on value of investments in power assets and the timing of such investments.

2 Generation Asset Valuation with Spikes in Price

It is well known that in the presence of price uncertainty the traditional discounted cash flow (DCF) approach tends to undervalue a real asset by ignoring the “optionality” available to the asset owner (e.g. Dixit and Pindyck 1994). [1], [4], [8], [16], [17], and [18] provide good surveys and a variety of applications on the real options approach to evaluation of flexibility, strategy, and investment project. In a fully integrated functioning financial and physical market for electricity, the future operating profits of an electricity generating unit can be approximated by a series of electricity financial instruments. Thus one is able to apply financial methods as developed by Black and Scholes (1973) and Merton (1973) to value a power plant via valuing the proper set of financial instruments that match the payoff of the plant. Such an approach is taken in Deng, et. al. (2001) for

obtaining the value of power generating assets. Specifically, they construct a “spark spread option” (defined in section 2.2) based valuation model for fossil-fuel power plants. They demonstrate that the option-based approach better explains the observed market valuation than does the DCF based valuation⁴. However, since their model is based on a simple mean-reverting futures price model without explicitly modeling price jumps, they cannot perform analysis on the effects by price jumps and spikes on the valuation of power generating capacity.

We extend the spark spread valuation model proposed in Deng, et. al. (2001) by adopting a more realistic electricity spot price model which explicitly take into account the jumps and spikes. Based on the mean-reverting jump-diffusion power price process, we demonstrate that the spark spread option based valuation can be implemented via an analytic solution approach outlined in [5]. This greatly shortens the computational time and makes it feasible to perform extensive sensitivity analysis on the generating capacity value with respect to varying power price model parameters. Moreover, by using a spot price model, our valuation model can accommodate a large set of time granularity such as hours, days, weeks and quarters in addition to months, over which the operational options of a power asset are defined when evaluating the operational options. It makes our model more flexible for implementation than a futures-price based model.

2.1 A Power Price Model with Spikes

Due to the silent presence of jumps and spikes in the historical electricity prices, several jump-diffusion processes have been proposed to model electricity spot price in [2], [5] and [13]. One reason for modeling the electricity spot price instead of the forward curve is that the physical power markets for spot trading have been established at more and more geographic locations whereas the

⁴The DCF valuations underestimate, by nearly a factor of four, the sale prices of several power plants divested by a southern California utility in 1998 (See [6]).

financial futures markets are still limited to a small portion of the locations. Moreover, in certain regions, the power spot markets are relatively more liquid than the corresponding futures markets, especially for electricity futures with maturity beyond 12-month. Thus it would be a little easier to calibrate a spot price model than a forward curve model using the market price data, for instance, the spot price model can be calibrated to match only the liquidly traded futures prices but not necessarily the entire forward curve.

To reflect the key features of mean-reversion, jump and seasonality in electricity spot price, we adopt the following price model as specified in [5] for our asset valuation model. Let $X_t = \ln S_t^E$ where S_t^E is the electricity spot price, and let $Y_t = \ln S_t^G$ where S_t^G is the spot price of a generating fuel, e.g. natural gas. The spot price is typically considered as hourly price or daily price obtained by taking the average of twenty-four hourly prices. There are two types of jumps in the log-price process of electricity X_t : a type-1 jump representing an upwards jump and a type-2 jump representing a downwards jump. By choosing the intensity functions properly for the jump processes, we can mimic the spikes in the power prices.

Under regularity conditions, X_t and Y_t are characterized by the following stochastic differential equations (SDE) under a proper probability measure Q ,

$$d \begin{pmatrix} X_t \\ Y_t \end{pmatrix} = \begin{pmatrix} \kappa_1(t)(\theta_1(t) - X_t) \\ \kappa_2(t)(\theta_2(t) - Y_t) \end{pmatrix} dt + \begin{pmatrix} \sigma_1(t) & 0 \\ \rho(t)\sigma_2(t) & \sqrt{1 - \rho^2(t)}\sigma_2(t) \end{pmatrix} dW_t + \sum_{i=1}^2 \Delta Z_t^i \quad (1)$$

where $\kappa_1(t)$ and $\kappa_2(t)$ are the mean-reverting coefficients; $\theta_1(t)$ and $\theta_2(t)$ are the long term means of log-price of electricity X_t and natural gas Y_t , respectively; $\sigma_1(t)$ and $\sigma_2(t)$ are instantaneous volatility rates of X_t and Y_t ; $\rho(t)$ is the instantaneous correlation coefficient between X and Y ; W_t

is a \mathcal{F}_t -adapted standard Brownian motion under Q in \mathbb{R}^2 ; Z^j is a compound Poisson process in \mathbb{R}^2 with the Poisson arrival intensity being $\lambda_j(t)$ ($j = 1, 2$). ΔZ^j denotes the random jump size of a type- j jump in \mathbb{R}^2 ($j = 1, 2$), which is assumed to be exponentially distributed with mean μ_j^j ($j = 1, 2$). Note that the parameters $\kappa_1(t)$, $\theta_2(t)$, $\sigma_2(t)$, $\sigma_2(t)$, and $\sigma_2(t)$ are all functions of time t thus model (1) is capable of capturing the seasonality in electricity and natural gas prices. Price processes in (1) belong to the affine jump-diffusion family as described in [9] and the transform techniques developed in [9] can be applied. The generalized Fourier transform function of the jump-size distribution is

$$\phi_J^j(c_1, t) \equiv \frac{1}{1 - \mu_j^j c_1} \quad (j = 1, 2) \quad (2)$$

where c_1 is a complex constant.

We plot a typical sample path of the electricity price model (1), simulated daily over a little more than three years, with a set of parameters estimated using historical price data at the PJM market in figure 2 (the detail of parameter estimation will be given in section 2.2). The solid curve with dots is the simulated price path and the dashed curve is the PJM historical price. Figure 2 shows that the spot price model (1) captures most of the empirical features of PJM data fairly well.

2.2 Spark Spread Valuation

A “spark spread” option on electricity and a fuel commodity pays the option holder the positive part of the price difference between the electricity price S_t^E and the adjusted fuel cost $K_H S_t^G$ at maturity time t , namely, $\max(S_t^E - K_H S_t^G, 0)$, where K_H is a contract parameter called “strike heat rate”.

Consider a fossil-fuel electric power plant that transforms the fuel into electricity, its economic

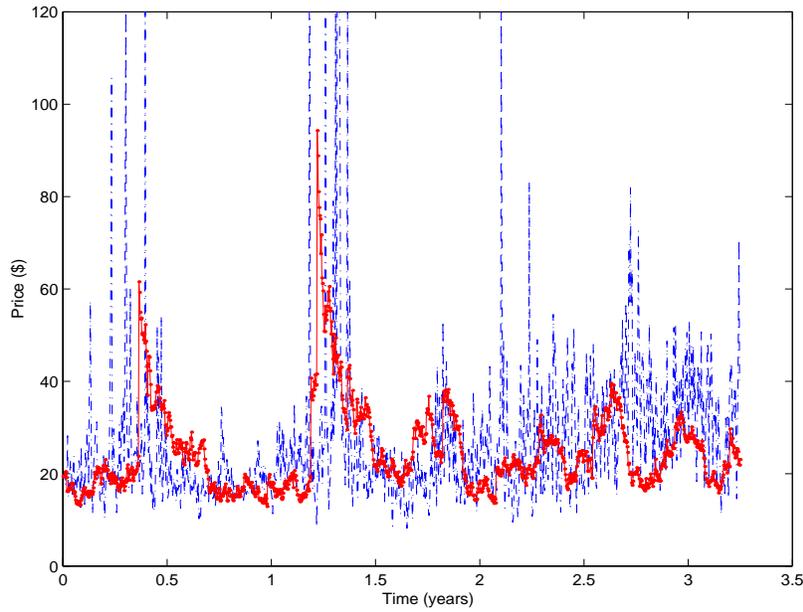


Figure 2: PJM Spot Price: Historical Data vs. Simulated Data

value is determined by the spread between the market price of electricity and the fuel that is used to generate it. The quantity of fuel that a generation asset requires to generate each unit of electricity depends on the asset's efficiency. This efficiency is summarized by the asset's **operating heat rate**, which is defined as the number of millions of British thermal units (MMBtus) of the input fuel required to generate one megawatt hour (MWh) of electricity. The lower the operating heat rate (denoted by H), the more efficient a power generation asset. The operating heat rate of a generation unit varies with the operating conditions (such as output levels) and can be affected by the weather temperature as well. It may even change over time. However, as a simplifying assumption, we will consider operating heat rate of a power plant to be a constant through time in the valuation model (see [7] for valuing a power plant incorporating non-constant heat rate and other operating characteristics).

The right to operate a generation asset with operating heat rate H that burns generating fuel G at time t shall yield a comparable financial payoff, assuming no operational constraints, to that

of a spark spread option with strike heat rate H written on generating fuel G maturing at the same time t . The equivalence between the value derived from the right to operate a generation asset during certain time period and that of a portfolio of appropriately defined spark spread options is the essence of the spark spread valuation model for valuing a generation asset.

In the following analysis, we make several simplifying assumptions (e.g., see [6]) about the operating characteristics of generation assets under consideration.

Assumption 1 Ramp-ups and ramp-downs of a power generating unit can be done with very little advance notice.

Assumption 2 A facility's operation (e.g., start-up/shutdown costs) and maintenance costs are constant over time.

Assumption 3 The fixed-cost associated with starting up or shutting down a power unit can be either neglected or amortized into variable costs.

Assumption 4 A facility's operating heat rate does not change much as the output level varies.

Given the fact that a typical gas turbine combined cycle co-generation plant has a response time (ramp up/down) of several hours and the variable costs (e.g. operation and maintenance) do not vary much over time, these assumptions are reasonable for the purpose of constructing a first-order approximation to the value of a power generating unit. Moreover, Deng and Oren (2003) investigate the impacts by start-up cost, ramp-up time and output dependent heat rate on power plant valuation and they find that the magnitude of mis-valuation of those relatively efficient power plants is small due to making the above simplifying assumptions.

Under these assumptions, we evaluate the right to operate a power generation asset over its remaining useful life by summing up the value of a proper set of spark spread options with maturity

time spanning the same life domain of the asset. This provides us with an estimate to the value of the underlying power asset.

The **time- t capacity right** of a fossil-fuel fired electric power plant is defined as the right to convert K_H units of generating fuel into one unit of electricity by running the plant at time t , where K_H is the plant's operating heat rate. Then, the payoff of one share of time- t capacity right is $\max(S_E^t - K_H S_G^t, 0)$, where S_E^t and S_G^t are the spot prices of electricity and generating fuel at time t , respectively.

Let $u(t)$ denote the value of one share of the time- t capacity right. $u(t)$ can be valued using different electricity derivatives depending on the fuel type. For a natural gas fired power plant, the value of $u(t)$ is given by the corresponding spark spread call option on electricity and natural gas with a strike heat rate of K_H ; while for a coal-fired power plant, it often has a long-term coal supply contract which guarantees the supply of coal at a predetermined price c , and therefore the payoff of the time- t capacity right degenerates to that of a call option with strike price $K_H \cdot c$.

The **virtual value** of a fossil-fuel power plant, denoted by V , is given by integrating the value of the plant's time- t capacity right over the remaining life $[0, T]$ of the power plant, namely,

$$V = K \int_0^T u(t) dt \quad (3)$$

where K is the capacity of the power plant. $u(t)$ is usually a function of the initial prices of electricity and input fuel (X_0 and Y_0), the heat rate, and the maturity t . Under the jump-diffusion spot price model 1, we employ the Fourier transform approach outlined in [5] to value $u(t)$ in closed-form up to a Fourier inversion. V is then obtained by numerical integration.

Under **Assumptions 1-4**, the virtual value less the present value of the future O&M costs is very close to the true value of generating capacity. In what follows we will investigate the

implications by power price spikes on generating capacity valuation and the sensitivity of the valuation with respect to changing parameters in model (1). Since the O&M costs are assumed to be constants, we set them to be zero.

We calculate the virtual capacity value for a hypothetical gas-fired power plant using spark spread valuation with spot price parameters given in Table 1. The electricity price parameters are estimated based on futures price data at PJM with the constraint that the intensities of upwards and downwards jumps are identical, namely, $\lambda_1 = \lambda_2$. For simplicity, we assume all parameters are constants instead of time-dependent functions in the numerical examples (i.e., seasonality is not reflected). Specifically, the theoretical futures prices of different maturities can be computed in closed form. The parameters are then obtained by minimizing the root-mean squared errors between the theoretical and market futures prices of chosen maturities. The natural gas price parameters are estimated in the same fashion using the futures price data at Henry Hub. We assume the instantaneous correlation between log-prices of electricity and gas ρ to be 0.3.

κ_1	4.0399	κ_2	3.6917
θ_1	3.604	θ_2	0.7893
σ_1	0.6369	σ_2	0.488
ρ	0.3		
λ_1	7.665	λ_2	7.665
μ_1	0.1155	μ_2	-0.015
S_1^0	21.7	S_2^0	3.16

Table 1: Spot Price Model Parameters for Capacity Valuation

Suppose the power plant will be operated for 15 years. Its capacity is 300 MW. The risk-free rate is 4.5%. The heat rate Hr ranges from 7.5 MMBtu/MWh to 13.5 MMBtu/MWh. For each value of Hr , we calculate 780 ($= 52weeks \times 15years$) weekly spark spread options values with strike heat rate $K_H = Hr$ and then sum up the options values to get the value of the power plant with operating heat rate being Hr . The computational results indicate that the value of the

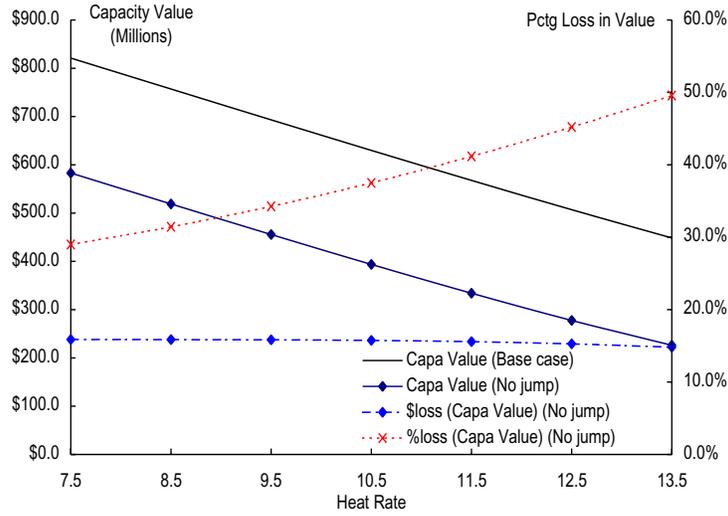


Figure 3: Capacity Value with/without Jumps

hypothetical plant ranges from \$821 millions to \$448 millions as the hear rate varies from 7.5 to 13.5 MMBtu/MWh (See Table 2). The results are also illustrated by the downwards-sloping plain

Heat Rate (MMBtu/MWh)	7.5	8.5	9.5	10.5	11.5	12.5	13.5
Capacity Value (\$Millions)	821.1	756.9	693.1	629.9	567.7	507.0	448.5

Table 2: Capacity Value with Parameters in Table 1 (Base Case)

solid curve in Figure 3. The x-axis represents the heat rate levels and the primary y-axis on the left is for capacity value. We note that these capacity values will serve as the base case for the later sensitivity analysis of the capacity valuation.

2.2.1 Impact of Price Jump on Capacity Value

We first look at how much of the value of a power plant can be attributed to the jumps in electricity price. We re-calculate the capacity value for heat rate ranging from 7.5 MMBtu/MWh to 13.5 MMBtu/MWh with $\lambda_1 = \lambda_2 = 0$ in equation (1). The values are plotted in Figure 3 as the solid curve with diamonds against the primary y-axis on the left. The absolute capacity value loss is a decreasing concave function of the heat rate ranging from \$238 millions ($Hr = 7.5$) to \$222 millions

($Hr = 13.5$) (the dashed curve with diamonds in Figure 3). The absolute value loss in percentage to the base value is an increasing convex function of the heat rate and the percentage losses are plotted as the dashed curve with crosses against the secondary y-axis on the right in Figure 3. We see that the less efficient a power plant the more portion of its capacity value attributed to the jumps in the spot price process. The presence of jumps can make up as much as 50% of the capacity value of the very inefficient power plants (e.g. $Hr = 13.5$ MMBtu/MWh).

We next examine what would happen to capacity value if we do not explicitly model the jumps in the spot price but instead using a large volatility parameter to account for the price volatility caused by jumps. We consider the simple alternative mean-reverting price model obtained by setting $\lambda_1 = \lambda_2 = 0$ in (1). For this alternative model, we keep the non-jump-related parameters (except for the electricity volatility σ_1) the same as those in Table 1 but choose σ_1 so as to match the capacity value for a particular heat rate Hr' under this alternative model with the corresponding capacity value under model (1). We then illustrate the change in capacity value under the alternative power price model for power plants with other heat rates.

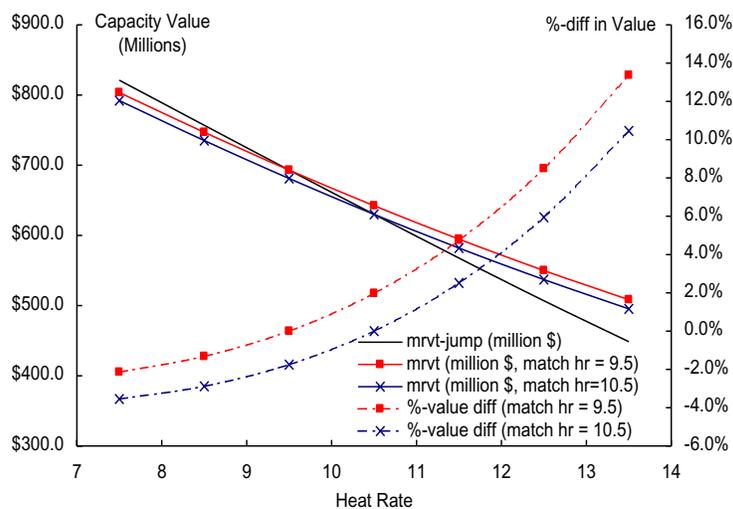


Figure 4: Capacity Value: Jump vs. Mean-reverting

A line search in σ_1 shows that the capacity value at $Hr = 9.5$ is matched for $\sigma_1 = 1.8219$ under the alternative mean-reverting price model. The capacity value at different heat rate levels under the mean-reverting model are plotted in Figure 4 by the solid curve with squares. It intersects the plain solid curve, which is the capacity value curve under price model (1), at $Hr = 9.5$. By lumping the price volatility caused by jumps into the diffusion volatility, the simple mean-reverting price model leads to under-valuation of capacity (up to 2% at $Hr = 7.5$) for efficient generating units but over-valuation of capacity (up to 13% at $Hr = 13.5$) for inefficient units. The percentage of difference in valuation is plotted by the dashed curve with squares. The solid and dashed curves with crosses are for the case where the capacity value is matched at $Hr = 10.5$. The same observations on over-valuation and under-valuation of capacity hold true.

2.2.2 Sensitivity of Capacity Value to Model Parameters

When implementing the spark spread valuation model, we rely on a set of spot price parameters that are estimated using historical power price data. As the parameter estimation errors are unavoidable in all statistical procedures, it is important to understand the robustness and sensitivity of the capacity valuation results with respect to changes in the parameters of (1).

- Volatility σ_1 and correlation coefficient ρ : To see how sensitive the capacity value is to the changing volatility parameter σ_1 of the power price process, we vary σ_1 by $\pm 20\%$ holding other parameters in Table 1 the same and then compute the changes in capacity value with respect to the base case values reported in Table 2. We obtain both the dollar value changes and the percentages of value change. In the left panel of Figure 5, we plot these changes due to a 20% increase or a 20% decrease in the power price volatility σ_1 , respectively. The solid curves show the dollar value changes with respect to the base case in Table 2 on the left-side

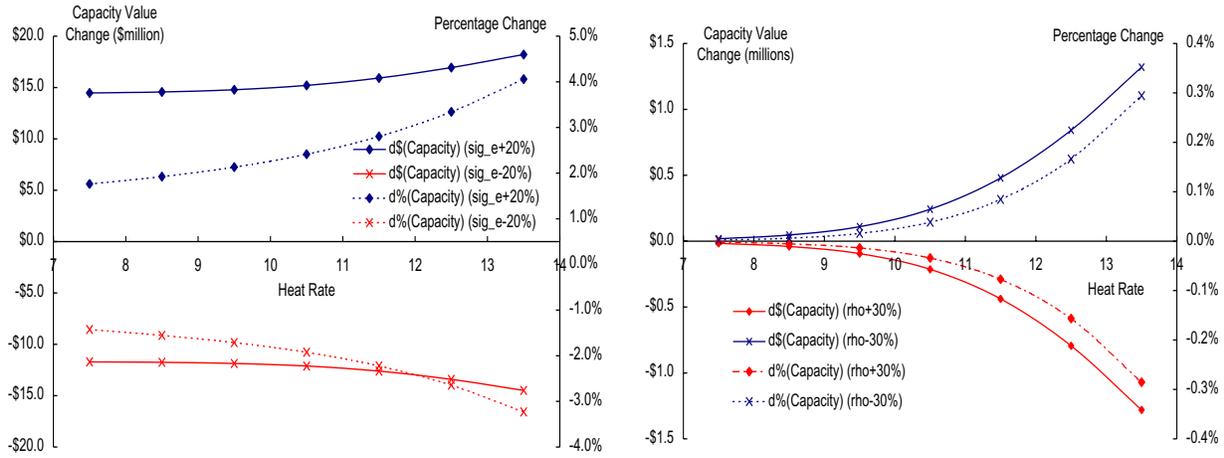


Figure 5: Sensitivity of Capacity Value: Varying σ_1 (left panel) and ρ (right panel)

axis and the dashed curves show the percentage changes with respect to the base case on the right-side axis. Specifically, the solid curve with diamonds illustrates that the increase in capacity value due to a 20% increase in σ_1 is from \$14.5 millions to \$17.2 millions over the heat rate interval [7.5, 13.5]. The corresponding percentage increase in capacity value is from 1.8% to 4.1% (see the dashed curve with diamonds in the left panel of Figure 5). On the other hand, the solid curve with crosses plots the decrease in capacity value due to a 20% decrease in σ_1 which ranges from \$11.7 millions to \$14.5 millions over the same heat rate interval. The corresponding percentage decrease in capacity value is from 1.4% to 3.2% (see the dashed curve with crosses in the left panel of Figure 5).

To get the sensitivity of capacity value with respect to the correlation coefficient between power and natural gas prices, we vary ρ by $\pm 30\%$ and hold other parameters unchanged. The dollar value changes and the Percentage value changes are plotted in the right panel of Figure 5. The range of the dollar value change is from \$0.02 millions to \$1.32 millions and the range of the percentage value change is from 0.002% to 0.29% over the heat rate interval [7.5, 13.5].

While an increase (decrease) in the power price volatility σ_1 causes the capacity value to increase (decrease), an increase (decrease) in the correlation between power and gas prices ρ leads to decreasing (increasing) capacity value. The capacity value is far less sensitive to changing electricity-to-gas correlation than it is to changing power price volatility.

- Mean-reverting coefficient κ_1 : We next examine the effects of changing mean-reverting coefficient of electricity price κ_1 on capacity valuation. When varying κ_1 by $\pm 20\%$, the dollar value change and the percentage value change vary from \$45.6 millions to \$71.8 millions and from 5.6% to 16%, respectively. Solid curves in Figure 6 represent the absolute value changes across different heat rate levels. The dashed curves plot the percentage changes of capacity value.

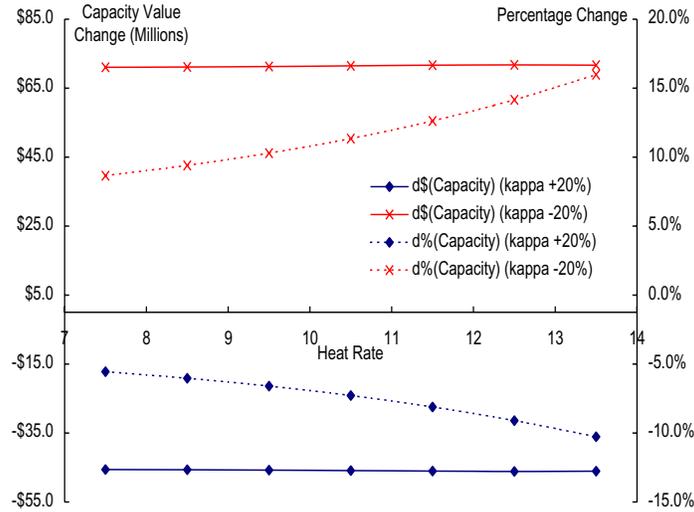


Figure 6: Sensitivity of Capacity Value: Varying κ_1

Similar to the case of correlation coefficient ρ , an increase (or, a decrease) in power mean-reverting coefficient κ_1 leads to a decrease (or, an increase) in capacity valuation. However, changing κ_1 has a much stronger effect on capacity valuation than changing ρ as illustrated by Figure 6 and Figure 5.

- Jump rate λ_1 and average jump size μ_1 : Finally, we investigate how changing jump parameters affects the capacity valuation results. We vary the price jump rate λ_1 and the average upwards jump size μ_1 by $\pm 20\%$. The left and right panels of Figure 7 demonstrate the effects of a 20% variation in jump rate and jump size on capacity value, respectively. A 20% change in either jump rate or jump size causes very significant absolute dollar value changes and the changes are of similar magnitudes. The same is true with the percentage value changes.

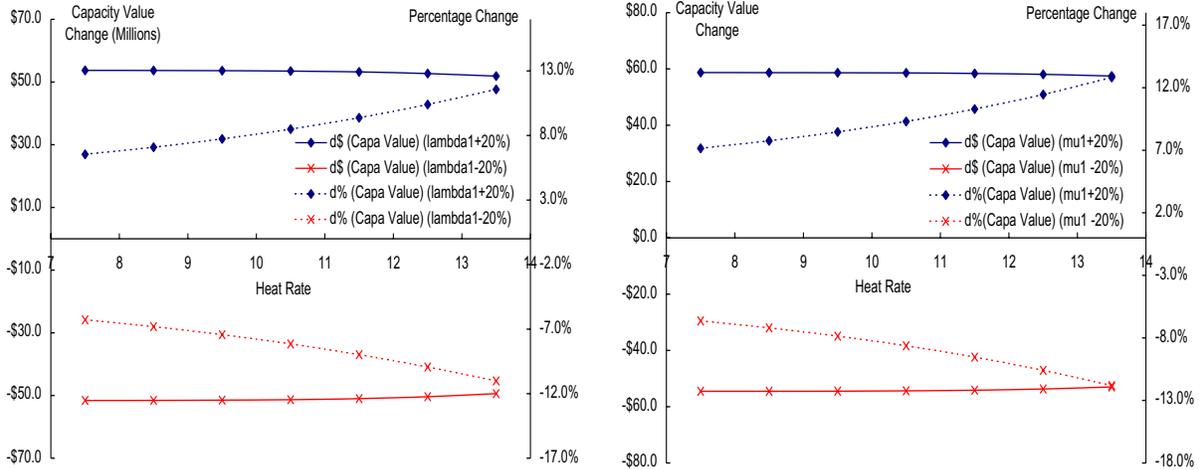


Figure 7: Sensitivity of Capacity Value: Varying λ_1 (left panel) and μ_1 (right panel)

To see which factor, the jump rate or the jump size, plays a more important role in influencing the capacity value, we simultaneously increase λ_1 by 20% and decrease μ_1 by 20% and calculate the capacity value change. Although such simultaneous parameter change shall have no impact on the expect drift rate of the power price, it causes the capacity value to slightly decrease as illustrated by the solid curve with diamonds in Figure 8. The dashed curve with diamonds plots the corresponding percentage decrease in capacity value at different heat rate levels. On the other hand, if we decrease λ_1 by 20% and increase μ_1 by 20% at the same time, then the capacity value is slightly increased. The value increments and the percentages of such increments are plotted by the solid and dashed curves with crosses,

respectively, in Figure 8. The implication of this observation is that power plants are valued more in an environment where power prices contain less-frequent but larger-size jumps than in an environment where power prices have more-frequent but smaller-size jumps.

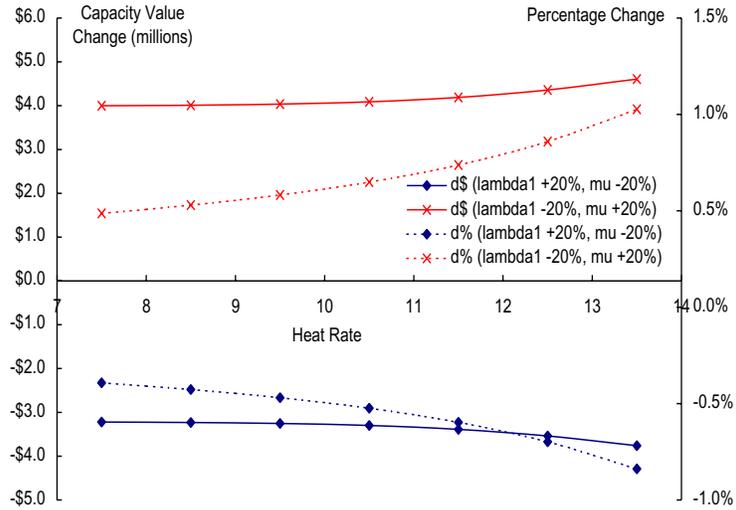


Figure 8: Sensitivity of Capacity: Simultaneously Varying λ_1 and μ_1

3 Value of Investment Opportunity and When to Invest

In the previous section, we value the power generating capacity by viewing the capacity as a real option whose payoff structure can be replicated by a bunch of financial options. We now turn to following related questions: given the opportunity to incur a sunk investment cost K to install the capacity and realize the value V , what is the value of such an investment opportunity and when is the best time to exercise that investment option. The existing literatures suggest that without jumps in the investment value process V the value of an investment opportunity depends on the convenience yield and volatility of the investment value process and a firm should wait to invest until the value V rises to a threshold level V^* . Recall that the value of a power plant at time 0 is

given by (3). The value of a similar plant to be constructed at time t is thus given by

$$V_t = K \int_t^{t+T} u(X_t, s-t) ds. \quad (4)$$

To emphasize the dependence of $u(s-t)$ (for $s \geq t$) on X_t , we replace $u(s-t)$ with $u(X_t, s-t)$ in (4). Since X_t in (1) is a jump diffusion process, V_t defined by (4) is also a jump diffusion process due to the generalized Ito's formula for a jump diffusion process (see [9]). We thus model the investment value process V_t as a jump diffusion process that is similar to X_t .

We consider that the value of investment, V_t , evolves according to a regime-switching process which alternates back and forth between “high” and “low” states through jumps of random size. Such a regime-switching setting is appropriate, for example, in the current deregulated electricity industry. When the spot price of electricity is unusually high, firms are attracted to invest in building new plants. This may result in excess capacity for the subsequent years causing the value process of investing in new capacity to drop into “low” state. The low state will prevail until events such as decommissioning of a nuclear plant or persistent load growth which causes the value process to jump back into “high” state.

Specifically, let $X_t \equiv \ln V_t$ and U_t be a 0-1 valued regime state variable evolving according to a continuous-time Markov chain:

$$dU_t = 1_{\{U_t=0\}} \cdot \zeta(U_t) dN_t^{(0)} + 1_{\{U_t=1\}} \cdot \zeta(U_t) dN_t^{(1)} \quad (5)$$

where 1_A is an indicator function for event A , $N_t^{(i)}$ is a Poisson process with arrival intensity $\lambda^{(i)}$ ($i = 0, 1$) and $\zeta(0) = -\zeta(1) = 1$. $M(t)$ is the corresponding compensated continuous-time Markov

chain defined as:

$$dM_t = -\lambda^{(U_t)}\zeta(U_t)dt + dU_t. \quad (6)$$

We model X_t as the following process

$$dX_t = (r - \delta - \frac{\sigma^2}{2})dt + \sigma dB_t + \iota(U_{t-})dM_t \quad (7)$$

where B_t is a standard Brownian motion in R^1 ; r is the risk free interest rate; δ is the convenience yield of the installed capacity; and $\iota(U)$ ($U = 0$ or 1) is a random variable with a distribution function of $v_U(z)$ representing the jump size associated with the regime switching jumps. As we are primarily interested in the effects of jumps on the value of investment opportunities and the timing of investment, we decide not to model seasonality in the investment value process (7) by setting all parameters to be constants and leave the investigation of seasonality for future work.

Let $F^i(X_t)$ denote the value of an investment opportunity when the regime state is i ($i = 0, 1$).

By applying the Hamilton-Bellman-Jacobi equation in each state i , we have

$$\begin{cases} (r - \delta - \frac{\sigma^2}{2})F'_0(x) + \frac{1}{2}\sigma^2 F''_0(x) + \lambda_0 \int_{-\infty}^{+\infty} [F_1(x+z) - F_0(x)]dv_0(z) = rF_0(x) \\ (r - \delta - \frac{\sigma^2}{2})F'_1(x) + \frac{1}{2}\sigma^2 F''_1(x) + \lambda_1 \int_{-\infty}^{+\infty} [F_0(x+z) - F_1(x)]dv_1(z) = rF_1(x). \end{cases} \quad (8)$$

We conjecture that the solutions have the following form

$$F_i(x) = \exp(\alpha_i + \beta x) \quad i = 0, 1.$$

By further assuming $\iota(0)$ and $-\iota(1)$ are exponential random variables with mean μ_0 and μ_1 , re-

spectively, we simplify (8) to

$$\begin{cases} (r - \delta - \frac{\sigma^2}{2})\beta + \frac{1}{2}\sigma^2\beta^2 + \lambda_0(\frac{e^{\alpha_1 - \alpha_0}}{1 - \mu_0\beta} - 1) = r \\ (r - \delta - \frac{\sigma^2}{2})\beta + \frac{1}{2}\sigma^2\beta^2 + \lambda_1(\frac{e^{\alpha_0 - \alpha_1}}{1 + \mu_1\beta} - 1) = r \end{cases} \quad (9)$$

Intuitively, a firm would only exercise the investment option in the “high” states. In the “low” states a firm is always better off by waiting since it knows the value of investment will eventually jump up. Therefore we have the value matching and smooth pasting conditions in the “high” state $i = 1$ only.

$$F_1(x^*) = \exp(\alpha_1 + \beta x^*) = \exp(x^*) - K \quad (10)$$

$$F_1'(x^*) = \beta \exp(\alpha_1 + \beta x^*) = \exp(x^*) \quad (11)$$

From (9), (10), and (11), we can numerically solve for $(\alpha_0, \alpha_1, \beta, x^*)$. In particular, V^* , the threshold level for triggering investment is given by

$$V^* = \exp(x^*) = \frac{\beta}{\beta - 1}K. \quad (12)$$

In what follows we set the investment cost K equal to 1, $r = 4\%$, $\delta = 5\%$, $\sigma = 0.4$, $\lambda_0 = 1.42$, $\lambda_1 = 2.95$, $\mu_0 = 8\%$, and $\mu_1 = 11\%$. Figure (9) plots the value curve of the investment opportunities $F_i(V)$ and the threshold V^* for investing under these parameters.

To contrast the above results with those from a jump-diffusion value process with two types of random up and down jumps, we introduce the following alternative investment value process

$$dX_t = (r - \delta - \frac{\sigma^2}{2})dt + \sigma dB_t + \sum_{i=0}^1 \Delta Z_t^i$$

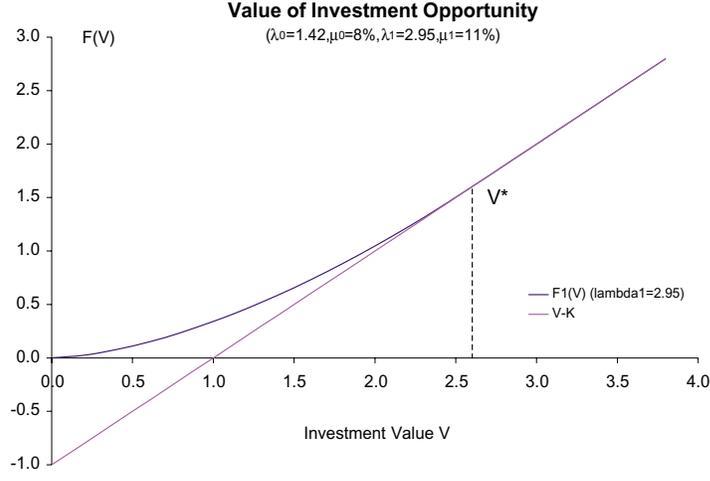


Figure 9: Value of Investment Opportunity

where $X_t = \ln V_t$, Z_t^i is a compound Poisson process in R^1 with a constant intensity of λ_i and jump size distributed as an exponential random variable with mean μ_i ($\mu_0 \geq 0$, $\mu_1 \leq 0$). The Hamilton-Bellman-Jacobi equation for the value function of the investment opportunity, $F(X_t)$, is given by

$$(r - \delta - \frac{\sigma^2}{2})F'(x) + \frac{1}{2}\sigma^2 F''(x) + \sum_{i=0}^1 \lambda_i \int_{-\infty}^{+\infty} [F(x+z) - F(x)] dv_i(z) = rF(x) \quad (13)$$

Conjecture the solution to be of form $\exp(\alpha + \beta x)$, then (13) boils down to

$$(r - \delta - \frac{\sigma^2}{2})\beta + \frac{1}{2}\sigma^2\beta^2 + \sum_{i=0}^1 \lambda_i (\frac{1}{1 - \mu_i\beta} - 1) = r \quad (14)$$

Along with the value matching condition (10) and the smooth pasting condition (11) we can solve for α , β and V^* . Indeed, V^* is again given by (12) but the β is solved from (14) instead.

In the case where the value process has random up and down jumps, firms are induced to invest only when the effect of downwards jumps dominates. Figure 10 shows the values of the opportunity

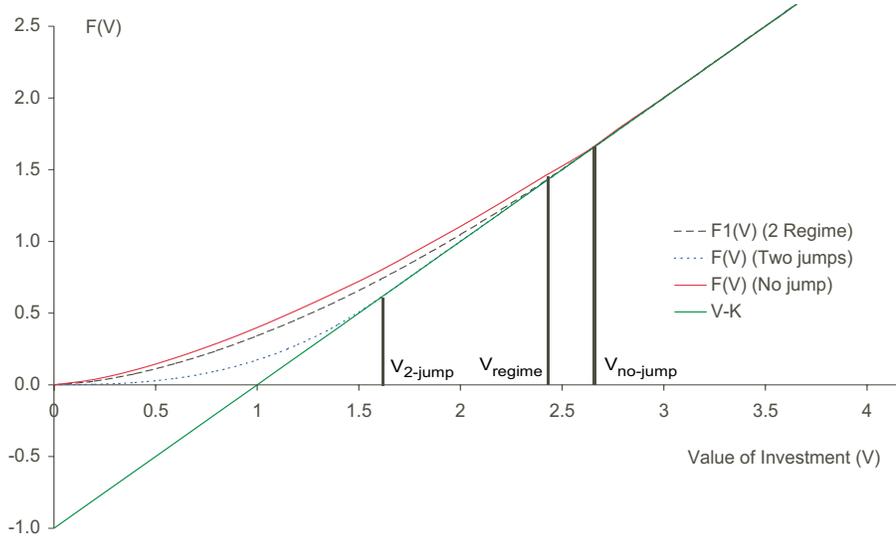


Figure 10: Comparison of Value to Invest under Different Models

to invest under the regime-switching, two types of random jumps and no-jump ($\lambda_0 = \lambda_1 = 0$) cases as well as the investment threshold V^* in the 3 cases denoted by V_{regime} , V_{2-jump} , and $V_{no-jump}$, respectively. For the particular set of parameters, the investment threshold is the highest in no-jump, lowest in random jump, and in between for the regime-switching case.

4 Implication of Jumps on Capacity Valuation and the Value of Investment Opportunity

As we have seen, an accurate power price model is an essential part of the spark spread option based capacity valuation model. A jump-diffusing model is more realistic than a simple mean-reverting model for modeling the power price. A mis-specified power price model could result in more than 10% valuation errors for the not-so-efficient power plants. Those existing power plants

being divested by utility companies are good examples of such inefficient plants. When valuing the divested generating assets, one shall understand that the valuation results are quite sensitive to the power price modeling assumptions. The simple mean-reverting power price model tends to overvalue the inefficient generating assets. However, when it comes to evaluate an investment in building a new power plant which is very efficient, a mean-reverting power price model would undervalue the investment but to a lesser extent than the overvaluation.

Employing the power price model 1, we demonstrate the significance of power price jumps on the value of power assets by setting jump intensities to zero in (1). The computational results indicate that jumps could contribute as much as up to 50% of the value of an inefficient power plant. The managerial insight that comes out of the observations on how price jumps and spikes impact capacity valuation is that, in the near-term when the effects of jumps and spikes on capacity value are significant, even a very inefficient power plant is quite valuable.

In examining the sensitivity of capacity value with respect to changing model parameters, we find that, given the same percentage of variation, the capacity value is very sensitive to changes in mean-reverting coefficient κ_1 , and jump parameters such as the frequency λ_1 and size μ_1 ; modestly sensitive to changes in power volatility σ_1 ; but not sensitive to changes in gas-to-electricity correlation ρ at all. For instance, 30% change in ρ results in less than 0.5% capacity value change over the heat rate range of [7.5, 13.5]. Sensitivity analysis on jump parameters reveals that a power plant is valued more in an environment where power prices contain less-frequent but larger-size jumps than in an environment where power prices have more-frequent but smaller-size jumps.

On valuing an opportunity to invest in power generating capacity, we find that the jumps and spikes in the capacity value process have significant impacts on the investment timing decisions. While the upwards jumps in the price spikes increase the options values embedded in the installed

capacity, we illustrate that the presence of downwards jumps in the value process of an investment reduces the value of the opportunity to invest and induces firms to wait shorter before they invest. We compute the value of an opportunity to invest under three different modeling setups: the regime-switching model for the capacity value process; the jump-diffusion model with two types of random jumps; and no-jump model (i.e., $\lambda_0 = \lambda_1 = 0$). In the case where the capacity value process has random up and down jumps, firms are induced to invest only when the effect of downwards jumps dominates. We can see for the particular set of parameters the investment threshold is the highest in the no-jump case, lowest in the random-jump case, and in between in the regime-switching case.

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