

# ISyE 8803E Game Theory

Spring 2008

## Assignment 3

Issued: February 20, 2003

Due: February 27, 2003

Do two of the following three problems:

### Problem 1

Let  $F_i : X \mapsto 2^{Y_i}$ ,  $Y_i \subset \mathbb{R}^{n_i}$  for  $i = 1, \dots, k$ . Let  $\prod_{i=1}^k F_i : X \mapsto 2^{\prod_{i=1}^k Y_i}$  be defined by

$$\left( \prod_{i=1}^k F_i \right) (x) \equiv \prod_{i=1}^k F_i(x)$$

Show the following:

1. If  $F_i$  is upper hemi-continuous at  $x$  for each  $i$ , and  $F_i(x)$  is compact for each  $i$ , then  $\prod_{i=1}^k F_i$  is upper hemi-continuous at  $x$ , and  $\left( \prod_{i=1}^k F_i \right) (x)$  is compact.
2. If  $F_i$  is lower hemi-continuous at  $x$  for each  $i$ , then  $\prod_{i=1}^k F_i$  is lower hemi-continuous at  $x$ .
3. If  $F_i$  is closed at  $x$  for each  $i$ , then  $\prod_{i=1}^k F_i$  is closed at  $x$ .
4. If  $F_i$  has open graph for each  $i$ , then  $\prod_{i=1}^k F_i$  has open graph.

### Problem 2

For any sets  $A, B \subset Y$ , and addition  $+: Y \times Y \mapsto Y$  defined and continuous, let  $A + B \equiv \{a + b : a \in A, b \in B\}$ . Let  $F_i : X \mapsto 2^Y$  for  $i = 1, \dots, k$ . Let  $\sum_{i=1}^k F_i : X \mapsto 2^Y$  be defined by

$$\left( \sum_{i=1}^k F_i \right) (x) \equiv \sum_{i=1}^k F_i(x)$$

Show the following:

1. If  $F_i$  is upper hemi-continuous at  $x$  for each  $i$ , and  $F_i(x)$  is compact for each  $i$ , then  $\sum_{i=1}^k F_i$  is upper hemi-continuous at  $x$ , and  $\left( \sum_{i=1}^k F_i \right) (x)$  is compact.
2. If  $F_i$  is lower hemi-continuous at  $x$  for each  $i$ , then  $\sum_{i=1}^k F_i$  is lower hemi-continuous at  $x$ .

3. If  $F_i$  has open graph for each  $i$ , then  $\sum_{i=1}^k F_i$  has open graph.

**Problem 3**

Let  $F : X \mapsto 2^Y$ . Let  $\text{conv}(F) : X \mapsto 2^Y$  be defined by

$$(\text{conv}(F))(x) \equiv \text{conv}(F(x))$$

Show the following:

1. If  $F$  is upper hemi-continuous at  $x$ , and  $F(x)$  is compact, then  $\text{conv}(F)$  is upper hemi-continuous at  $x$ .
2. If  $F$  is lower hemi-continuous at  $x$ , then  $\text{conv}(F)$  is lower hemi-continuous at  $x$ .
3. If  $F$  has an open graph, then  $\text{conv}(F)$  has an open graph.
4. Even if  $F$  is compact valued and closed,  $\text{conv}(F)$  may not be closed.