ISyE 8872 Topics in Nonlinear Optimization

Fall 2001

Assignment 8

Issued: November 29, 2001

Due: December 13, 2001, 2:50pm

Problem 1
Let f, $g \in \operatorname{Conv} \mathbb{R}^n$. Suppose that $\operatorname{ri}(\operatorname{dom}(f)) = \operatorname{ri}(\operatorname{dom}(g))$ and that f and g agree on this set. Show that $\operatorname{cl}(f) = \operatorname{cl}(g)$.

Problem 2
Let $f \in \overline{\operatorname{Conv}} \mathbb{R}^n$. For $r \in \mathbb{R}$, define

$$S_r(f) := \{x \in \mathbb{R}^n \ : \ f(x) \leq r\}.$$

1. Show that if the set $S_r(f)$ is nonempty for some $r \in \mathbb{R}$, then

   $$[S_r(f)]_\infty = \{d \in \mathbb{R}^n : (d, 0) \in [\operatorname{epi}(f)]_\infty\}.$$

   Using this fact, show the following results:

2. If $S_r(f)$ is nonempty and bounded for some $r \in \mathbb{R}$, then it is a (possibly empty) compact set for every $r \in \mathbb{R}$.

3. Assume that $C \subset \mathbb{R}^n$ is a nonempty closed convex set such that $C \cap \operatorname{dom}(f) \neq \varnothing$. Suppose that the set of minimizing points for the minimization problem

   $$\inf\{f(x) \ : \ x \in C\}$$

   is nonempty and bounded. Then, for every $r \in \mathbb{R}$, the set

   $$\{x \in C : f(x) \leq r\}$$

   is a (possibly empty) compact convex set.

Problem 3
Recall that a function $h : \mathbb{R}^n \mapsto \mathbb{R} \cup \{-\infty, +\infty\}$ is convex if its epigraph $\operatorname{epi}(h) \equiv \{(x, r) \in \mathbb{R}^n \times \Re \ : \ h(x) \leq r\}$ is a convex set.

1. Prove that a function $h : \mathbb{R}^n \mapsto \mathbb{R} \cup \{-\infty, +\infty\}$ is convex if and only if it satisfies the inequality $h(\lambda x + (1 - \lambda)y) \leq \lambda h(x) + (1 - \lambda)h(y)$ for any points $x, y \in \operatorname{dom}(h)$ and any $\lambda \in (0, 1)$.

2. Prove that if the function $h : \mathbb{R}^n \mapsto \mathbb{R} \cup \{-\infty, +\infty\}$ is convex then its domain is convex.

3. Prove that if $h(x^0) > -\infty$ for some $x^0 \in \operatorname{ri}(\operatorname{dom}(h))$ then h never takes the value $-\infty$.

Problem 4
Bertsekas, Problem 5.2.1

Problem 5
Bertsekas, Problem 5.4.4