ISyE 8872 Topics in Nonlinear Optimization

Fall 2001

Assignment 5

Issued: October 2, 2001

Due: October 9, 2001

Problem 1
Let $S \subset \mathbb{R}^n$, $S \neq \varnothing$, and let K be the conical hull of S. Show that any nonzero $x \in K$ can be represented as a conical combination of at most n vectors in S, and that these vectors can be chosen to be linearly independent.

Problem 2
Let C_1 and C_2 be two nonempty convex sets satisfying $\mathrm{ri}\,(C_1) \cap \mathrm{ri}\,(C_2) = \varnothing$. Show that they can be properly separated, that is, there exists $s \in \mathbb{R}^n$ such that

$$\sup_{y_1 \in C_1} \langle s, y_1 \rangle \;\leq\; \inf_{y_2 \in C_2} \langle s, y_2 \rangle$$

and

$$\inf_{y_1 \in C_1} \langle s, y_1 \rangle \;<\; \sup_{y_2 \in C_2} \langle s, y_2 \rangle$$

Problem 3
Let C_1 and C_2 be two nonempty convex sets satisfying

$$\inf_{y_1 \in C_1, y_2 \in C_2} \|y_2 - y_1\| \;>\; 0$$

Show that there exists $s \in \mathbb{R}^n$, $r \in \mathbb{R}$ such that

$$\sup_{y_1 \in C_1} \langle s, y_1 \rangle \;<\; r \;<\; \inf_{y_2 \in C_2} \langle s, y_2 \rangle$$

that is, the hyperplane $H_{s,r}$ strictly separates C_1 and C_2.

Problem 4
Consider polyhedron

$$P \;:=\; \left\{ x \;:\; x = \sum_{i=1}^{m} \alpha_i y^i, \sum_{i=1}^{m} \alpha_i = 1, \alpha_i \geq 0 \;\forall i = 1, \ldots, m \right\}$$

Show that P is the convex hull of its extreme points.