ISyE 8813C Game Theory

Fall 2013

Assignment 6

Issued: October 31, 2013

Due: November 14, 2013

Problem 1
A set $\{x^0, x^1, \ldots, x^m\} \subset \mathbb{R}^n$ is affinely independent if $\sum_{i=0}^m \lambda_i = 0$ and $\sum_{i=0}^m \lambda_i x^i = 0$ imply that $\lambda_0 = \lambda_1 = \cdots = \lambda_m = 0$. Show that $\{x^0, x^1, \ldots, x^m\} \subset \mathbb{R}^n$ is affinely independent if and only if $\{x^1 - x^0, \ldots, x^m - x^0\}$ is linearly independent.

Problem 2
Suppose that $\{x^0, x^1, \ldots, x^m\} \subset \mathbb{R}^n$ is affinely independent. Show that for any $x \in \text{co}(x^0, x^1, \ldots, x^m)$, there is a unique vector $\lambda \in [0,1]^{m+1}$ such that $x = \sum_{i=0}^m \lambda_i x^i$ and $\sum_{i=0}^m \lambda_i = 1$. The number λ_i is called the ith barycentric coordinate of x with respect to $\{x^0, x^1, \ldots, x^m\}$. (This property is more general: For any x in the affine hull of $\{x^0, x^1, \ldots, x^m\} \subset \mathbb{R}^n$, where $\{x^0, x^1, \ldots, x^m\}$ is affinely independent, there is a unique vector $\lambda \in \mathbb{R}^{m+1}$ such that $x = \sum_{i=0}^m \lambda_i x^i$ and $\sum_{i=0}^m \lambda_i = 1$.)

Problem 3
Show that homeomorphism is an equivalence relation.

Problem 4
Let e^i denote the ith standard basis vector in $\mathbb{R}^{m+1}$, $i = 0, \ldots, m$. Let $\Delta_m \equiv \text{co}(e^0, e^1, \ldots, e^m)$ denote the standard m-simplex. Let $S \equiv \text{co}(x^0, x^1, \ldots, x^m) \subset \mathbb{R}^n$ denote any m-simplex. Show that Δ_m and S are homeomorphic.

Problem 5
Brouwer Fixed-Point Theorem: Let $K \subset \mathbb{R}^n$ be compact and convex. Let $f : K \mapsto K$ be continuous. Then f has a fixed point in K.

1. Give an example to show that f may not have a fixed point if K is not compact, but the other assumptions continue to hold.

2. Give an example to show that f may not have a fixed point if K is not convex, but the other assumptions continue to hold.

3. Give an example to show that f may not have a fixed point if f is not continuous, but the other assumptions continue to hold.