ISyE 8872 Topics in Nonlinear Optimization

Fall 2001

Assignment 4

Issued: September 11, 2001

Due: September 25, 2001

Problem 1
Show that for any nonempty set $S \subseteq \mathbb{R}^n$,

$$\begin{aligned} &\operatorname{aff}(S) = \operatorname{aff}(\operatorname{cl}(S)) = \operatorname{aff}(\operatorname{co}(S)) = \operatorname{aff}(\overline{\operatorname{co}}(S)) \\ &\overline{\operatorname{co}}(S) = \overline{\operatorname{co}}(\operatorname{cl}(S)) = \overline{\operatorname{co}}(\operatorname{co}(S)) \end{aligned}$$

Problem 2
Suppose that $C_i \subseteq \mathbb{R}^{n_i}$ is a convex set for $i = 1, 2$. Show that

$$\begin{aligned} \operatorname{aff}(C_1) \times \operatorname{aff}(C_2) &= \operatorname{aff}(C_1 \times C_2) \\ \operatorname{ri}(C_1) \times \operatorname{ri}(C_2) &= \operatorname{ri}(C_1 \times C_2) \end{aligned}$$

Problem 3
Let $\varnothing \neq S \subseteq \mathbb{R}^n$ and $x \in \mathbb{R}^n$ be given. Show that:

$$\begin{aligned} \operatorname{aff}(S - x) &= \operatorname{aff}(S) - x \\ \operatorname{ri}(S - x) &= \operatorname{ri}(S) - x \end{aligned}$$

Morever, show that if $x \in \operatorname{aff}(S)$ then $\operatorname{lin}(S - x) = \operatorname{aff}(S) - x$.

Problem 4
Let $f : \Re^n \mapsto \Re^m$ be an affine function.

1. Let $C \subset \Re^n$ be a convex set. Show that

$$\operatorname{ri}\,(f(C)) = f(\operatorname{ri}(C))$$

2. Let $D \subset \Re^m$ be a convex set such that $f^{-1}(\operatorname{ri}(D)) \neq \varnothing$. Show that

$$\operatorname{ri}\,\left(f^{-1}(D)\right) = f^{-1}(\operatorname{ri}(D))$$