ISyE 8872 Topics in Nonlinear Optimization

Fall 2001

Assignment 3

Issued: September 4, 2001

Due: September 11, 2001

Problem 1
Let $x^0, x^1, \ldots, x^k \in \mathbb{R}^n$ be fixed. Show that the following statements are equivalent:

(1) $\dim(\mathrm{aff}\,\{x^0, x^1, \ldots, x^k\}) = k$

(2) for any $x \in \mathrm{aff}\,\{x^0, x^1, \ldots, x^k\}$, $\dim(\mathrm{lin}\,\{x^0 - x, x^1 - x, \ldots, x^k - x\}) = k$

(3) the vectors $x^1 - x^0, \ldots, x^k - x^0$ are linearly independent

(4) if $\alpha_0 x^0 + \cdots + \alpha_k x^k = 0$ and $\alpha_0 + \cdots + \alpha_k = 0$, then $\alpha_0 = \cdots = \alpha_k = 0$

(5) the vectors $(x^0, 1), (x^1, 1), \ldots, (x^k, 1)$ are linearly independent

Problem 2
Let $A \subseteq \mathbb{R}^n$ be an affine manifold. By definition, a set $B \subset A$ is an *affine basis* for A if $\mathrm{aff}\,(B) = A$ and the elements of B are affinely independent. Use what you already know about linear subspaces to show that:

(1) A has an affine basis;

(2) every affine basis for A has cardinality equal to $\dim(A) + 1$;

(3) B is an affine basis for A if and only if B is a minimal element with respect to the property that $\mathrm{aff}\,(B) = A$;

(4) if $S \subseteq \mathbb{R}^n$ is such that $\mathrm{aff}\,(S) = A$, then S contains an affine basis for A;

(5) every $x \in A$ can be expressed in a unique way as an affine combination of the elements of an affine basis B.

Problem 3
Let $S \subseteq \mathbb{R}^n$, $S \neq \varnothing$. Show that for any $x \in \mathrm{aff}\,(S)$, $\mathrm{lin}\,(S - x) = \mathrm{aff}\,(S) - x$.