ISyE 8801B Game Theory

Fall 2003

Assignment 6

Issued: November 17, 2003

Due: November 24, 2003

Problem 1

Friedman, Section 4.1.1.2, p.118, Lemma 4.1: Consider a stationary multiperiod game A with n players. Suppose that the following assumptions hold:

Assumption 1: The feasible sets X_i are nonempty, compact, and convex for all players $i \in \{1, \ldots, n\}$.

Assumption 2: Let $X \equiv \prod_{i=1}^{n} X_i = X_1 \times \cdots \times X_n$. The single period payoff functions $g_i : X \times X \mapsto \mathbb{R}$ are bounded and continuous.

Assumption 3: Let $X_{-i} \equiv \prod_{j \neq i} X_j$. For any $(x_{-i}(t-1), x_{-i}(t)) \in X_{-i} \times X_{-i}$, the conditional single period payoff functions $g_i(x_{-i}(t-1), x_{-i}(t); \cdot, \cdot) : X_i \times X_i \mapsto \mathbb{R}$ defined by $g_i(x_{-i}(t-1), x_{-i}(t); x_i(t-1), x_i(t)) \equiv g_i((x_{-i}(t-1); x_i(t-1)), (x_{-i}(t); x_i(t)))$ are concave.

Assumption 4: Discount factors $\alpha_i \in [0, 1)$ for all players $i \in \{1, \ldots, n\}$.

Assumption 5: Players know the past decisions of all players.

Assumption 6: Each player i chooses a policy π_i in an attempt to maximize

$$G_i(\pi) \equiv \sum_{t=1}^{\infty} \alpha_i^t g_i(x(t-1), x(t))$$

where $\pi = (\pi_1, \ldots, \pi_n)$, $x(t) = (x_1(t), \ldots, x_n(t))$, $x(0)$ is given, and $\{x(t)\}_{t=1}^{\infty}$ is the sequence of decisions produced by policy combination π.

Given any pair $(\bar{x}(t-1), \bar{x}(t+1)) \in X \times X$ of decision combinations, consider a single stage game $B(\bar{x}(t-1), \bar{x}(t+1))$ with n players and the same feasible sets X_i as in game A. The objective functions $h_i(\bar{x}(t-1), \cdot, \bar{x}(t+1)) : X \mapsto \mathbb{R}$ of game $B(\bar{x}(t-1), \bar{x}(t+1))$ are defined by $h_i(\bar{x}(t-1), x, \bar{x}(t+1)) \equiv g_i(\bar{x}(t-1), x) + \alpha_i g_i(x, \bar{x}(t+1))$. The result claimed in Lemma 4.1 of Friedman is as follows: Open loop strategy combination $\pi^* = (x^*(0) = x(0), x^*(1), x^*(2), \ldots)$ is an equilibrium point of stationary multiperiod game A if and only if $x^*(t)$ is an equilibrium point of single stage game $B(x^*(t-1), x^*(t+1))$ for all $t \in \{1, 2, \ldots\}$. Show with a counterexample that the claimed result does not hold.