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Name: _____

ISyE 2027 E
Test 1

Calculators, notes, and books are not allowed. Put your name on both sides of this sheet. Please stop when time is up. You may leave terms like $\binom{52}{5}$ and e^{-2} in your answers.

1. (30 points) Suppose $\mathbb{P}(A) = 3/10$ and $\mathbb{P}(AB) = 2/10$.
   (a) Compute $\mathbb{P}(B^c A)$.
   (b) Compute $\mathbb{P}(B \mid A)$.
   (c) If $\mathbb{P}(B) = 2/3$, what is $P(A \mid B)$?

2. (30 points) If a person is infected with a particular disease, the blood test will be positive with probability 95%. If a person is not infected, the blood test will be positive with probability 0.5%. One percent of the population is infected with this disease.
   (a) What is the probability that a randomly selected person tests positive?
   (b) What is the probability that an infected person will test negative?
   (c) Suppose a randomly selected person tests positive. Is the probability that they are infected closest to 0, 1/4, 1/3, 1/2, 2/3, 3/4 or 9/10?

3. (30 points) Someone removed all of the diamonds from the deck leaving 13 red cards and 26 black cards, which is 39 cards total. Assume that you are dealt five cards from this well-shuffled deck.
   (a) What is the probability of being dealt two pairs?
   (b) What is the probability of being dealt a full house (i.e., three of one rank and two of another rank)?
   (c) What is the probability of two pairs and all 5 cards are black?

4. (30 points) Suppose $\mathbb{P}\{X = k\} = (3/4)(1/4)^{k-1}$ for $k = 1, 2, \ldots$
   (a) Compute $\mathbb{P}\{X = 1 \mid X \leqslant 2\}$.
   (b) Compute $\mathbb{P}\{X > 5\}$, and simplify your answer.
   (c) Compute $\mathbb{E}[X]$.

5. (30 points) An insurance policy pays \$100 per day for up to 2 days of hospitalization and \$50 per day for each day of hospitalization thereafter. The number of days of hospitalization N is a discrete random variable with p.m.f.

$$\mathbb{P}\{N = k\} = (5 - k)/10 \text{ for } k \in \{1, 2, 3, 4\}. \tag{1}$$

   (a) Let $g(k)$ represent the payment for k days of hospitalization for $k \in \{0, 1, 2, \ldots\}$. Give an equation defining g.
   (b) Compute the expected number of days of hospitalization.
   (c) Compute the expected hospitalization payment.