ISyE 8813C Game Theory

Fall 2013

Assignment 3

Issued: September 19, 2013

Due: September 26, 2013

Problem 1
Consider closed sets $F, G \subset \mathbb{R}^m$, and suppose that $F \cap G = \varnothing$. Show that there exist open sets $A, B \subset \mathbb{R}^m$ such that $F \subset A$, $G \subset B$, and $A \cap B = \varnothing$.

Problem 2
Let $X \subset \mathbb{R}^m$, $Y \subset \mathbb{R}^k$, and $F : X \mapsto 2^Y$.

1. Show that if F is open, then F is lower hemi-continuous.
2. The following statement comes from Proposition 11.9(d) in Border (1985): "If F is upper hemi-continuous at x and $F(x)$ is a singleton, then F is lower hemi-continuous at x."
   (a) Give a counterexample to the statement above.
   (b) Correct the statement, and prove the correctness of your statement.
3. Show that if $F^{-}(\{y\})$ is open for all $y \in Y$, then F is lower hemi-continuous.

Problem 3
Let $X \subset \mathbb{R}^m$, $Y \subset \mathbb{R}^k$, and $F : X \mapsto 2^Y$. Show that F is lower hemi-continuous at x if and only if for all sequences $\{x_k\}_k \subset X$ such that $x_k \to x$ as $k \to \infty$, and all $y \in F(x)$, there is a sequence $\{y_k\}_k \subset Y$ such that $y_k \in F(x_k)$ for all k and $y_k \to y$ as $k \to \infty$.