ISyE 8872 Topics in Nonlinear Optimization

Fall 2001

Assignment 7

Issued: November 15, 2001

Due: November 27, 2001

Problem 1
Let $f_1, f_2 \in \operatorname{Conv} \mathbb{R}^n$ be such that $\operatorname{dom}(f_1) \cap \operatorname{dom}(f_2) \neq \varnothing$.

(1) Show that $\operatorname{cl}(f_1) + \operatorname{cl}(f_2) \leq \operatorname{cl}(f_1 + f_2)$.

(2) Show that if $\operatorname{ri}(\operatorname{dom}(f_1)) \cap \operatorname{ri}(\operatorname{dom}(f_2)) \neq \varnothing$ then $\operatorname{cl}(f_1) + \operatorname{cl}(f_2) = \operatorname{cl}(f_1 + f_2)$.

(3) Give an example to show that the equality in (2) fails if $\operatorname{ri}(\operatorname{dom}(f_1)) \cap \operatorname{ri}(\operatorname{dom}(f_2)) = \varnothing$.

(4) State without proof analogous results for (1) and (2) in the case where m functions $f_1, \ldots, f_m \in \operatorname{Conv} \Re^n$ are given.

Problem 2
Let $f : \mathbb{R}^n \to \mathbb{R} \cup \{\infty\}$. Show that:

1. if $\operatorname{dom}(f)$ is closed and f restricted to $\operatorname{dom}(f)$ is lower semi-continuous then f is lower semi-continuous;

2. if $f \in \operatorname{Conv} \mathbb{R}^n$ and $\operatorname{dom}(f)$ is an affine manifold then $f \in \overline{\operatorname{Conv}} \mathbb{R}^n$.

Problem 3
Let $f : \mathbb{R}^n \to \mathbb{R} \cup \{\pm\infty\}$. Show that:

1. for all $x \in \mathbb{R}^n$, $\operatorname{cl}(f)(x) = \sup\{g(x) : g \in \mathcal{C}\}$ where $\mathcal{C}$ is the collection of all lower semi-continuous functions $g : \mathbb{R}^n \to \Re \cup \{\pm\infty\}$ satisfying $g \leq f$;

2. $\operatorname{cl}(f)$ is the largest function in $\mathcal{C}$, that is, $\operatorname{cl}(f) \in \mathcal{C}$ and $\operatorname{cl}(f) \geq g$ for all $g \in \mathcal{C}$.