ISyE 8813C   Game Theory

Fall 2013

Assignment 2

Issued: August 29, 2013

Due: September 5, 2013

Problem 1
Consider a simultaneous play game with players $i = 1, \ldots, n$. Each player i has feasible set X_i of decisions x_i. Assume that X_i is countable. Let $X := X_1 \times \cdots \times X_n$. Let $X_{-i} := X_1 \times \cdots \times X_{i-1} \times X_{i+1} \times \cdots \times X_n$, and let $x_{-i} := (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$. Each player i has objective function $f_i : X \mapsto \mathbb{R}$ that the player wants to maximize. Let $\mathcal{P}(X_i)$ denote the set of probability distributions on X_i. Let $(p_1^*, \ldots, p_n^*) \in \mathcal{P}(X_1) \times \cdots \times \mathcal{P}(X_n)$ denote a mixed strategy Nash equilibrium. Let $X_i^* := \{x_i \in X_i \,:\, p_i^*(x_i) > 0\}$.

1. Show that

$$X_i^* \;\subset\; \arg\max \left\{ \sum_{x_{-i} \in X_{-i}} f_i(x_i, x_{-i}) p_1^*(x_1) \cdots p_{i-1}^*(x_{i-1}) p_{i+1}^*(x_{i+1}) \cdots p_n^*(x_n) \;:\; x_i \in X_i \right\}$$

   that is, every decision $x_i \in X_i$ such that $p_i^*(x_i) > 0$ is a best deterministic response for player i.

2. Recall that $\mathcal{P}(X_i^*)$ denotes the set of all probability distributions on X_i that put all its probability on decisions in X_i^*. Show that

$$\mathcal{P}(X_i^*) \;\subset\; \arg\max \left\{ \sum_{x \in X} f_i(x_i, x_{-i}) p_1^*(x_1) \cdots p_{i-1}^*(x_{i-1}) p_i(x_i) p_{i+1}^*(x_{i+1}) \cdots p_n^*(x_n) \;:\; p_i \in \mathcal{P}(X_i) \right\}$$

   that is, every probability distribution $p_i \in \mathcal{P}(X_i)$ such that $p_i(X_i^*) = 1$ is a best mixed strategy response for player i.

3. Show that $p^* \in \mathcal{P}(X)$ given by $p^*(x) = p_1^*(x_1) \cdots p_n^*(x_n)$ is a correlated equilibrium. That is, every mixed strategy Nash equilibrium defines a correlated equilibrium.

4. Let $p^* \in \mathcal{P}(X)$ denote any correlated equilibrium. Show that p^* is a coarse correlated equilibrium.

5. Let $\hat{p}_i \in \mathcal{P}(X_{-i})$ be given by $\hat{p}_i(x_{-i}) = p_1^*(x_1) \cdots p_{i-1}^*(x_{i-1}) p_{i+1}^*(x_{i+1}) \cdots p_n^*(x_n)$. Show that $(\hat{p}_1, \ldots, \hat{p}_n)$ is an endogenous equilibrium.

6. Consider any endogenous correlated equilibrium $(\hat{p}_1, \ldots, \hat{p}_n) \in \mathcal{P}(X_{-1}) \times \cdots \times \mathcal{P}(X_{-n})$. Show that $(\hat{p}_1, \ldots, \hat{p}_n)$ is Bayes concordant.

Problem 2

Consider the following 2-player game (as you well know, the rules of the game require simultaneous play, and each player wants to maximize the player's score given in the following table):

	Rock	Paper	Scissors
Rock	(0,0)	(-1,1)	(1,-1)
Paper	(1,-1)	(0,0)	(-1,1)
Scissors	(-1,1)	(1,-1)	(0,0)

1. Identify the set of pure strategy Nash equilibria.
2. Identify the set of mixed strategy Nash equilibria.
3. Consider the repeated Rock-Paper-Scissors game, in which each player uses Cournot adjustment. Describe all the sequences of strategy profiles that may result (the sequence may depend on the initial strategy profile).
4. What are the limits of the (marginal) empirical frequencies of strategies for the Cournot adjustment sequences? Does any of these limits coincide with a mixed strategy Nash equilibrium? Prove your claim.
5. What are the limits of the (joint) empirical frequencies of strategy profiles for the Cournot adjustment sequences? Does any of these limits coincide with a correlated equilibrium? Prove your claim.