ISyE 8813C Game Theory

Fall 2013

Assignment 5

Issued: October 10, 2013

Due: October 24, 2013

Problem 1
Let $F_i : X \mapsto 2^{Y_i}$, $Y_i \subset \mathbb{R}^{n_i}$ for $i = 1, \ldots, k$. Let $\prod_{i=1}^k F_i : X \mapsto 2^{\prod_{i=1}^k Y_i}$ be defined by

$$\left(\prod_{i=1}^k F_i \right)(x) \equiv \prod_{i=1}^k F_i(x)$$

Show the following:

1. If F_i is upper hemi-continuous at x for each i, and $F_i(x)$ is compact for each i, then $\prod_{i=1}^k F_i$ is upper hemi-continuous at x, and $\left(\prod_{i=1}^k F_i \right)(x)$ is compact.

2. If F_i is lower hemi-continuous at x for each i, then $\prod_{i=1}^k F_i$ is lower hemi-continuous at x.

3. If F_i is outer semi-continuous (closed) at x for each i, then $\prod_{i=1}^k F_i$ is outer semi-continuous (closed) at x.

4. If F_i has open graph for each i, then $\prod_{i=1}^k F_i$ has open graph.

Problem 2
For any sets $A, B \subset Y$, and addition $+ : Y \times Y \mapsto Y$ defined and continuous, let $A + B \equiv \{a + b : a \in A, b \in B\}$. Let $F_i : X \mapsto 2^Y$ for $i = 1, \ldots, k$. Let $\sum_{i=1}^k F_i : X \mapsto 2^Y$ be defined by

$$\left(\sum_{i=1}^k F_i \right)(x) \equiv \sum_{i=1}^k F_i(x)$$

Show the following:

1. If F_i is upper hemi-continuous at x for each i, and $F_i(x)$ is compact for each i, then $\sum_{i=1}^k F_i$ is upper hemi-continuous at x, and $\left(\sum_{i=1}^k F_i \right)(x)$ is compact.

2. If F_i is lower hemi-continuous at x for each i, then $\sum_{i=1}^k F_i$ is lower hemi-continuous at x.

3. If F_i has open graph for each i, then $\sum_{i=1}^{k} F_i$ has open graph.

Problem 3

Let $F : X \mapsto 2^Y$, $Y \subset \mathbb{R}^n$. Let $\text{conv}(F) : X \mapsto 2^Y$ be defined by

$$(\text{conv}(F))(x) \equiv \text{conv}(F(x))$$

Show the following:

1. If F is upper hemi-continuous at x, and $F(x)$ is compact, then $\text{conv}(F)$ is upper hemi-continuous at x.
2. If F is lower hemi-continuous at x, then $\text{conv}(F)$ is lower hemi-continuous at x.
3. If F has an open graph, then $\text{conv}(F)$ has an open graph.
4. Even if F is compact valued and closed, $\text{conv}(F)$ may not be closed.