ISyE 8872 Topics in Nonlinear Optimization

Fall 2001

Assignment 6

Issued: October 18, 2001

Due: October 30, 2001

Problem 1
Let $f : \mathbb{R}^n \mapsto \mathbb{R} \cup \{+\infty\}$ be not identically equal to $+\infty$. Show that the following properties are equivalent:

(i) f is convex;

(ii) $\mathrm{epi}\,(f)$ is a convex set in $\mathbb{R}^n \times \mathbb{R}$;

(iii) $\mathrm{epi}_s(f)$ is a convex set in $\mathbb{R}^n \times \mathbb{R}$.

Problem 2
Let f be a real-valued differentiable function defined on an open set of $\mathbb{R}^n$, which contains the convex set $S \subset \mathbb{R}^n$, $S \neq \varnothing$. Show that:

(a) f is strongly convex with modulus $c > 0$ on S if and only if the function $g(x) := f(x) - c\,\|x\|^2/2$ is convex on S;

(b) f is strongly convex with modulus $c > 0$ on S if and only if

$$f(x) \geq f(x^0) + \nabla f(x^0)^T (x - x^0) + \frac{1}{2}\, c\, \|x - x^0\|^2,$$

for all $x, x^0 \in S$;

(c) f is strongly convex with modulus $c > 0$ on S if and only if

$$[\nabla f(x) - \nabla f(x^0)]^T (x - x^0) \geq c\, \|x - x^0\|^2,$$

for all $x, x^0 \in S$.