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<center>ISyE 2027 Test 3</center>

Calculators, notes, and books are not allowed. Put your name on both sides of this sheet. Please stop when time is up. You may leave terms like $\binom{52}{5}$ and e^{-2} in your answers.

1. (30 points) For each of the following, which approximation would be most appropriate: Gaussian with a continuity correction, Gaussian without a continuity correction, a Poisson approximation, or none of the above.

   (a) The number of grand prize winning tickets in the next mega-millions lottery.
   (b) The total weight of 100 bales of hay.
   (c) The total number of people in the next 25 cars.

2. (30 points) Let X, Y, and Z be independent Poisson random variables with means 2, 3, and 4, respectively.

   (a) Compute $\mathbb{P}\{X = 1\}$.
   (b) Compute $\mathbb{P}\{X + Y = 1\}$.
   (c) Compute $\mathrm{Cov}(X + Y, Y + Z)$.

3. (30 points)

   The length of time T to process a part has c.d.f. $F(t) = t^3/8$ for $0 \leqslant t \leqslant 2$.

   (a) Determine the probability that the processing time is less than 1.
   (b) Compute the mean processing time.
   (c) Compute $\mathbb{E}\left[\frac{1}{T^2}\right]$.

4. (30 points) Suppose we have a miniload automated storage/retrieval system with a storage rack that is 100 feet long and 50 feet high. Assume that there are two motors: one moving the mast horizontally at 2 feet per second and one moving the load handler vertically at 1 foot per second. Let T be the length of time for the load handler to go from the origin to a random location (X, Y) in the rack. Assume that (X, Y) is uniformly distributed over the rack face.

   (a) Give an expression for T in terms of X, Y and the speeds of the two motors. Assume that T is the longer of the time to move the mast horizontally and the time to move the load handler vertically.
   (b) Determine the c.d.f. of T.
   (c) Compute $\mathbb{P}\{X^2 + Y^2 < 49\}$.

5. (30 points) There are two servers. Hiba has just started service at server 1, while Daniella has just started service at server 2. Hiba's service time at server 1 is exponentially distributed with mean 1/4 hour. Daniella's service time at server 2 is exponentially distributed with mean 1/6 hour. Daniella's service time and Hiba's service time are independent.

   (a) What is the probability that Hiba finishes service before Daniella?
   (b) What is the expected time *in minutes* until the first person finishes?
   (c) What is the expected time *in minutes* until the second person finishes?

6. (30 points) Peter needs to process 25 jobs. The processing times of the 25 jobs are i.i.d. with mean 4 minutes and variance 9 minutes^2.

   (a) What is the mean time in minutes to process all 25 jobs?
   (b) What is the standard deviation in minutes of the time to process all 25 jobs?
   (c) Accurately approximate the probability that Peter finishes all 25 jobs within 2 hours and 10 minutes.

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