ISyE 8813C Game Theory

Fall 2013

Assignment 4

Issued: October 1, 2013

Due: October 10, 2013

Problem 1
Let $X \subset \mathbb{R}^m$ and $Y \subset \mathbb{R}^k$. Show the following:

1. Consider $F : X \mapsto 2^{\mathbb{R}^m}$. If F is upper hemi-continuous and closed valued, then the set $\{x \in X : x \in F(x)\}$ of fixed points of F is a (possibly empty) closed subset of X.

2. Consider $F, G : X \mapsto 2^Y$. If F and G are upper hemi-continuous and closed valued, then the set $\{x \in X : F(x) \cap G(x) \neq \varnothing\}$ is a (possibly empty) closed subset of X.

3. Consider $F : X \mapsto 2^Y$. If F is upper hemi-continuous, then the set $\{x \in X : F(x) \neq \varnothing\}$ is a closed subset of X.

4. Consider $F : X \mapsto 2^Y$. If F is lower hemi-continuous, then the set $\{x \in X : F(x) \neq \varnothing\}$ is an open subset of X.

Problem 2
Let $X \subset \mathbb{R}^m$, $Y \subset \mathbb{R}^k$, and $F : X \mapsto 2^Y$. Let $\bar{F} : X \mapsto 2^Y$ be defined by

$$\bar{F}(x) \quad \equiv \quad \text{closure (in } Y\text{) of } F(x)$$

Show the following:

1. F is lower hemi-continuous at x if and only if $\bar{F}$ is lower hemi-continuous at x.

2. If F is upper hemi-continuous at x, then $\bar{F}$ is upper hemi-continuous at x.

Problem 3
Let $G : X \mapsto 2^Y$ and $F : Y \mapsto 2^Z$. Let $F \circ G : X \mapsto 2^Z$ be defined by

$$(F \circ G)(x) \quad \equiv \quad \bigcup_{y \in G(x)} F(y)$$

Show the following:

1. If F and G are upper hemi-continuous, then $F \circ G$ is upper hemi-continuous.

2. If F and G are lower hemi-continuous, then $F \circ G$ is lower hemi-continuous.

3. Show by example that it may happen that F and G are closed, but $F \circ G$ is not closed.