Uncertainty Analysis in Using Markov Chain Model to Predict Roof Life Cycle Performance

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ABSTRACT

Making decisions on building maintenance policies is an important topic in facility management. To evaluate different maintenance policies and make rational selection, both performance and maintenance cost of building components need to be of concern. For roofing sytem Markov Chain model has been developed to simulate the stochastic degrading process to evaluate the life cycle perfornance and cost. [Van Winden and Dekker 1998; Lounis et al. 1999] Taking value in a discrete state space, this model is especially appropriate when scaled rating regular inspections and related mainteance policies are implemented in large organizations. [Van Winden and Dekker 1998]

However, many parameters in this Markov Chain model are associated with variance of significant magnitude. The propagation of these variances through the model will result in uncertainties in predicted life cycle performance and cost results. Without a solid uncertainty analysis on the simulation, decisions based on these simulation results can be unrealiable. In this paper we provide methods to estimate the range of parameter values and represent them in a probabilistic framwork. Monte Carlo method is used to analyze simulation output (life cycle cost and performance) variance propagated from these parameters through the model. These probablisitc informnation can be used to make better informed decisions.

An example is provided to illustrate the Markov Chain model development, parameter identification method, Monte-Carlo uncertainty assessment and decision making with probabilistic information. It is shown that the uncertainty propagating through this process is not negligible and may significantly influence or even change the final decision

KEYWORDS

Uncertainty Assessment, Markov Chain Model, Life Cycle Performance, Life Cycle Cost, Monte Carlo Method.

1 INTRODUCTION

Making decisions on building maintenance policies is an important topic in facility management. To rationalize selections among different maintenance strategies, two difficult aspects should be concerned. First, simulating the degradation process is a complex but necessary task. This process is probabilistic in nature due to the uncertain environmental factors in the service life duration and the variability among each individual. Hence it is desirable to simulate and predict this process in the framework of stochastic models. However, the validation and parameter identification of a stochastic model depends on the availability and format of data, coming either from controlled experiments or field tests. Since the service life interval is tens-of-years in duration, the standard lab test way is to conduct accelerated degradation experiments and make inference on the real service life through the lab testing results.[Masters 1989; Martin et al. 1996] The accessibility to field data is quite limited due to the extended time period. Therefore it is necessary to discuss how to make inference based on limited informaiton with different data format and assess its uncertainty. Secondly, since most of field data is collected by inspectors, the reliability of inspection methods needs to be tested to ensure the constistence and objectivity of inspection process. Executable and reliable methods are under development. [Saunders et al. 1998]. This paper will focus on the first aspect and assume the reliable inspection when field data is used.

In the field of roofing system degradation, there are systematical methods implemented by large scale facility organizations to record the condition of unit in an ordinal scale. In corresponding to the ordinal data inspected at regular time interval, Markov Chain modeling technique is developed to represent and predict the degrading behavior of roofing system. [Van Winden and Dekker 1998; Lounis et al. 1999]. Through the data collected from Dutch GSA, Van Winden and Dekker [1998] justify the feasibility of this model in getting insight in the relation between the maintenance budget and the overall performance, and therefore guiding the budget distribution decisions. In BELCAM project, Lounis el al. [1999] integrate Markov Chain model to optimize maintenance priority assignment for a large roofing system network. It is shown through these researches that Markov Chain model is appropriate in making predicitons and supporting decision making. However, in both cases the parameters (the coefficients in transition matrix) are provided by authors, and there are few discussions on the method to determine their values. If we have sufficient information from the inspection records to determine the transition matrix, the parameter range should be studied due to the statistical nature of data format and the variance among individual units. Furthermore, since the inspection system has not worked for sufficient time period, we do not have enough data to directly infer the parameters. Therefore the method to infer parameters under incomplete or unknown field data needs to be discussed. Besides the parameter identification problem, the uncertainty assessment of model prediction is another unaddressed topic. Van Winden and Dekka [1998] are aware of the fact that the prediction involves too many uncertainties, and thus cautiously point out that the prediction serves as a high level guideline in comparing different policies. In Lounis et al' paper [1999], since the goal is to establish a formal method to assign relative maintenance priority among a number of roofing systems, the uncertainty involved in the Markov chain model would not weaken the algorithm. In other words, although the predictions based on Markov model may be sensitive to the deviation caused by parameter variance, the decision is not too much influenced by this deviation. However, in a more general background, as will be shown in the further exposition through an example, prediction uncertainties can be influential to rational decision makings and therefore should be of concern.

2. MARKOV CHAIN MODEL DESCRIPTION

2.1 Basics

The following discrete scale value for roofing system will be used in this paper.

Condition Rating	Condition/State Description	Damage

1	Excellent	0-10%
2	Very Good	11-25%
3	Good	26-40%
4	Fair	41-55%
5	Poor	56-70%
6	Very Poor	71-85%
7	Failed	>85%

Table 1. Condition Assessment Scales (Modified from Lounis et al. 1998)

Based on such a scale system, if we observe the roofing system at fixed time interval, a random variable S_n can be used to represent the system state at nth observation. S_n takes value from 1 to 7 with certain probability, and the collection of all the random variables $\{S_1, S_2, \ldots, S_n\}$ constitutes a stochastic process. The applicability of Markov chain to represent and predict the roofing system degrading process has been discussed by previous literature. [Van Winden and Dekker 1998; Lounis et al. 1999] Such a Markov process can be described through the following formula: [Ross 2000]

$$S_n = r\,P^{(n)} = rP^n \qquad (1)$$

$$E(S) = \sum_{i=1}^{n} i * S_n(i) \qquad (2)$$

S_n – State vector at time step n
$S_n(i)$ – the ith component in vector S_n, meaning the probability for the system to take value i.
P – Transition matrix, where P_{ij} represents the probability of process going from state i to j.
r – Initial state vector.
$E(S_n)$ – Expected value of system state. It will be used to represent the predicted system state as a result presented to decision makers

In this article, we assume the transition matrix P takes the following form with non zero items p_i (i=1 to n, n is the number of states): $p_{ij}=p_i$, when $j=i+1$; $p_{ij}=1-p_i$, when $j=i$; $p_n=1$;$p_{ij}=0$ otherwise. The implication is that it is only possible for this system to staty in the current state or go to the adjacent next state for the next step. To justify this statement, it is important to assume that the inspection interval to be small enough so that the state transition will not exceed more than 1 rating class between intervals. This is realistic in practice since the inspection interval is about 2-3 years. It is to be pointed out that this assumption is for illustrating simplicity and is not a restrictive requirement for applying the method developed in this paper.

2.2 Maintenance policy representation

The maintenance policy situated in such a scaling system can always be described as "When the system hit state i, recover it back to state j". This action can be represented in matrix format. For example, M in (3) represents the following policy: whenever the state reaches 4 or upper, restore it back to state 2. With a maintanence policy matrix M the system performance can be predicted with (4). [Augenbroe and Park 2002]

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (3)$$

$$s_n = r*(M*P)^n \qquad (4)$$

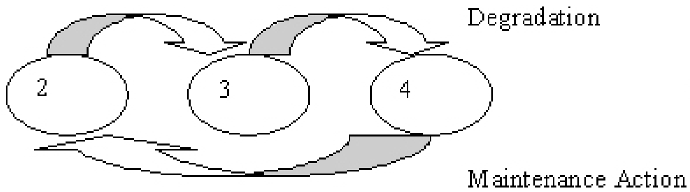
The assumption in this model is that the maintenance takes place right after the inspection. In a realistic world, maintenance action may not be conducted immediately. For example, if maintenance is taken m time periods later after the inspection, the transition matrix can be written as P^m*M.

2.3 Identify the Period of the Markov Chain Model

If there is no maintenance, then system will deteriorate toward the "fail" state eventually. However, with appropriate maintence interventions the system behaves periodically in the long run. For example, with the maintanence policy *M* in (3) and the transition matrix *P* shown in Fig. 1, the system will behave as following:

1->2->3->4->2->3->4->2->….

The system will not hit states higher than 4 because whenever it hits 4, it is restored back to state 2. As shown in fig1, "2->3->4" becomes a period in the long run.



$$P=\begin{pmatrix} 0.95 & 0.05 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.93 & 0.07 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.91 & 0.09 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.88 & 0.12 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.83 & 0.17 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.75 & 0.25 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Figure 1. Periodic Behavior of Aging Process

2.4 Calculating the life cycle performance and cost

Once the period of system is identified, the whole life cycle process can be devided to two :

1) From initial state to the first periodic state. For the example in Fig. 1, it is from 1 to 2.
2) Periodic behavior thereafter. For example in Fig. 1, it is 2->3->4.

The expected time duration of phase 1) is the time for $E(S_n)$ to reach the periodic state. For phase 2), the stationary distribution of the process is established by the following: [Ross 2000]

$$\pi = \pi * (M * P) \qquad (5)$$

We can consider π_j as the proportion of time the system stay in state *j* after the periodic behavior starts. Suppose we consider the life cycle behavior of *N* years, and it takes N_1 years to reach periodic phase, then the system will be in the periodic phase for $N-N_1$ years. For any periodic state *i*, the system will be in it for $(N-N_1) * \pi_j$ years.

Given the information on how many years the system are expected to spend in each state, it is easy to calculate the life cycle performance and cost by the following formula:

$$LCP = \frac{\sum_{i=1}^{n} i * Ti}{N}, LCC = \frac{\sum_{i=1}^{n} Ci * Ti}{N} \qquad (6)$$

T_i – years in state *i*;
N – concerned years;
C_i- The cost associated with the state i to be restored to certain goal prescribed by M. If there is no action associated with this state, it is 0.
LCP – Expected life cycle performance
LCC – Expected life cycle maintenance cost per year ($/Year)

3. ESTIMATE COEFFICIENT IN P MATRIX

If we have sufficient scale rated in-field data, it is straightforward to compute the coefficients in *P* matrix:

$$p_i = \frac{\sum_{t=1}^{m} p_i^t}{t},\; s_i^2 = \frac{\sum_{t=1}^{m} (p_i^t - p_i)^2}{n+1} \qquad (7)$$

However this method subjects to the following constraints: since p_i^t reflects the overall probability of the roofing systems, the observed roofing systems should be of the same kind and expose to similar circumstances. This requirement is too restrictive to make it a realistic method. Another difficulty is that inspected data under certain maintenance policy may never reach certain states. For example, roofing systems shown in Fig. 1 will never reach state 5, 6 and 7, and the data collection work from this organization will not directly yield useful data to estimate p_5 and p_6.

It is more practical to infer parameters from a service life curve, coming either from lab test or expert opinion. When it comes to expert opinion, intuiation is to let expert directly estimate the coefficient in Markov chain model. However, this estimation requires the expert to be farmiliar with both the assumptions of Markov chain model and the specific transition behavior. Therefore it is more reasonable to let experts serve the information similar with lab test result: how does a certain kind of roofing system behave over time? The expert or lab tester should be asked to supply a curve similar to "expert judgement" curve shown in Fig. 2.

Given this curve coefficients in *P* can be derived based on (8) [Abraham and Wirahadikusumah 1999; Ractutanu and Sundquist 2002]. The idea is to find the parameters set so that the prediction based on this set best matches the observations.

$$Min(\sum |Y(t) - E[S(t,P]|) \qquad (8)$$

Y(t)- estimated condition state at time *t*, provided by expert.
E[S(t,P)] - Expected value of roofing condition at time t as predicted by the Markov Chain model with probability matrix *P*, compute from (1) and (2).

To illustrate the idea, given the expert judgment on service life condition under natural degradation as the sequence of points shown in Fig. 2, we can optimize (8) and derive the *P* matrix as shown in (9). The comparison of expert judgement and model prediction is demonstrated in Fig. 2.

$$P = \begin{pmatrix} 0.95 & 0.05 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.93 & 0.07 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.91 & 0.09 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.88 & 0.12 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.83 & 0.17 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.75 & 0.25 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad (9)$$

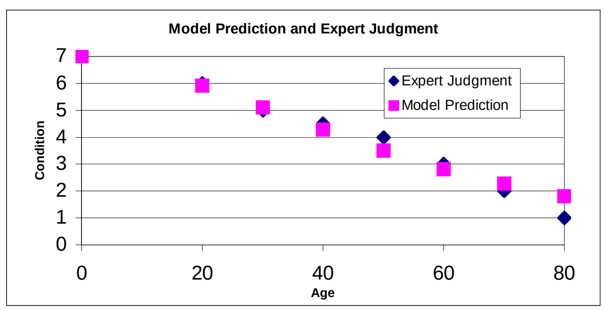


Figure 2. Expert Judgment Data and Model Prediction

4. Uncertainty Assessment through Monte-Carlo Method

This section is to evaluate the uncertaitnies propogated through the coefficients in *P* and *C* matrix. These parameters can not be accurately measured with negligible variance; rather, they are estimated or inferred from estimations. They will not take the exact estimated value but take values around it with some probablisitc distributions. The most commonly used technique in evaluating the variance of simulation results casued by parameter uncertainties is Monte-Carlo method: first associate all parameters with certain probability distributions; then their values are randomly generated for simulation. It is mathematically proved that the prediction result approaches the normal distribution no matter how the assumed parameter distributions look like. Practically, it is suggested that the minimum number of simulation required is 60-80 times. [Lomas and Eppel 1992]. In this paper we will estimate the varianace of *LCC* and *LCP* propogated by matrix *C* and *P*.

First we need to estimate the input parameter distributions. If the service life curve comes from lab tese, its variance should be supplied. If it comes from the expert estimation, the same expert should be asked to also provide the 95% confidence interval boundary. It is more natural for an expert to give the boundary corresponding to the certain state, that is to say, he will normally judge that it will take 20 years for the system to fall to "Very good" status, and he is 95% sure that it will take 18 to 22 years for this transition. To represent it, we would denote the estimation point as *(i, n)* where i is state and n is the time interval to reach it. Therefore the expert judgment is in fact a sequence of points. Suppose for every *i* with 95% probability it takes 0.9*n* to 1.1n years to reach this state, then the probabilistic distribution for each point would be $(i, N(n, (0.0561n)^2))$.

To estimate the variance of C_{ij} depends also on the expert opinion when statistical data is not available. If C_{ij} reasonably represents the budget cost from state I to state j, then the probabilistic distribution of C_{ij} represents the actual cost from state I to state j. For example, if we use the scale provided by Lounis and the material is evaluated as being in state 3, the damage extent can be any where between 56%-70%. The actual cost to improve the material from state 3 to state 4 (which is defined as 41%-55%) will vary from case to case. Suppose the expert judges that with 95% probability that the cost will fall into the interval of $(0.8C_{ij}, 1.2C_{ij})$, then we can calculate that $C_{ij} = N(\overline{C_{ij}}, (0.0612\overline{C_{ij}})^2)$ by elementary probability knowledge.

Monte Carlo simulation can be performed based on the given information. After the simulation *LCC* and *LCA* cen be represented in a probabilistic distribution. An example will be demonstrated later.

5. DECISION MAKING

To illustrate the previous algorithm and how the probabilistic information influence the decision making, a system with an estimated service life as Fig. 1 is studied. Its corresponding *P* matrix has been inferred earlier. Its maintenance cost matrix is as (10), and the coefficients are the percentage of the cost of new roofing system. [Van Winden and Dekker 1998]

$$\overline{C} = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 0 & 0 & 0 & 0 & 0 \\ 15 & 4 & 2 & 0 & 0 & 0 & 0 \\ 20 & 15 & 10 & 10 & 0 & 0 & 0 \\ 50 & 40 & 25 & 15 & 10 & 0 & 0 \\ 65 & 55 & 37 & 23 & 15 & 10 & 0 \\ 80 & 70 & 50 & 30 & 20 & 15 & 10 \end{pmatrix} \qquad (10)$$

Suppose the uncertainty assessment by the expert is conducted and the sequence of service life is estimated as $(i, N(n, (0.0561n)^2)$, while the cost is estimated as $C_{ij} = N(\overline{C_{ij}}, (0.0612\overline{C_{ij}})^2)$. Suppose the concerned time period is 60 years, and the decision maker wants to develop some understanding on the cost and performance between the following two policies: 1) *Policy 1*: Do nothing; 2) *Policy 2:* maintenance policy represented by *M* in (3).

For the first policy, the life cycle cost is the fixed as A. *LCC=A/60*. The performance index of the system can be estimated directly through the service life estimation without resort to Markov Chain model. The result is shown as following Fig.3. $LCP=N(5.17,0.31^2)$.

For the second policy, the simulation results are shown in Fig. 4 and 5. They can be represented as *LCP=N(5.53,0.34)* and *LCC=N(11.17,1.33)*.

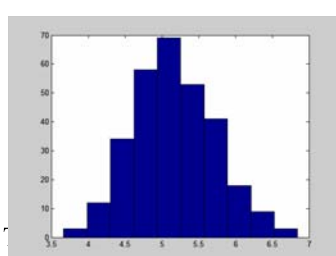


Figure 3. LCP for Policy 1

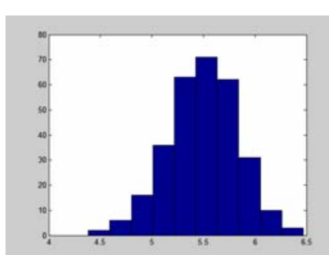


Figure 4. LCP for Policy 2

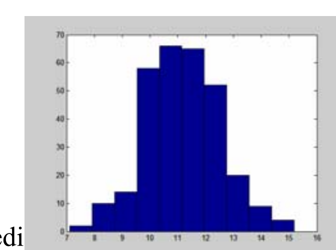


Figure 5. LCC for Policy 2

In the following it is supposed that the expected budget is 10/year and the required performance is 5, we will illustrate how the additional probabilistic information will affect the decision making in selecting the policies. Therefore the concerned consequence of each maintenance policy can be written as: $\{\{C_1;\ not\ C_1\};\ \{C_2\ ;not\ C_2\}\}$ where C_1 represent the event *{LCP>5}* and C_2 represent the event *{LCC<10}*. Table 2 and 3 show the results both from probablistic and deterministic method.

	Deterministic		Probabilistic	
	LCP	*LCC*	*LCP*	*LCC*
Policy 1	*5.17*	*0*	$N(5.17,0.31^2)$	*0*
Policy 2	*5.53*	*11.17*	$N(5.53,0.34^2)$	$N(11.17,1.33^2)$

Table2. Result from deterministic and probablistic method

	P{C1: (LCP>5)}	*P{C2:(LCC<10)}*
Policy1	*0.7083*	*1*
Policy2	*0.9405*	*0.8105*

Table 3. Inference from probabilistic information

Without uncertainty analysis we will reject policy 2 immediately because its *LCC* value exceeds required value. However, by looking into the probablisitc information in table 3 decisions will be supported from utility theory. There are systematic methods to construct the utility value but in this paper we consider them as given. [De Wit 2001] In this simple example, we assign the same utility value, 1 when the criterion is satisfied; and 0, when it is not satisfied. Informally speaking, the utility values represent the relative importance decision maker assign for the satisfaction of different criterion.

With the quantified utility value, we can formulate our problem as a simple decision making problem: the action space is *{Policy1, Policy2}*; the consequence of the action is C_1 or C_2, shown in Table 3; and the utility value of each consequence is given as $U(C_1)=U(C_2)=1$, $U(not\ C_1)=U(not\ C_2)=0$. Thus the utility value can be computed as following:

$$EU(Policy)=(U(C_1)*P(C_1)+U(C_2)*P(C_2)+U(not\ C_1)*P(not\ C_1)+U(not\ C_2)*P(not\ C_2)$$

Where *EU(Policy)* : expected utility value of certain action.

$$EU(Policy1)=0.7083*1+1*1=1.7083$$
$$EU(Policy2)=0.9405+0.8105=1.7510$$

Therefore the rational decision should be *Policy2* because it results in larger expected value. Through this example we show that the Monte Carlo analysis can introduce further information and influence our decision. Under certain conditions such as in this example, the rational decision will differ from the decision induced from the deterministic information.

7. Conclusion

This paper is analyzing the uncertainties propogated through Markov chain model in predicting the performance and maintance cost of roofing system. It is demonstrated that thte magnitude of uncrtainty has significant impact in the selection of maintenance policies. Therefore it suggests that uncertainty analysis is neessary for rational decision making in this field. This paper focuses on the method under the context of roofing system, but it can be easily extended to the fileds of other building components.

For the future work, the method and analysis presented in this paper need to be supported by practical data. The realistic probabilistic distributions of service life field data, maintenance cost data are all crucial information to estimate the variance of simulation results.

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