

ISyE 2027 E
Test 3

Calculators, notes, and books are not allowed. Put your name on both sides of this sheet. Please stop when time is up. You may leave terms like $\binom{52}{5}$ and e^{-2} in your answers.

1. (30 points) What would be a reasonable guess for the distribution of each of the following random variables?
 - (a) The location of the valve on Chris' right front tire.
 - (b) The length of time until the next train derailment in the U.S.
 - (c) The number of train derailments during the rest of 2018.
2. (30 points) Suppose that the c.d.f. of X is $F(t) = t^2/9$ for $0 \leq t \leq 3$.
 - (a) Compute $\mathbb{P}\{X \leq 1 \mid X < 2\}$.
 - (b) Compute $\mathbb{E}[X]$.
 - (c) Compute $\mathbb{E}[X^2]$.
3. (30 points) For Halloween, small packets of jelly beans are produced containing black and orange jelly beans. Let X denote the number of black and Y the number of orange jelly beans in a randomly selected packet. Assume that the joint p.m.f. of X and Y is given by $\mathbb{P}\{X = i, Y = j\} = c$ for $i \geq 0$, $j \geq 0$, and $4 \leq i + j \leq 5$; otherwise, the probability is zero.
 - (a) What is the probability of 5 jelly beans in a randomly selected packet?
 - (b) What is the probability that there is an equal number of black and orange jelly beans in a randomly selected packet?
 - (c) Would the covariance between X and Y be positive, negative, or zero?
4. (30 points) Suppose that the total time needed to retrieve an item in minutes is $T = 2L/5 + 6$ where the distance traveled to the item L has an exponential distribution with mean 10 meters.
 - (a) Compute the variance of T .
 - (b) Compute the covariance of T and L .
 - (c) Given that the picker has traveled 8 meters without reaching the item's location, what is the expected remaining distance to the item's location?
5. (30 points) Suppose that someone needs to grade 64 tests. The average time to grade each test is 10 minutes with a variance of 25 minutes². Assume that the times to grade the 64 tests are independent and identically distributed.
 - (a) What is the squared coefficient of variation for the time to grade a single test?
 - (b) What is the standard deviation of the time to grade all 64 tests?
 - (c) Accurately approximate the probability that the grading can be done within ten hours?